NEW

PLANE AND SPHERICAL

TRIGONOMETRY

BY

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PREFACE.

In revising his Plane and Spherical Trigonometry, the author has effected many important improvements. The attention of teachers is specially invited to the following features of the new work:

1. The proofs of the functions of 0°, 90°, 180°, and 270°; §§ 22 to 25.

2. The proofs of the functions of 120°, 135°, etc.; § 27.

3. The method of finding the values of the remaining functions of an angle when the value of any one is given; § 28.

4. The proofs of the functions of $(-A)$, and $(90° + A)$, in terms of those of A; §§ 29, 30.

5. The method of solution in the examples of §§ 34 and 35.

6. The general demonstration of the formulæ for $\sin(x+y)$ and $\cos(x+y)$; § 42.

7. The discussion of the line values of the functions, and their application in tracing the changes in the six principal functions of an angle as the angle increases from 0° to 360°; §§ 60, 61.

8. The discussion of trigonometric equations in § 62.

9. The solution of right triangles by Natural Functions; see Ex. 1, page 54.

10. The discussion of the ambiguous case in the solution of oblique triangles; §§ 117 to 120.

11. The proof of the formulæ for the values of x in the cubic equation $x^3 - ax - b = 0$; § 126.

12. The *geometrical* proof of the important theorems of § 133.

13. The demonstration of the formulæ for right spherical triangles *before* those for oblique spherical triangles; see Chapters XI. and XII.

14. The reduction of the number of cases in the complete demonstration of the fundamental theorems for spherical right triangles, to three, by application of the theorems of § 133; see § 136.

15. The solution of Quadrantal and Isosceles Spherical triangles: §§ 149, 150.

16. The discussion of the ambiguous cases in the solution of oblique spherical triangles: §§ 165, 166: especially the rules given on pages 108 and 111 for determining the number of solutions.

At the end of Chapter XII. will be found a collection of formulæ in form for convenient reference.

The revised work contains a much greater number of examples than the old; they have been selected with great care, and are with few exceptions new.

The results have been worked out by aid of the author's new Six Place Logarithmic Tables, which contain also a Table of Natural Functions, and an Auxiliary Table for Small Angles. The Trigonometry can be obtained either with or without the Tables.

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PLANE TRIGONOMETRY.

SPHERICAL TRIGONOMETRY.

PLANE TRIGONOMETRY.

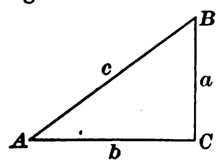
I. TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES.

1. Trigonometry treats of the properties and measurement of angles and triangles.

In *Plane Trigonometry* we consider *plane* figures only.

2. Definitions of the Trigonometric Functions of Acute Angles.

Let BAC be any acute angle.



From any point in either side, as B, draw a perpendicular to the other side, forming the right triangle ABC.

We then have the following definitions, applicable to either of the acute angles A or B:

In any right triangle,

The **sine** *of either acute angle is the ratio of the opposite side to the hypotenuse.*

The **cosine** *is the ratio of the adjacent side to the hypotenuse.*

The **tangent** *is the ratio of the opposite side to the adjacent side.*

The **cotangent** *is the ratio of the adjacent side to the opposite side.*

The **secant** *is the ratio of the hypotenuse to the adjacent side.*

The **cosecant** *is the ratio of the hypotenuse to the opposite side.*

We also have the following definitions:

The **versed sine** *of an angle is* 1 *minus the cosine of the angle.*

The **coversed sine** *is* 1 *minus the sine.*

The eight ratios defined above are called the *Trigonometric Functions* of the angle.

Representing the sides BC, CA, and AB by a, b, and c, respectively, and employing the usual abbreviations, we have:

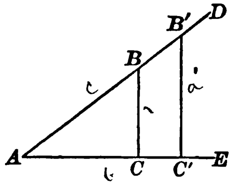
$$\sin A = \frac{a}{c}. \qquad \tan A = \frac{a}{b}. \qquad \sec A = \frac{c}{b}. \qquad \text{vers}\, A = 1 - \frac{b}{c}.$$

$$\cos A = \frac{b}{c}. \qquad \cot A = \frac{b}{a}. \qquad \csc A = \frac{c}{a}. \qquad \text{covers}\, A = 1 - \frac{a}{c}.$$

$$\sin B = \frac{b}{c}. \qquad \tan B = \frac{b}{a}. \qquad \sec B = \frac{c}{a}. \qquad \text{vers}\, B = 1 - \frac{a}{c}.$$

$$\cos B = \frac{a}{c}. \qquad \cot B = \frac{a}{b}. \qquad \csc B = \frac{c}{b}. \qquad \text{covers}\, B = 1 - \frac{b}{c}.$$

3. It is important to observe that the values of the trigonometric functions depend solely on the magnitude of the angle, and are entirely independent of the lengths of the sides of the right triangle which contains it.



For let B and B' be any two points in the side AD of the angle DAE, and draw BC and $B'C'$ perpendicular to AE.

Then by the definition of § 2, we have

$$\sin A = \frac{BC}{AB}, \text{ and } \sin A = \frac{B'C'}{AB'}.$$

But since the right triangles ABC and $AB'C'$ are similar, their homologous sides are proportional; whence,

$$\frac{BC}{AB} = \frac{B'C'}{AB'}.$$

Thus the two values obtained for $\sin A$ are equal.

4. We have from § 2,

$$\sin A = \frac{a}{c}, \ \cos A = \frac{b}{c}, \ \sin B = \frac{b}{c}, \text{ and } \cos B = \frac{a}{c}.$$

Whence, $a = c \sin A = c \cos B$, and $b = c \sin B = c \cos A$.

That is, *in any right triangle, either side about the right angle is equal to the hypotenuse multiplied by the sine of the opposite angle, or by the cosine of the adjacent angle.*

Again, $\tan A = \frac{a}{b}$, $\cot A = \frac{b}{a}$, $\tan B = \frac{b}{a}$, and $\cot B = \frac{a}{b}$.

Whence, $a = b \tan A = b \cot B$, and $b = a \tan B = a \cot A$.

That is, *in any right triangle, either side about the right angle is equal to the tangent of the opposite angle, or the cotangent of the adjacent angle, multiplied by the other side.*

5. We have from § 2,

$$\sin A = \frac{a}{c} = \cos B. \qquad \sec A = \frac{c}{b} = \csc B.$$

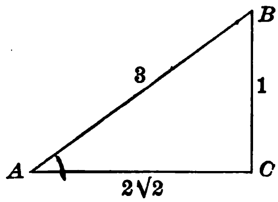
$$\tan A = \frac{a}{b} = \cot B. \qquad \text{vers } A = 1 - \frac{b}{c} = \text{covers } B.$$

As B is the complement of A, these results may be stated as follows:

The sine, tangent, secant, and versed sine of any acute angle are respectively the cosine, cotangent, cosecant, and coversed sine of the complement of the angle.

6. To Find the Values of the Other Seven Functions of an Acute Angle, when the Value of Any One is Given.

1. Given $\csc A = 3$; find the values of the remaining functions of A.



We may write the equation $\csc A = \frac{3}{1}$.

Since the cosecant is the hypotenuse divided by the opposite side, we may regard A as one of the acute angles of the right triangle ABC, in which the hypotenuse $AB = 3$, and the opposite side $BC = 1$.

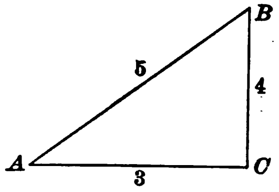
Whence by Geometry, $AC = \sqrt{\overline{AB}^2 - \overline{BC}^2} = \sqrt{9 - 1} = \sqrt{8} = 2\sqrt{2}$.

Then by the definitions of § 2,

$$\sin A = \frac{1}{3}. \qquad \tan A = \frac{1}{2\sqrt{2}}. \qquad \sec A = \frac{3}{2\sqrt{2}}. \qquad \text{vers } A = 1 - \frac{2\sqrt{2}}{3}.$$

$$\cos A = \frac{2\sqrt{2}}{3}. \qquad \cot A = 2\sqrt{2}. \qquad \text{covers } A = 1 - \frac{1}{3} = \frac{2}{3}.$$

2. Given vers $A=\frac{2}{5}$; find the value of cot A.



Since vers $A=1-\cos A$, we have $\cos A=1-\text{vers}\,A=1-\frac{2}{5}=\frac{3}{5}$.

Then, in the right triangle ABC, we take the adjacent side $AC=3$, and the hypotenuse $AB=5$.

Whence, $BC=\sqrt{\overline{AB}^2-\overline{AC}^2}=\sqrt{25-9}=\sqrt{16}=4.$

Then by definition, $\cot A=\frac{3}{4}$.

EXAMPLES.

In each of the following, find the values of the remaining functions:

3. $\sin A=\frac{3}{5}$. **5.** $\cot A=\frac{7}{24}$. **7.** $\cos A=\frac{3\sqrt{3}}{14}$. **9.** $\sec A=x$.

4. vers $A=\frac{8}{13}$. **6.** $\csc A=7$. **8.** covers $A=\frac{2}{17}$. **10.** $\tan A=\frac{b}{a}$.

11. Given $\cot A=\frac{3}{2}$; find $\sin A$.

12. Given $\csc A=\frac{41}{40}$; find $\cos A$.

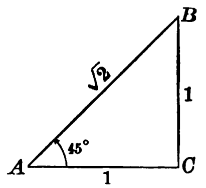
13. Given $\sec A=5$; find $\cot A$.

14. Given $\cos A=\frac{21}{29}$; find $\csc A$.

15. Given $\tan A=\frac{4\sqrt{2}}{7}$; find $\sec A$.

16. Given $\sin A=\frac{y}{x}$; find $\tan A$.

7. Functions of 45°.



Let ABC be an isosceles right triangle, AC and BC being each equal to 1.

Then $\angle A=45°$, and $AB=\sqrt{\overline{AC}^2+\overline{BC}^2}=\sqrt{1+1}=\sqrt{2}$.

Whence by definition,

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}.$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}.$

$\tan 45° = 1.$

$\cot 45° = 1.$

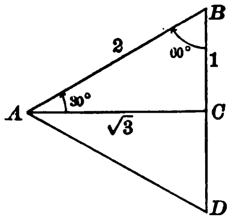
$\sec 45° = \sqrt{2}.$

$\csc 45° = \sqrt{2}.$

$\text{vers } 45° = 1 - \frac{1}{2}\sqrt{2} = \frac{2-\sqrt{2}}{2}.$

$\text{covers } 45° = 1 - \frac{1}{2}\sqrt{2} = \frac{2-\sqrt{2}}{2}.$

8. Functions of 30° and 60°.



Let ABD be an equilateral triangle having each side equal to 2.
Draw AC perpendicular to BD.
Then by Geometry, $BC = \frac{1}{2} BD = 1$, and $\angle BAC = \frac{1}{2} \angle BAD = 30°$.
Also, $AC = \sqrt{\overline{AB}^2 - \overline{BC}^2} = \sqrt{4-1} = \sqrt{3}.$
Then by definition, in the right triangle ABC,

$\sin 30° = \frac{1}{2} = \cos 60°.$

$\cos 30° = \frac{\sqrt{3}}{2} = \sin 60°.$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3} = \cot 60°.$

$\cot 30° = \sqrt{3} = \tan 60°.$

$\sec 30° = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3} = \csc 60°.$

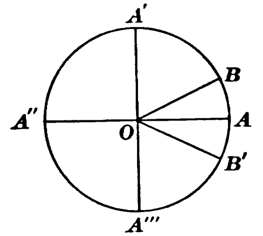
$\csc 30° = 2 = \sec 60°.$

$\text{vers } 30° = 1 - \frac{\sqrt{3}}{2} = \text{covers } 60°.$

$\text{covers } 30° = 1 - \frac{1}{2} = \frac{1}{2} = \text{vers } 60°.$

II. TRIGONOMETRIC FUNCTIONS OF ANGLES IN GENERAL.

9. In Geometry, we are, as a rule, concerned with angles which are less than two right angles; but in Trigonometry it is convenient to consider them as unrestricted in magnitude.



Let AA'' and $A'A'''$ be a pair of perpendicular diameters of the circle AA''.

Suppose a radius OB to start from the position OA, and revolve about the point O as a pivot, in a direction contrary to the motion of the hands of a clock.

When OB coincides with OA', it has generated an angle of 90°; when it coincides with OA'', of 180°; with OA''', of 270°; with OA, its first position, of 360°; with OA' again, of 450°; and so on.

We thus see that a significance may be attached to a positive angle of any number of degrees.

10. The interpretation of an angle as measuring the amount of rotation of a moving radius, enables us to distinguish between positive and negative angles.

Thus, if a positive angle indicates revolution from the position OA in a direction *contrary* to the motion of the hands of a clock, a negative angle may be taken as indicating revolution from the position OA in *the same* direction as the motion of the hands of a clock.

Thus, if the radius OB' starts from the position OA, and revolves about the point O as a pivot in the same direction as the motion of the hands of a clock, when it coincides with OA''' it has generated an angle of $-90°$; when it coincides with OA'', of $-180°$; with OA', of $-270°$; and so on.

We may then conceive of *negative* angles of any number of degrees.

It is immaterial which direction we consider the positive direction of rotation; but having at the outset adopted a certain direction as positive, our subsequent operations must be in accordance.

11. The fixed line OA from which the rotation is supposed to commence, is called the *initial line*, and the rotating radius in its final position is called the *terminal line.*

12. In designating an angle, we shall always write first the letter at the extremity of the initial line.

Thus, in designating the angle formed by the lines OA and OB, if we regard OA as the initial line, we should call it AOB; and if we regard OB as the initial line, we should call it BOA.

There are always two angles less than 360° in absolute value, one positive and the other negative, formed by the same initial and terminal line.

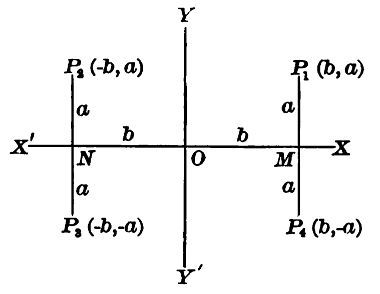
Thus, there are formed by OA and OB' the positive angle AOB' between 270° and 360°, and the negative angle AOB' between 0° and $-90°$.

We shall distinguish between such angles by referring to them as "the positive angle AOB'" and "the negative angle AOB'," respectively.

13. It is evident that the terminal lines of any two angles which differ by a multiple of 360° are coincident.

Thus, the angles 30°, 390°, $-330°$, etc., have the same terminal line.

14. Rectangular Co-ordinates.



Let P_1 be any point in the plane of the lines XX' and YY' intersecting at right angles at O, and draw P_1M perpendicular to XX'.

Then OM and P_1M are called the *rectangular co-ordinates* of P_1; OM is called the *abscissa*, and P_1M the *ordinate.*

The lines of reference, XX' and YY', are called the *axis of* X and the *axis of* Y, respectively, and O is called the *origin.*

It is customary to express the fact that the abscissa of a point is b, and its ordinate a, by saying that for the point in question $x = b$ and $y = a$; or, more concisely, we may refer to the point as "the point (b, a)," where the first term in the parenthesis is understood to be the abscissa, and the second term the ordinate.

15. If, in the figure of § 14, $OM = ON = b$, and P_1P_4 and P_2P_3 are drawn perpendicular to XX' so that $P_1M = P_2N = P_3N = P_4M = a$, the points P_1, P_2, P_3, and P_4 will have the same co-ordinates, (b, a).

To avoid this ambiguity, abscissas measured to the *right* of O are considered *positive*, and to the *left*, *negative*; and ordinates measured *above* XX' are considered *positive*, and *below*, *negative*.

Then the co-ordinates of the points will be as follows:

$$P_1,\ (b, a);\ P_2,\ (-b, a);\ P_3,\ (-b, -a);\ P_4,\ (b, -a).$$

16. If a point lies upon XX', its ordinate is zero; and if it lies upon YY', its abscissa is zero.

17. General Definitions of the Functions.

We will now give general definitions of the trigonometric functions, applicable to any angle whatever.

Take the initial line of the angle as the positive direction of the axis of X, the vertex being the origin.

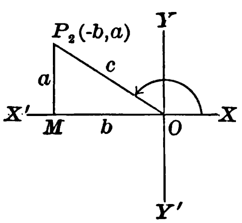
From any point in the terminal line, drop a perpendicular to the axis of X.

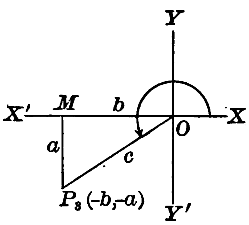
Find the co-ordinates of this point; then,

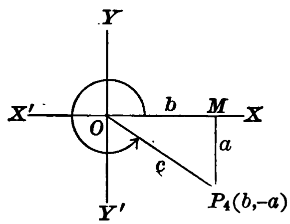
The **sine** *of the angle is the ratio of the* **ordinate** *of the point to its* **distance** *from the origin.*

The **cosine** *is the ratio of the* **abscissa** *to the* **distance**.
The **tangent** *is the ratio of the* **ordinate** *to the* **abscissa**.
The **cotangent** *is the ratio of the* **abscissa** *to the* **ordinate**.
The **secant** *is the ratio of the* **distance** *to the* **abscissa**.
The **cosecant** *is the ratio of the* **distance** *to the* **ordinate**.

18. We will now apply the definitions of § 17 to finding the functions of the angles XOP_2, XOP_3, and XOP_4 in the following figures:







Let P_2, P_3, and P_4 be any points on the terminal lines OP_2, OP_3, and OP_4, and draw P_2M, P_3M, and P_4M perpendicular to XX'.

Let $P_2M = P_3M = P_4M = a$, $OM = b$, and $OP_2 = OP_3 = OP_4 = c$.

Then the co-ordinates of P_2 are $(-b, a)$; of P_3, $(-b, -a)$; of P_4, $(b, -a)$.

Whence by definition,

$$\sin XOP_2 = \frac{a}{c}. \qquad \sin XOP_3 = \frac{-a}{c} = -\frac{a}{c}. \qquad \sin XOP_4 = \frac{-a}{c} = -\frac{a}{c}.$$

$$\cos XOP_2 = \frac{-b}{c} = -\frac{b}{c}. \qquad \cos XOP_3 = \frac{-b}{c} = -\frac{b}{c}. \qquad \cos XOP_4 = \frac{b}{c}.$$

$$\tan XOP_2 = \frac{a}{-b} = -\frac{a}{b}. \qquad \tan XOP_3 = \frac{-a}{-b} = \frac{a}{b}. \qquad \tan XOP_4 = \frac{-a}{b} = -\frac{a}{b}.$$

$$\cot XOP_2 = \frac{-b}{a} = -\frac{b}{a}. \qquad \cot XOP_3 = \frac{-b}{-a} = \frac{b}{a}. \qquad \cot XOP_4 = \frac{b}{-a} = -\frac{b}{a}.$$

$$\sec XOP_2 = \frac{c}{-b} = -\frac{c}{b}. \qquad \sec XOP_3 = \frac{c}{-b} = -\frac{c}{b}. \qquad \sec XOP_4 = \frac{c}{b}.$$

$$\csc XOP_2 = \frac{c}{a}. \qquad \csc XOP_3 = \frac{c}{-a} = -\frac{c}{a}. \qquad \csc XOP_4 = \frac{c}{-a} = -\frac{c}{a}.$$

Note 1. The definitions of § 17 are seen to include those of § 2.

The definitions of the versed sine and coversed sine, given in § 2, are sufficiently general to apply to any angle whatever.

Note 2. In all the figures of the present chapter, the small letters will be understood as denoting the *lengths* of the lines to which they are attached, without regard to their algebraic sign.

19. If the initial line of an angle coincides with OX, and its terminal line lies between OX and OY, the angle is said to be in the *first quadrant;* if the terminal line lies between OY and OX', the angle is said to be in the *second quadrant;* if between OX' and OY', in the *third quadrant;* if between OY' and OX, in the *fourth quadrant.*

Thus, any positive angle between 0° and 90°, or 360° and 450°, or any negative angle between $-270°$ and $-360°$, is in the first quadrant; any positive angle between 90° and 180°, or 450° and 540°, or any negative angle between $-180°$ and $-270°$, is in the second quadrant.

20. It follows from the definitions of § 17 that, *for any angle in the first quadrant, all the functions are positive.*

It is also evident by inspection of the results of § 18 that:

In the second quadrant, the sine and cosecant are positive, and the cosine, tangent, cotangent, and secant are negative.

In the third quadrant, the tangent and cotangent are positive, and the sine, cosine, secant, and cosecant are negative.

In the fourth quadrant, the cosine and secant are positive, and the sine, tangent, cotangent, and cosecant are negative.

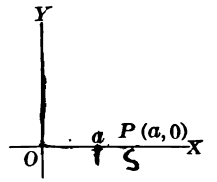
It is usual to express the above in tabular form, as follows:

Functions.	*First Quad.*	*Second Quad.*	*Third Quad.*	*Fourth Quad.*
Sine and cosecant	+	+	−	−
Cosine and secant	+	−	−	+
Tangent and cotangent	+	−	+	−

21. Since the terminal lines of any two angles which differ by a multiple of 360° are coincident (§ 13), it is evident that the trigonometric functions of two such angles are identical.

Thus, the functions of 50°, 410°, 770°, −310°, etc., are identical.

22. Functions of 0° and 360°.



The terminal line of 0° coincides with the initial line OX.

Let P be a point on OX such that $OP = a$.

Then by § 16, the co-ordinates of P are $(a, 0)$.

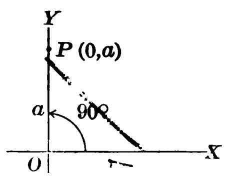
Whence by definition,

$$\sin 0° = \frac{0}{a} = 0. \qquad \tan 0° = \frac{0}{a} = 0. \qquad \sec 0° = \frac{a}{a} = 1.$$

$$\cos 0° = \frac{a}{a} = 1. \qquad \cot 0° = \frac{a}{0} = \infty. \qquad \csc 0° = \frac{a}{0} = \infty.$$

By § 21, the functions of 360° are the same as those of 0°.

23. Functions of 90°.



Let P be a point on OY such that $OP = a$.

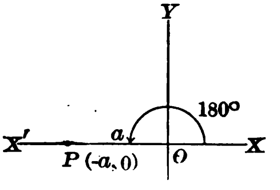
Then the co-ordinates of P are $(0, a)$.

Whence by definition,

$\sin 90° = \frac{a}{a} = 1.$ $\tan 90° = \frac{a}{0} = \infty.$ $\sec 90° = \frac{a}{0} = \infty.$

$\cos 90° = \frac{0}{a} = 0.$ $\cot 90° = \frac{0}{a} = 0.$ $\csc 90° = \frac{a}{a} = 1.$

24. Functions of 180°.

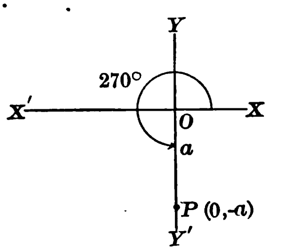


Let P be a point on OX' such that $OP = a$.
Then the co-ordinates of P are $(-a, 0)$.
Whence by definition,

$\sin 180° = \frac{0}{a} = 0.$ $\tan 180° = \frac{0}{-a} = 0.$ $\sec 180° = \frac{a}{-a} = -1.$

$\cos 180° = \frac{-a}{a} = -1.$ $\cot 180° = \frac{-a}{0} = \infty.$ $\csc 180° = \frac{a}{0} = \infty.$

25. Functions of 270°.



Let P be a point on OY' such that $OP = a$.
Then the co-ordinates of P are $(0, -a)$.
Whence by definition,

$\sin 270° = \frac{-a}{a} = -1.$ $\tan 270° = \frac{-a}{0} = \infty.$ $\sec 270° = \frac{a}{0} = \infty.$

$\cos 270° = \frac{0}{a} = 0.$ $\cot 270° = \frac{0}{-a} = 0.$ $\csc 270° = \frac{a}{-a} = -1.$

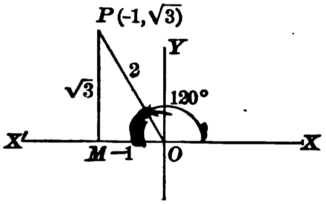
Note. No absolute meaning can be attached to such a result as $\cot 0° = \infty$; it merely signifies that as an angle approaches 0°, its cotangent increases without limit.

A similar interpretation must be given to the equations $\csc 0° = \infty$, $\tan 90° = \infty$, etc.

26. The results of the last four articles may be conveniently expressed in tabular form as follows:

Angle.	*Sin.*	*Cos.*	*Tan.*	*Cot.*	*Sec.*	*Csc.*
0°	0	1	0	∞	1	∞
90°	1	0	∞	0	∞	1
180°	0	-1	0	∞	-1	∞
270°	-1	0	∞	0	∞	-1
360°	0	1	0	∞	1	∞

27. Functions of 120°, 135°, 150°, etc.



Let OPM be a right triangle having OP, OM, and PM equal to 2, 1, and $\sqrt{3}$, respectively, and $\angle POM = 60°$. (Compare § 8.)

Then $\angle XOP = 120°$, and the co-ordinates of P are $(-1, \sqrt{3})$.

Whence by definition,

$$\sin 120° = \frac{\sqrt{3}}{2}. \qquad \tan 120° = -\sqrt{3}. \qquad \sec 120° = -2.$$

$$\cos 120° = -\frac{1}{2}. \qquad \cot 120° = -\frac{1}{\sqrt{3}} = -\frac{1}{3}\sqrt{3}. \qquad \csc 120° = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}.$$

In like manner may be proved the remaining values given in the following table, which are left as exercises for the student:

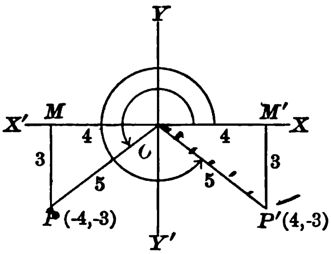
Angle.	*Sin.*	*Cos.*	*Tan.*	*Cot.*	*Sec.*	*Csc.*
120°	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	-2	$\frac{2}{3}\sqrt{3}$
135°	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	$-\sqrt{3}$	$-\frac{2}{3}\sqrt{3}$	2
210°	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$-\frac{2}{3}\sqrt{3}$	-2
225°	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	-2	$-\frac{2}{3}\sqrt{3}$
300°	$-\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	2	$-\frac{2}{3}\sqrt{3}$
315°	$-\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330°	$-\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	$-\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	-2

28. *Given the value of one function of an angle, to find the values of the remaining functions.* (Compare § 6.)

1. Given $\sin A = -\frac{3}{5}$; find the values of the remaining functions of A.

The example may be solved by a method similar to that of § 6; since the sine is the ratio of the ordinate to the distance, we may regard the point of reference as having its ordinate equal to -3, and its distance equal to 5.

There are *two* points, P and P', which are 3 units below the axis of X, and distant 5 units from O.



There are then *two angles*, XOP and XOP', in the third and fourth quadrants, respectively, either of which may be the angle A.

Now, $OM = OM' = \sqrt{\overline{OP}^2 - \overline{PM}^2} = \sqrt{25-9} = 4.$

Then the co-ordinates of P are $(-4, -3)$; and of P', $(4, -3)$.

Whence by definition:

Angle.	*Cos.*	*Tan.*	*Cot.*	*Sec.*	*Csc.*
XOP	$-\frac{4}{5}$	$\frac{3}{4}$	$\frac{4}{3}$	$-\frac{5}{4}$	$-\frac{5}{3}$
XOP'	$\frac{4}{5}$	$-\frac{3}{4}$	$-\frac{4}{3}$	$\frac{5}{4}$	$-\frac{5}{3}$

Thus the two solutions to the problem are:

$$\cos A = \mp\frac{4}{5},\ \tan A = \pm\frac{3}{4},\ \cot A = \pm\frac{4}{3},\ \sec A = \mp\frac{5}{4},\ \csc A = -\frac{5}{3};$$

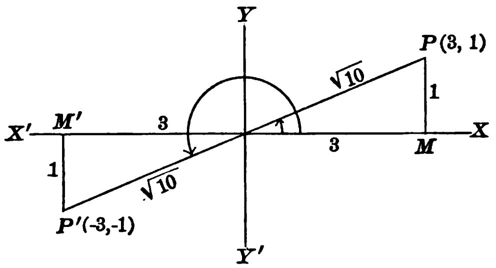
where the upper signs refer to XOP, and the lower signs to XOP'

2. Given $\cot A = 3$; find the values of the remaining functions of A.

The equation may be written either $\cot A = \frac{3}{1}$, or $\cot A = \frac{-3}{-1}$.

We may then regard the point of reference as having its abscissa equal to 3 and its ordinate equal to 1, or as having its abscissa equal to -3 and its ordinate equal to -1.

There are *two angles*, XOP and XOP', in the first and third quadrants, respectively, either of which satisfies the given condition.



Then $OP = OP' = \sqrt{\overline{OM}^2 + \overline{PM}^2} = \sqrt{9+1} = \sqrt{10}$.

Whence by definition:

Angle.	*Sin.*	*Cos.*	*Tan.*	*Sec.*	*Csc.*
XOP	$\frac{1}{\sqrt{10}}$	$\frac{3}{\sqrt{10}}$	$\frac{1}{3}$	$\frac{\sqrt{10}}{3}$	$\sqrt{10}$
XOP'	$-\frac{1}{\sqrt{10}}$	$-\frac{3}{\sqrt{10}}$	$\frac{1}{3}$	$-\frac{\sqrt{10}}{3}$	$-\sqrt{10}$

Thus the two solutions are:

$$\sin A = \pm\frac{1}{\sqrt{10}},\quad \cos A = \pm\frac{3}{\sqrt{10}},\quad \tan A = \frac{1}{3},\quad \sec A = \pm\frac{\sqrt{10}}{3},\quad \csc A = \pm\sqrt{10}.$$

Note. It must be clearly borne in mind, in examples like the above, that the "distance" is *always positive.*

EXAMPLES.

In each of the following, find the values of the remaining functions:

3. $\sec A = \frac{5}{4}$.

4. $\cot A = -\frac{12}{5}$.

5. $\sin A = \frac{15}{17}$.

6. $\cos A = -\frac{21}{29}$.

7. $\csc A = -\frac{25}{7}$.

8. $\tan A = \frac{9}{40}$.

9. $\sec A = -\frac{7}{2}$.

10. $\sin A = -\frac{1}{5}$.

11. $\tan A = -7$.

12. $\csc A = 3$.

13. $\cos A = \frac{a}{b}$.

14. $\cot A = x$.

29. Functions of $(-A)$ in terms of those of A.

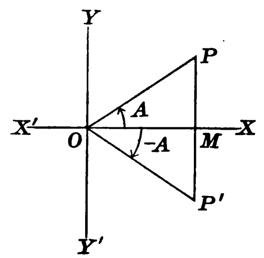


FIG. 1.

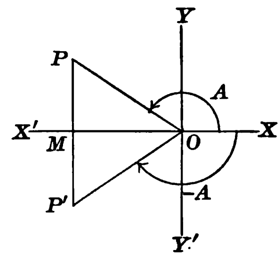


FIG. 2.

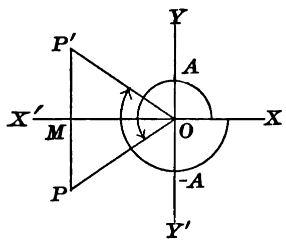


FIG. 3.

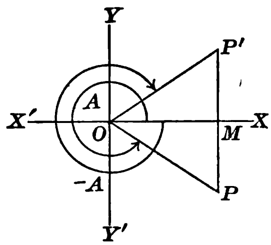


FIG. 4.

There may be four cases: A in the first quadrant (Fig. 1), A in the second quadrant (Fig. 2), A in the third quadrant (Fig. 3), or A in the fourth quadrant (Fig. 4).

In each figure, let the positive angle XOP represent the angle A, and the negative angle XOP' the angle $-A$.

Draw PM perpendicular to XX', and produce it to meet OP' at P'.

In the right triangles OPM and $OP'M$, the side OM is common, and $\angle POM = \angle P'OM$.

Hence, the triangles are equal, and $PM = P'M$ and $OP = OP'$.

Then in each figure,

$$\begin{aligned} \text{abscissa } P' &= \text{abscissa } P, \\ \text{ordinate } P' &= -\text{ordinate } P, \\ \text{and} \qquad \text{distance } P' &= \text{distance } P. \end{aligned}$$

Then,

$$\frac{\text{ord. } P'}{\text{dist. } P'} = -\frac{\text{ord. } P}{\text{dist. } P}. \quad \frac{\text{ord. } P'}{\text{abs. } P'} = -\frac{\text{ord. } P}{\text{abs. } P}. \quad \frac{\text{dist. } P'}{\text{abs. } P'} = \frac{\text{dist. } P}{\text{abs. } P}.$$

$$\frac{\text{abs. } P'}{\text{dist. } P'} = \frac{\text{abs. } P}{\text{dist. } P}. \quad \frac{\text{abs. } P'}{\text{ord. } P'} = -\frac{\text{abs. } P}{\text{ord. } P}. \quad \frac{\text{dist. } P'}{\text{ord. } P'} = -\frac{\text{dist. } P}{\text{ord. } P}.$$

Whence,

$$\left.\begin{aligned} \sin(-A) &= -\sin A. & \tan(-A) &= -\tan A. & \sec(-A) &= \sec A. \\ \cos(-A) &= \cos A. & \cot(-A) &= -\cot A. & \csc(-A) &= -\csc A. \end{aligned}\right\} \quad (1)$$

30. Functions of $(90° + A)$ in terms of those of A.

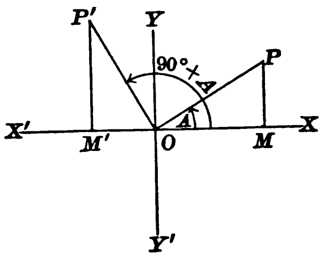


FIG. 1.

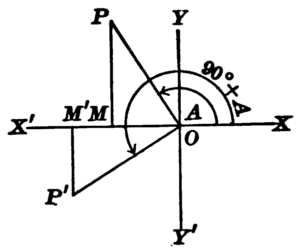


FIG. 2.

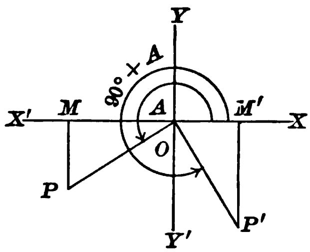


FIG. 3.

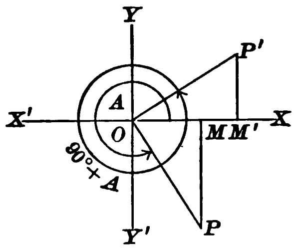


FIG. 4.

There may be four cases: A in the first quadrant (Fig. 1), A in the second quadrant (Fig. 2), A in the third quadrant (Fig. 3), or A in the fourth quadrant (Fig. 4).

In each figure, let the positive angle XOP represent the angle A, and the positive angle XOP' the angle $90° + A$.

Take $OP' = OP$, and draw PM and $P'M'$ perpendicular to XX'.

Since OP is perpendicular to OP', and OM to $P'M'$, $\angle POM = \angle OP'M'$.

Then the right triangles OPM and $OP'M'$ have the hypotenuse and an acute angle of one equal to the hypotenuse and an acute angle of the other.

Hence, the triangles are equal, and $PM = OM'$ and $OM = P'M'$.

Then in each figure,

$$\text{ordinate } P' = \text{abscissa } P,$$

$$\text{abscissa } P' = -\text{ordinate } P,$$

and

$$\text{distance } P' = \text{distance } P.$$

Then, $\dfrac{\text{ord. } P'}{\text{dist. } P'} = \dfrac{\text{abs. } P}{\text{dist. } P}$. $\qquad \dfrac{\text{abs. } P'}{\text{ord. } P'} = -\dfrac{\text{ord. } P}{\text{abs. } P}$.

$$\frac{\text{abs. } P'}{\text{dist. } P'} = -\frac{\text{ord. } P}{\text{dist. } P}. \qquad \frac{\text{dist. } P'}{\text{abs. } P'} = -\frac{\text{dist. } P}{\text{ord. } P}.$$

$$\frac{\text{ord. } P'}{\text{abs. } P'} = -\frac{\text{abs. } P}{\text{ord. } P}. \qquad \frac{\text{dist. } P'}{\text{ord. } P'} = \frac{\text{dist. } P}{\text{abs. } P}.$$

Or,
$$\left.\begin{array}{ll} \sin(90^\circ + A) = \cos A. & \cot(90^\circ + A) = -\tan A. \\ \cos(90^\circ + A) = -\sin A. & \sec(90^\circ + A) = -\csc A. \\ \tan(90^\circ + A) = -\cot A. & \csc(90^\circ + A) = \sec A. \end{array}\right\} \qquad (2)$$

31. The results of § 30 may be stated as follows:

The sine, cosine, tangent, cotangent, secant, and cosecant of any angle are equal, respectively, to the cosine, minus the sine, minus the cotangent, minus the tangent, minus the cosecant, and the secant, of an angle 90° less.

32. Functions of $(90^\circ - A)$ **in terms of those of** A.

By § 31,
$$\begin{aligned} \sin(90^\circ - A) &= \cos(-A) = \cos A \ (\S\ 29). \\ \cos(90^\circ - A) &= -\sin(-A) = \sin A. \\ \tan(90^\circ - A) &= -\cot(-A) = \cot A. \\ \cot(90^\circ - A) &= -\tan(-A) = \tan A. \\ \sec(90^\circ - A) &= -\csc(-A) = \csc A. \\ \csc(90^\circ - A) &= \sec(-A) = \sec A. \end{aligned}$$

These formulæ were proved for acute angles in § 5.

33. Functions of $(180^\circ - A)$ **in terms of those of** A.

By § 31,
$$\begin{aligned} \sin(180^\circ - A) &= \cos(90^\circ - A) = \sin A \ (\S\ 32). \\ \cos(180^\circ - A) &= -\sin(90^\circ - A) = -\cos A. \\ \tan(180^\circ - A) &= -\cot(90^\circ - A) = -\tan A. \\ \cot(180^\circ - A) &= -\tan(90^\circ - A) = -\cot A. \\ \sec(180^\circ - A) &= -\csc(90^\circ - A) = -\sec A. \\ \csc(180^\circ - A) &= \sec(90^\circ - A) = \csc A. \end{aligned}$$

34. By successive applications of the theorem of § 31, any function of a multiple of 90°, plus or minus A, may be expressed as a function of A.

1. Express $\sin(270^\circ + A)$ as a function of A.

By § 31, $\sin(270^\circ + A) = \cos(180^\circ + A) = -\sin(90^\circ + A) = -\cos A$.

If the multiple of 90° is greater than 270°, we may subtract 360°, or any multiple of 360°, from the angle, in accordance with § 21.

2. Express $\sec(990^\circ - A)$ as a function of A.

Subtracting twice 360°, or 720°, from the angle, we have

$$\sec(990^\circ - A) = \sec(270^\circ - A).$$

And by § 31, $\quad \sec(270^\circ - A) = -\csc(180^\circ - A) = -\csc A$ (§ 33).

If the multiple of 90° is negative, we may add 360°, or any multiple of 360°, to the angle.

3. Express $\tan(-180° + A)$ as a function of A.

Adding 360° to the angle, we have

$$\tan(-180° + A) = \tan(180° + A).$$

And by § 31, $\quad \tan(180° + A) = -\cot(90° + A) = \tan A.$

EXAMPLES.

Express each of the following as a function of A:

4. $\sin(180° + A)$.
5. $\cos(270° - A)$.
6. $\cot(450° + A)$.
7. $\csc(360° - A)$.
8. $\tan(540° - A)$.
9. $\sec(630° + A)$.
10. $\tan(-270° - A)$.
11. $\csc(-90° - A)$.
12. $\cot(-180° + A)$.
13. $\sin(-630° + A)$.
14. $\tan(-450° - A)$.
15. $\cos(-900° - A)$.
16. $\sin(810° - A)$.
17. $\csc(1080° + A)$.
18. $\sec(1260° + A)$.

35. By means of the theorem of § 31, any function of any angle, positive or negative, may be expressed as a function of a certain acute angle.

1. Express $\sin 317°$ as a function of an acute angle.

By § 31, $\sin 317° = \cos 227° = -\sin 137° = -\cos 47°$.

Since the complement of 47° is 43°, another form of the result is $-\sin 43°$ (§ 5).

Note. As in the examples of § 34, 360°, or any multiple of 360°, may be added to, or subtracted from, the angle.

EXAMPLES.

Express each of the following as a function of an acute angle:

2. $\cos 322°$.
3. $\tan 208°$.
4. $\sec 559°$.
5. $\csc 803° 45'$.
6. $\cot(-378°)$.
7. $\sin(-139° 5')$.

It is evident from the above that any function of any angle can be expressed as a function of a certain acute angle *less than* 45°.

Express each of the following as a function of an acute angle less than 45°:

8. $\cot 155°$.
9. $\sin 1138° 36'$.
10. $\sec 457°$.
11. $\cos 496° 20'$.
12. $\tan(-681°)$.
13. $\csc(-257°)$.

14. Find the value of $\csc(-210°)$.

Adding 360° to the angle, we have

$$\csc(-210°) = \csc 150°.$$

And by § 31, $\quad \csc 150° = \sec 60° = 2$ (§ 8).

Find the values of the following:

15. $\cot 405°$.
16. $\sin 480°$.
17. $\csc 600°$.
18. $\tan 690°$.
19. $\cos(-420°)$.
20. $\sec(-225°)$.

III. GENERAL FORMULÆ.

36. It follows immediately from the definitions of § 17 that, if x is any angle,

$$\left.\begin{array}{lll} \sin x = \dfrac{1}{\csc x}. & \tan x = \dfrac{1}{\cot x}. & \sec x = \dfrac{1}{\cos x}. \\ \cos x = \dfrac{1}{\sec x}. & \cot x = \dfrac{1}{\tan x}. & \csc x = \dfrac{1}{\sin x}. \end{array}\right\} \qquad (3)$$

37. *To prove the formula*

$$\tan x = \frac{\sin x}{\cos x}. \qquad (4)$$

I. When the angle x is acute (or in the first quadrant).

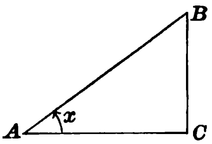


FIG. 1.

In the right triangle ABC, let BAC be the angle x.

By § 2,
$$\tan x = \frac{BC}{AC} = \frac{\frac{BC}{AB}}{\frac{AC}{AB}} = \frac{\sin x}{\cos x}.$$

II. When x is in the second, third, or fourth quadrant.

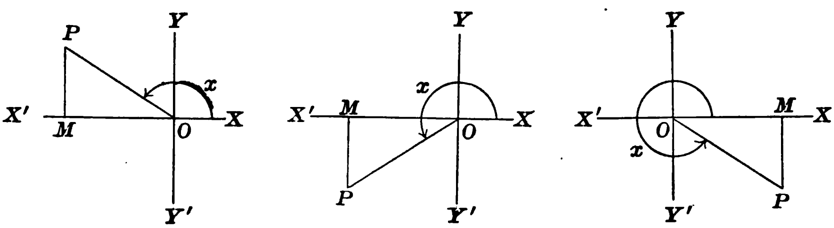


FIG. 2. FIG. 3. FIG. 4.

In each figure, let the positive angle XOP represent the angle x, and draw PM perpendicular to XX'.

Then in each figure, by the definitions of § 17,

$$\tan x = \frac{\text{ord. } P}{\text{abs. } P} = \frac{\frac{\text{ord. } P}{\text{dist. } P}}{\frac{\text{abs. } P}{\text{dist. } P}} = \frac{\sin x}{\cos x}.$$

38. *To prove the formula*

$$\cot x = \frac{\cos x}{\sin x}. \tag{5}$$

By (3), § 36, $\cot x = \dfrac{1}{\tan x} = \dfrac{1}{\dfrac{\sin x}{\cos x}}$ (§ 37) $= \dfrac{\cos x}{\sin x}.$

39. *To prove the formula*

$$\sin^2 x + \cos^2 x = 1. \tag{6}$$

Note. $\sin^2 x$ signifies $(\sin x)^2$; that is, the square of the sine of x.

I. When the angle x is acute (or in the first quadrant).

In Fig. 1, § 37, we have by Geometry,

$$\overline{BC}^2 + \overline{AC}^2 = \overline{AB}^2.$$

Dividing by $\overline{AB}^2$, $\left(\dfrac{BC}{AB}\right)^2 + \left(\dfrac{AC}{AB}\right)^2 = 1.$

Then by definition, $(\sin x)^2 + (\cos x)^2 = 1.$

That is, $\sin^2 x + \cos^2 x = 1.$

II. When x is in the second, third, or fourth quadrant.

In each of the figures 2, 3, and 4 of § 37, we have

$$\overline{PM}^2 + \overline{OM}^2 = \overline{OP}^2.$$

Dividing by $\overline{OP}^2$, $\dfrac{\overline{PM}^2}{\overline{OP}^2} + \dfrac{\overline{OM}^2}{\overline{OP}^2} = 1.$

But in either figure, $\dfrac{\overline{PM}^2}{\overline{OP}^2} = \sin^2 x$, and $\dfrac{\overline{OM}^2}{\overline{OP}^2} = \cos^2 x.$

Whence, $\sin^2 x + \cos^2 x = 1.$

Formula (6) may be written in the forms

$$\sin^2 x = 1 - \cos^2 x, \text{ and } \cos^2 x = 1 - \sin^2 x.$$

40. *To prove the formulæ*

$$\sec^2 x = 1 + \tan^2 x, \tag{7}$$

and

$$\csc^2 x = 1 + \cot^2 x. \tag{8}$$

By (6), $1 = \cos^2 x + \sin^2 x.$ (A)

Dividing by $\cos^2 x$, $\dfrac{1}{\cos^2 x} = 1 + \dfrac{\sin^2 x}{\cos^2 x}.$

Whence by (3) and (4), $\sec^2 x = 1 + \tan^2 x.$

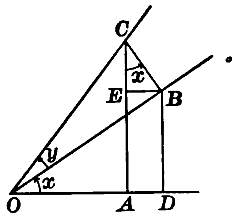
Again, dividing (A) by $\sin^2 x$, we have

$$\frac{1}{\sin^2 x} = 1 + \frac{\cos^2 x}{\sin^2 x}.$$

Whence by (3) and (5), $\csc^2 x = 1 + \cot^2 x$.

41. *To find the values of* $\sin(x+y)$ *and* $\cos(x+y)$ *in terms of the sines and cosines of* x *and* y.

I. When x and y are acute, and $x+y$ acute.



Let AOB and BOC denote the angles x and y, respectively.

Then, $\angle AOC = x + y$.

From any point C in OC draw CA and CB perpendicular to OA and OB; and draw BD and BE perpendicular to OA and AC.

Since EC and BC are perpendicular to OA and OB, the angles BCE and AOB are equal; that is, $\angle BCE = x$.

Now, $$\sin(x+y) = \frac{AC}{OC} = \frac{BD + CE}{OC} = \frac{BD}{OC} + \frac{CE}{OC}.$$

But, $$\frac{BD}{OC} = \frac{BD}{OB} \times \frac{OB}{OC} = \sin x \cos y,$$

and $$\frac{CE}{OC} = \frac{CE}{BC} \times \frac{BC}{OC} = \cos x \sin y.$$

Whence, $$\sin(x+y) = \sin x \cos y + \cos x \sin y. \tag{9}$$

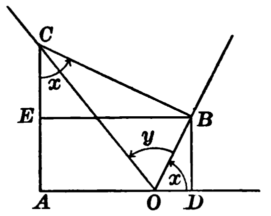
Again, $$\cos(x+y) = \frac{OA}{OC} = \frac{OD - BE}{OC} = \frac{OD}{OC} - \frac{BE}{OC}.$$

But, $$\frac{OD}{OC} = \frac{OD}{OB} \times \frac{OB}{OC} = \cos x \cos y,$$

and $$\frac{BE}{OC} = \frac{BE}{BC} \times \frac{BC}{OC} = \sin x \sin y.$$

Whence, $$\cos(x+y) = \cos x \cos y - \sin x \sin y. \tag{10}$$

II. When x and y are acute, and $x + y$ obtuse.



Let DOB and BOC denote the angles x and y, respectively.

Then, $\angle DOC = x + y$.

From any point C in OC draw CB perpendicular to OB, and CA perpendicular to DO produced; and draw BD and BE perpendicular to OD and AC.

Since EC and BC are perpendicular to OD and OB, the angles BCE and DOB are equal; that is, $\angle BCE = x$.

Then by § 17, $\sin DOC = \dfrac{AC}{OC} = \dfrac{BD + CE}{OC} = \dfrac{BD}{OC} + \dfrac{CE}{OC}$.

But, $$\frac{BD}{OC} = \frac{BD}{OB} \times \frac{OB}{OC} = \sin x \cos y,$$

and $$\frac{CE}{OC} = \frac{CE}{BC} \times \frac{BC}{OC} = \cos x \sin y.$$

Whence, $\sin (x + y) = \sin x \cos y + \cos x \sin y$.

Again, $$\cos DOC = \frac{-OA}{OC} = \frac{OD - BE}{OC} = \frac{OD}{OC} - \frac{BE}{OC}.$$

But, $$\frac{OD}{OC} = \frac{OD}{OB} \times \frac{OB}{OC} = \cos x \cos y,$$

and $$\frac{BE}{OC} = \frac{BE}{BC} \times \frac{BC}{OC} = \sin x \sin y.$$

Whence, $\cos (x + y) = \cos x \cos y - \sin x \sin y$.

42. Formulæ (**9**) and (**10**) are very important, and it is necessary to prove them *for all values of x and y.*

They have already been proved when x and y are any two acute angles; or, what is the same thing, when they are any two angles in the *first quadrant.*

Now let a and b be any assigned values of x and y, for which (**9**) and (**10**) are true.

By (**2**), § 30, $\sin[90° + (a+b)] = \cos(a+b)$,

and $\cos[90° + (a+b)] = -\sin(a+b)$.

Whence, by (**9**) and (**10**),

$$\sin[90° + (a+b)] = \cos a \cos b - \sin a \sin b, \tag{A}$$

and

$$\cos[90° + (a+b)] = -\sin a \cos b - \cos a \sin b. \tag{B}$$

But by (**2**), § 30, $\cos a = \sin(90° + a)$, and $-\sin a = \cos(90° + a)$.

Then, (A) and (B) may be written in the forms

$$\sin[(90° + a) + b] = \sin(90° + a)\cos b + \cos(90° + a)\sin b,$$

and

$$\cos[(90° + a) + b] = \cos(90° + a)\cos b - \sin(90° + a)\sin b;$$

which are in accordance with (**9**) and (**10**).

Therefore, if (**9**) and (**10**) hold for any assigned values of x and y, they also hold when one of the angles is increased by 90°.

But they have been proved to hold when x and y are both in the first quadrant; hence, they hold when x is in the second quadrant and y in the first.

And since they hold when x is in the second quadrant and y in the first, they hold when x and y are both in the second quadrant; and so on.

Hence, (**9**) and (**10**) hold for any values of x and y whatever, positive or negative.

43. By (**9**), $\sin[x + (-y)] = \sin x \cos(-y) + \cos x \sin(-y)$

$= \sin x \cos y + \cos x(-\sin y)$, by (**1**), § 29.

Whence,

$$\sin(x - y) = \sin x \cos y - \cos x \sin y. \tag{11}$$

By (**10**), $\cos[x + (-y)] = \cos x \cos(-y) - \sin x \sin(-y)$

$= \cos x \cos y - \sin x(-\sin y)$.

Whence,

$$\cos(x - y) = \cos x \cos y + \sin x \sin y. \tag{12}$$

44. By (**4**), $\tan(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)}$

$= \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$, by (**9**) and (**10**).

Dividing each term of the fraction by $\cos x \cos y$,

$$\tan(x+y) = \frac{\dfrac{\sin x \cos y}{\cos x \cos y} + \dfrac{\cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y}{\cos x \cos y} - \dfrac{\sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\tan x + \tan y}{1 - \tan x \tan y}. \tag{13}$$

In like manner, we may prove

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}. \qquad (14)$$

Again, by (5), $$\cot(x+y)=\frac{\cos(x+y)}{\sin(x+y)}$$

$$=\frac{\cos x\cos y-\sin x\sin y}{\sin x\cos y+\cos x\sin y}.$$

Dividing each term of the fraction by $\sin x\sin y$,

$$\cot(x+y)=\frac{\frac{\cos x\cos y}{\sin x\sin y}-\frac{\sin x\sin y}{\sin x\sin y}}{\frac{\sin x\cos y}{\sin x\sin y}+\frac{\cos x\sin y}{\sin x\sin y}}$$

$$=\frac{\cot x\cot y-1}{\cot y+\cot x}. \qquad (15)$$

In like manner, we may prove

$$\cot(x-y)=\frac{\cot x\cot y+1}{\cot y-\cot x}. \qquad (16)$$

45. From (9), (10), (11), and (12), we have

$$\sin(a+b)=\sin a\cos b+\cos a\sin b. \qquad (A)$$

$$\sin(a-b)=\sin a\cos b-\cos a\sin b. \qquad (B)$$

$$\cos(a+b)=\cos a\cos b-\sin a\sin b. \qquad (C)$$

$$\cos(a-b)=\cos a\cos b+\sin a\sin b. \qquad (D)$$

Adding and subtracting (A) and (B), and then (C) and (D),

$$\sin(a+b)+\sin(a-b)=2\sin a\cos b.$$

$$\sin(a+b)-\sin(a-b)=2\cos a\sin b.$$

$$\cos(a+b)+\cos(a-b)=2\cos a\cos b.$$

$$\cos(a+b)-\cos(a-b)=-2\sin a\sin b.$$

Let $a+b=x$, and $a-b=y$.
Then, $a=\frac{1}{2}(x+y)$, and $b=\frac{1}{2}(x-y)$.
Substituting these values, we have

$$\sin x+\sin y=2\sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y). \qquad (17)$$

$$\sin x-\sin y=2\cos\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y). \qquad (18)$$

$$\cos x+\cos y=2\cos\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y). \qquad (19)$$

$$\cos x-\cos y=-2\sin\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y). \qquad (20)$$

46. By (**17**) and (**18**), we have

$$\frac{\sin x + \sin y}{\sin x - \sin y} = \frac{2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)}{2\cos\frac{1}{2}(x+y)\sin\frac{1}{2}(x-y)}$$

$$= \tan\tfrac{1}{2}(x+y)\cot\tfrac{1}{2}(x-y)$$

$$= \frac{\tan\frac{1}{2}(x+y)}{\tan\frac{1}{2}(x-y)}, \text{ by (3)}. \qquad (21)$$

47. By (**9**) and (**11**), we have

$$\sin(x+y)\sin(x-y) = (\sin x\cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$$

$$= \sin^2 x\cos^2 y - \cos^2 x \sin^2 y$$

$$= \sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y \quad (\S\ 39)$$

$$= \sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 x \sin^2 y$$

$$= \sin^2 x - \sin^2 y. \qquad (22)$$

The result may also be written

$$\sin(x+y)\sin(x-y) = 1 - \cos^2 x - (1-\cos^2 y) \quad (\S\ 39)$$

$$= \cos^2 y - \cos^2 x. \qquad (23)$$

In like manner, we may prove

$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x. \qquad (24)$$

48. Functions of $2x$.

Putting $y = x$ in (**9**), we have

$$\sin 2x = \sin x\cos x + \cos x \sin x$$

$$= 2\sin x\cos x. \qquad (25)$$

Putting $y = x$ in (**10**), we obtain

$$\cos 2x = \cos^2 x - \sin^2 x. \qquad (26)$$

We also have by § 39,

$$\cos 2x = (1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x, \qquad (27)$$

and

$$\cos 2x = \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1. \qquad (28)$$

Putting $y = x$ in (**13**) and (**15**), we have

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}, \qquad (29)$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}. \qquad (30)$$

49. Functions of $\frac{1}{2}x$.

From (**27**) and (**28**) we have, by transposition,

$$2\sin^2 x = 1 - \cos 2x, \text{ and } 2\cos^2 x = 1 + \cos 2x.$$

Putting $\frac{1}{2}x$ in place of x, and therefore x in place of $2x$, we have

$$2\sin^2\tfrac{1}{2}x = 1 - \cos x, \tag{31}$$

$$2\cos^2\tfrac{1}{2}x = 1 + \cos x. \tag{32}$$

Again, putting $\frac{1}{2}x$ in place of x in (**25**),

$$2\sin\tfrac{1}{2}x\cos\tfrac{1}{2}x = \sin x. \tag{A}$$

Dividing (**31**) by (A), we have, by (**4**),

$$\tan\tfrac{1}{2}x = \frac{1-\cos x}{\sin x}. \tag{33}$$

Dividing (**32**) by (A), $$\cot\tfrac{1}{2}x = \frac{1+\cos x}{\sin x}. \tag{34}$$

50. Functions of $3x$.

We have, $\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x\sin x$, by (**9**)

$$\begin{aligned}
&= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x \quad (\S\ 48)\\
&= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x \quad (\S\ 39)\\
&= 3\sin x - 4\sin^3 x.
\end{aligned} \tag{35}$$

Also, $\cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$, by (**10**)

$$\begin{aligned}
&= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x \quad (\S\ 48)\\
&= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x) \quad (\S\ 39)\\
&= 4\cos^3 x - 3\cos x.
\end{aligned} \tag{36}$$

Again, $\tan 3x = \tan(2x+x) = \dfrac{\tan 2x + \tan x}{1-\tan 2x\tan x}$, by (**13**)

$$= \frac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \left(\dfrac{2\tan x}{1-\tan^2 x}\right)\tan x}, \text{ by } (\mathbf{29})$$

$$= \frac{2\tan x + (1-\tan^2 x)\tan x}{1-\tan^2 x - 2\tan^2 x} = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}. \tag{37}$$

EXERCISES.

51. 1. Prove the relation $\sec^2 x\csc^2 x = \sec^2 x + \csc^2 x$.

By (**3**), $\sec^2 x\csc^2 x = \dfrac{1}{\cos^2 x\sin^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x\sin^2 x}$, by (**6**)

$$= \frac{\sin^2 x}{\cos^2 x\sin^2 x} + \frac{\cos^2 x}{\cos^2 x\sin^2 x}$$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \sec^2 x + \csc^2 x.$$

2. Prove the relation $\dfrac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x.$

By (18) and (19), $\dfrac{\sin 3x - \sin x}{\cos 3x + \cos x} = \dfrac{2\cos\frac{1}{2}(3x+x)\sin\frac{1}{2}(3x-x)}{2\cos\frac{1}{2}(3x+x)\cos\frac{1}{2}(3x-x)} = \tan x.$

3. Prove the relation $\dfrac{\tan(x+y) - \tan x}{1 + \tan(x+y)\tan x} = \tan y.$

By (14), $\dfrac{\tan(x+y) - \tan x}{1 + \tan(x+y)\tan x} = \tan[(x+y) - x] = \tan y.$

Prove the following relations:

4. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}.$

5. $\dfrac{\cos(x+y)}{\cos(x-y)} = \dfrac{1 - \tan x \tan y}{1 + \tan x \tan y}.$

6. $\dfrac{\cos x + \cos y}{\cos x - \cos y} = -\cot\frac{1}{2}(x+y)\cot\frac{1}{2}(x-y).$

7. $\sin(x+y+z) = \sin x\cos y\cos z + \cos x\sin y\cos z$
$\qquad + \cos x\cos y\sin z - \sin x\sin y\sin z.$

8. $\cos(x+y+z) = \cos x\cos y\cos z - \sin x\sin y\cos z$
$\qquad - \sin x\cos y\sin z - \cos x\sin y\sin z.$

9. $\tan(60°+x) - \cot(30°-x) = 0.$

10. $\dfrac{\csc^2 A}{\csc^2 A - 2} = \sec 2A.$

11. $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x} = \sec x.$

12. $\left(\dfrac{\tan x + 1}{\tan x - 1}\right)^2 = \dfrac{1 + \sin 2x}{1 - \sin 2x}.$

13. $\dfrac{\sin 5x + \sin x}{\cos 5x + \cos x} = \tan 3x.$

14. $\dfrac{\sin 3x - \sin 5x}{\cos 3x - \cos 5x} = -\cot 4x.$

15. $\sin 4x = 4\sin x\cos x - 8\sin^3 x\cos x.$

16. $\cos 4x = 1 - 8\cos^2 x + 8\cos^4 x.$

17. By putting $x = 45°$ and $y = 30°$ in (11) and (12), prove
$$\sin 15° = \tfrac{1}{4}(\sqrt{6} - \sqrt{2}),\ \cos 15° = \tfrac{1}{4}(\sqrt{6} + \sqrt{2}).$$

18. By putting $x = 30°$ in (33) and (34), prove
$$\tan 15° = 2 - \sqrt{3},\ \cot 15° = 2 + \sqrt{3}.$$

19. Using the results of Ex. 17, prove
$$\sec 15° = \sqrt{6} - \sqrt{2},\ \csc 15° = \sqrt{6} + \sqrt{2}.$$

20. By putting $x = 45°$ in (31) and (32), prove
$$\sin 22\tfrac{1}{2}° = \tfrac{1}{2}\sqrt{2 - \sqrt{2}},\ \cos 22\tfrac{1}{2}° = \tfrac{1}{2}\sqrt{2 + \sqrt{2}}.$$

21. By putting $x = 45^\circ$ in (**33**) and (**34**), prove

$$\tan 22\tfrac{1}{2}^\circ = \sqrt{2} - 1,\ \cot 22\tfrac{1}{2} = \sqrt{2} + 1.$$

22. By putting $x = 22\tfrac{1}{2}^\circ$ in (**7**) and (**8**), and using the result of Ex. 21, prove

$$\sec 22\tfrac{1}{2}^\circ = \sqrt{4 - 2\sqrt{2}},\ \csc 22\tfrac{1}{2}^\circ = \sqrt{4 + 2\sqrt{2}}.$$

Prove the following relations:

23. $\tan(45^\circ + x) - \tan(45^\circ - x) = 2 \tan 2x.$

24. $\cos^4 x - \sin^4 x = \cos 2x.$

25. $\dfrac{1}{\csc x - \cot x} = \cot \tfrac{1}{2} x.$

26. $\sin^2(x + y) - \sin^2(x - y) = \sin 2x \sin 2y.$

27. $\dfrac{\tan(x - y) + \tan y}{1 - \tan(x - y)\tan y} = \tan x.$

28. $\cos 5A \cos 3A + \sin 5A \sin 3A = \cos 2A.$

29. $\sin(A + B)\cos(A - B) - \cos(A + B)\sin(A - B) = \sin 2B.$

30. $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 2 \cot 2x.$

31. $\sin 2x = \dfrac{2 \tan x}{1 + \tan^2 x}.$

32. $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}.$

33. $\dfrac{\sin 4x + \sin 3x}{\cos 3x - \cos 4x} = \cot \tfrac{1}{2} x.$

34. $\sin 50^\circ + \sin 10^\circ = \sin 70^\circ.$

35. $\dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x.$

36. $2 \cos 3x \sin x = \sin 4x - \sin 2x.$

37. $\cos 5x = 5 \cos x - 20 \cos^3 x + 16 \cos^5 x.$

38. $\dfrac{\cos x}{1 - \sin x} = \dfrac{\cot \frac{1}{2} x + 1}{\cot \frac{1}{2} x - 1}.$

39. $\tan 4x = \dfrac{4 \tan x - 4 \tan^3 x}{1 - 6 \tan^2 x + \tan^4 x}.$

40. $(\sin x + \cos x)(2 - \sin 2x) = 2(\sin^3 x + \cos^3 x).$

41. $(\sin x - \sin y)^2 + (\cos x - \cos y)^2 = 4 \sin^2 \dfrac{x - y}{2}.$

42. $\dfrac{1 + \sin x - \cos x}{1 + \sin x + \cos x} = \tan \tfrac{1}{2} x.$

43. $\dfrac{\sin 3x - \cos 3x}{\sin x + \cos x} = 2 \sin 2x - 1.$

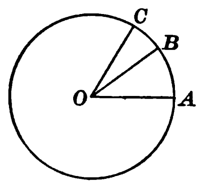
IV. MISCELLANEOUS THEOREMS.

52. Circular Measure of an Angle.

An angle is measured by finding its ratio to another angle, adopted arbitrarily as the unit of measure.

The usual unit of measure for angles is the degree, which is an angle equal to the ninetieth part of a right angle.

Another method of measuring angles, and one of great importance, is known as the *Circular Method;* in which the unit of measure is *the angle at the centre of a circle subtended by an arc whose length is equal to the radius.*



Thus, let AOB be any angle; and let AOC be the unit of circular measure; that is, the angle at the centre subtended by an arc whose length is equal to OA.

Then, circular measure $AOB = \dfrac{\angle AOB}{\angle AOC}$.

But by Geometry, $\dfrac{\angle AOB}{\angle AOC} = \dfrac{\text{arc } AB}{\text{arc } AC} = \dfrac{\text{arc } AB}{OA}$.

Whence, circular measure $AOB = \dfrac{\text{arc } AB}{OA}$.

That is, *the circular measure of an angle is the ratio of its subtending arc to the radius of the circle.*

53. By § 52, the circular measure of a right angle is the ratio of one-fourth the circumference to the radius.

But if R denotes the radius, the circumference of the circle is $2\pi R$.

Whence, circular measure of $90° = \dfrac{\frac{1}{4} \text{ of } 2\pi R}{R} = \dfrac{\pi}{2}$.

It follows from the above that the circular measure of 180° is π; of 60°, $\frac{\pi}{3}$; of 45°, $\frac{\pi}{4}$; etc.

That is, *an angle expressed in degrees may be reduced to circular measure by finding its ratio to* 180°, *and multiplying the result by* π.

Thus, since 115° is $\dfrac{23}{36}$ of 180°, the circular measure of 115° is $\dfrac{23\pi}{36}$.

54. Conversely, *an angle expressed in circular measure may be reduced to degrees by multiplying by* 180° *and dividing by* π; or, more briefly, *by substituting* 180° *for* π.

Thus, $\frac{7\pi}{15} = \frac{7}{15}$ of $180° = 84°$.

55. In the circular method, such expressions may occur as "the angle $\frac{2}{3}$," "the angle 1," etc.

These refer to the unit of circular measure; thus, the angle $\frac{2}{3}$ signifies an angle whose subtending arc is two-thirds of the radius.

The angle 1, that is, the angle whose subtending arc is equal to the radius, or the unit of circular measure, reduced to degrees by the rule of § 54, gives

$$\frac{180°}{\pi} = \frac{180°}{3.14159\cdots} = 57.2958°, \text{ approximately.}$$

Then the rule of § 54 may be modified as follows:

An angle expressed in circular measure may be reduced to degrees by multiplying by 57.2958°.

Thus, the angle $\frac{2}{3} = \frac{2}{3} \times 57.2958° = 38.1972° = 38°\,11'\,49.92''$.

EXAMPLES.

56. Express each of the following in circular measure:

1. 120°.	**3.** 67° 30′.	**5.** 86° 24′.	**7.** 163° 7′ 30″.
2. 315°.	**4.** 146° 15′.	**6.** 53° 20′.	**8.** 88° 53′ 20″.

Express each of the following in degree measure:

9. $\frac{5\pi}{6}$.	**11.** $\frac{23\pi}{64}$.	**13.** $\frac{1}{4}$.	**15.** $\frac{\pi - 1}{6}$.
10. $\frac{11\pi}{24}$.	**12.** $\frac{3}{2}$.	**14.** $\frac{5}{3}$.	**16.** $\frac{3\pi + 2}{5}$.

57. Inverse Trigonometric Functions.

The expression $\sin^{-1} x$, called the *inverse sine* of x, or the *anti-sine* of x, signifies *the angle whose sine is* x.

Thus, the statement that the sine of the angle x is equal to y may be expressed in either of the ways

$$\sin x = y, \text{ or } x = \sin^{-1} y.$$

In like manner, $\cos^{-1} x$ signifies the angle whose cosine is x; $\tan^{-1} x$, the angle whose tangent is x; etc.

Note. The student must be careful not to confuse the above notation with the *exponent* -1; the -1 power of $\sin x$ is expressed $(\sin x)^{-1}$, and not $\sin^{-1} x$.

It is evident that the sine of the angle whose sine is x is x; that is, $\sin(\sin^{-1}x) = x$.

In like manner, $\cos(\cos^{-1}x) = x$; $\tan(\tan^{-1}x) = x$; etc.

58. By aid of the principles of § 57, we may derive from any formula involving direct functions a relation between inverse functions.

1. From the formula $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$, prove

$$\tan^{-1}a + \tan^{-1}b = \tan^{-1}\frac{a+b}{1-ab}.$$

Let $\qquad \tan x = a,$ and $\tan y = b.$

Then by § 57, $\qquad x = \tan^{-1}a,$ and $y = \tan^{-1}b.$

Substituting these values in the given formula,

$$\tan(\tan^{-1}a + \tan^{-1}b) = \frac{a+b}{1-ab}.$$

Whence, $$\tan^{-1}a + \tan^{-1}b = \tan^{-1}\frac{a+b}{1-ab}.$$

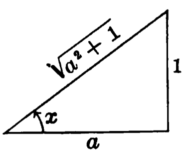
2. Prove the relation $\quad \cot^{-1}a - \sec^{-1}b = \cos^{-1}\dfrac{a+\sqrt{b^2-1}}{b\sqrt{a^2+1}}.$

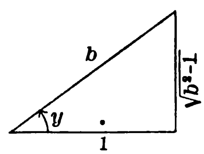
Let $\qquad \cot^{-1}a = x,$ and $\sec^{-1}b = y.$

Then, $\qquad \cot x = a,$ and $\sec y = b.$

Now, $$\cos(x-y) = \cos x \cos y + \sin x \sin y. \qquad \text{(A)}$$

To find the values of the sines and cosines of x and y, we may use the method of § 6.





In the right triangle containing the angle x, the adjacent side is a, and the opposite side 1; then, the hypotenuse is $\sqrt{a^2+1}$.

In the right triangle containing the angle y, the hypotenuse is b, and the adjacent side 1; then, the opposite side is $\sqrt{b^2-1}$.

Substituting the values of $\cos x$, $\cos y$, $\sin x$, and $\sin y$ in (A), we have

$$\cos(x-y) = \frac{a}{\sqrt{a^2+1}}\cdot\frac{1}{b} + \frac{1}{\sqrt{a^2+1}}\cdot\frac{\sqrt{b^2-1}}{b} = \frac{a+\sqrt{b^2-1}}{b\sqrt{a^2+1}}.$$

Whence, $\quad x - y$ or $\cot^{-1}a - \sec^{-1}b = \cos^{-1}\dfrac{a+\sqrt{b^2-1}}{b\sqrt{a^2+1}}.$

EXAMPLES.

3. From the formula $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$, prove
$$2 \cot^{-1} a = \cot^{-1} \frac{a^2 - 1}{2a}.$$

4. From the formula $\cos 2x = 1 - 2 \sin^2 x$, prove
$$2 \sin^{-1} a = \cos^{-1} (1 - 2a^2).$$

5. From the formula $\sin 2x = 2 \sin x \cos x$, prove
$$2 \cos^{-1} a = \sin^{-1} (2a\sqrt{1 - a^2}).$$

6. From the formula $\cos (x + y) = \cos x \cos y - \sin x \sin y$, prove
$$\cos^{-1} a + \cos^{-1} b = \cos^{-1} (ab - \sqrt{1 - a^2}\sqrt{1 - b^2}).$$

7. From the formula $\sin 3x = 3 \sin x - 4 \sin^3 x$, prove
$$3 \sin^{-1} a = \sin^{-1} (3a - 4a^3).$$

Prove the following relations:

8. $\cot^{-1} a + \cot^{-1} b = \cot^{-1} \frac{ab - 1}{a + b}.$

9. $2 \cos^{-1} a = \cos^{-1} (2a^2 - 1).$

10. $\sin^{-1} a - \sin^{-1} b = \sin^{-1} (a\sqrt{1 - b^2} - b\sqrt{1 - a^2}).$

11. $3 \tan^{-1} a = \tan^{-1} \frac{3a - a^3}{1 - 3a^2}.$

12. $\cot^{-1} (a - b) - \cot^{-1} (a + b) = \cot^{-1} \frac{a^2 - b^2 + 1}{2b}.$

13. $\sin^{-1} a + \cos^{-1} b = \tan^{-1} \frac{ab + \sqrt{1 - a^2}\sqrt{1 - b^2}}{b\sqrt{1 - a^2} - a\sqrt{1 - b^2}}.$

14. $\sec^{-1} a - \csc^{-1} b = \cos^{-1} \frac{\sqrt{a^2 - 1} + \sqrt{b^2 - 1}}{ab}.$

15. $\tan^{-1} a + \cos^{-1} \frac{1}{a} = \sin^{-1} \frac{a + \sqrt{a^2 - 1}}{a\sqrt{a^2 + 1}}.$

16. $\tan^{-1} \frac{a}{a - 1} - \tan^{-1} \frac{a + 1}{a} = \tan^{-1} \frac{1}{2a^2}.$

17. $2 \sin^{-1} a = \tan^{-1} \frac{2a\sqrt{1 - a^2}}{1 - 2a^2}.$

18. $\tan^{-1} a + 2 \tan^{-1} b = \tan^{-1} \frac{a(1 - b^2) + 2b}{1 - b^2 - 2ab}.$

59. The following table expresses the value of each of the six principal functions of an angle in terms of the other five:

$\sin A$	......	$\sqrt{1-\cos^2 A}$	$\dfrac{\tan A}{\sqrt{1+\tan^2 A}}$	$\dfrac{1}{\sqrt{1+\cot^2 A}}$	$\dfrac{\sqrt{\sec^2 A-1}}{\sec A}$	$\dfrac{1}{\csc A}$
$\cos A$	$\sqrt{1-\sin^2 A}$	......	$\dfrac{1}{\sqrt{1+\tan^2 A}}$	$\dfrac{\cot A}{\sqrt{1+\cot^2 A}}$	$\dfrac{1}{\sec A}$	$\dfrac{\sqrt{\csc^2 A-1}}{\csc A}$
$\tan A$	$\dfrac{\sin A}{\sqrt{1-\sin^2 A}}$	$\dfrac{\sqrt{1-\cos^2 A}}{\cos A}$	......	$\dfrac{1}{\cot A}$	$\sqrt{\sec^2 A-1}$	$\dfrac{1}{\sqrt{\csc^2 A-1}}$
$\cot A$	$\dfrac{\sqrt{1-\sin^2 A}}{\sin A}$	$\dfrac{\cos A}{\sqrt{1-\cos^2 A}}$	$\dfrac{1}{\tan A}$	......	$\dfrac{1}{\sqrt{\sec^2 A-1}}$	$\sqrt{\csc^2 A-1}$
$\sec A$	$\dfrac{1}{\sqrt{1-\sin^2 A}}$	$\dfrac{1}{\cos A}$	$\sqrt{1+\tan^2 A}$	$\dfrac{\sqrt{1+\cot^2 A}}{\cot A}$	......	$\dfrac{\csc A}{\sqrt{\csc^2 A-1}}$
$\csc A$	$\dfrac{1}{\sin A}$	$\dfrac{1}{\sqrt{1-\cos^2 A}}$	$\dfrac{\sqrt{1+\tan^2 A}}{\tan A}$	$\sqrt{1+\cot^2 A}$	$\dfrac{\sec A}{\sqrt{\sec^2 A-1}}$	......

The reciprocal forms were proved in § 36.

The others may be derived by aid of §§ 36, 37, 38, 39, and 40, and are left as exercises for the student.

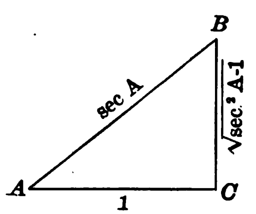
As an illustration, we will give a proof of the formula

$$\cos A = \frac{\sqrt{\csc^2 A - 1}}{\csc A}.$$

By § 39, $\cos A = \sqrt{1-\sin^2 A} = \sqrt{1 - \dfrac{1}{\csc^2 A}} = \dfrac{\sqrt{\csc^2 A - 1}}{\csc A}.$

They may also be conveniently proved by the method of § 6; thus, let it be required to prove the formula for each of the other functions in terms of the secant.

We have $\qquad \sec A = \dfrac{\sec A}{1}.$



Since the secant is the ratio of the hypotenuse to the adjacent side, we take $AB = \sec A$, and $AC = 1$; whence, $BC = \sqrt{\overline{AB}^2 - \overline{AC}^2} = \sqrt{\sec^2 A - 1}.$

Then by definition,

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}, \qquad \tan A = \sqrt{\sec^2 A - 1}, \qquad \csc A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}.$$

$$\cos A = \frac{1}{\sec A}, \qquad \cot A = \frac{1}{\sqrt{\sec^2 A - 1}},$$

60. Line Values of the Functions.

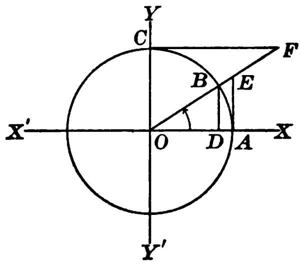


Fig. 1.

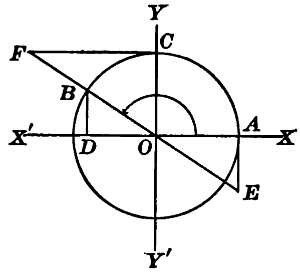


Fig. 2.

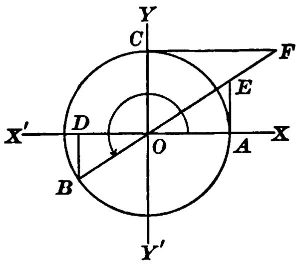


Fig. 3.

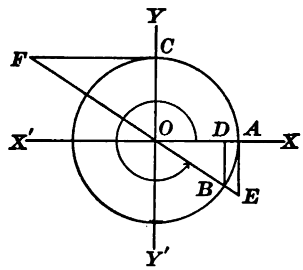


Fig. 4.

Let AOB be any angle. With O as a centre, and a radius equal to 1, describe the circle AB; draw BD and AE perpendicular to XX', and CF perpendicular to YY'. Then by § 17, the functions of AOB are:

	Sin.	*Cos.*	*Tan.*	*Cot.*	*Sec.*	*Csc.*
Fig. 1.	$\frac{BD}{OB}$	$\frac{OD}{OB}$	$\frac{BD}{OD}$	$\frac{OD}{BD}$	$\frac{OB}{OD}$	$\frac{OB}{BD}$
Fig. 2.	$\frac{BD}{OB}$	$-\frac{OD}{OB}$	$-\frac{BD}{OD}$	$-\frac{OD}{BD}$	$-\frac{OB}{OD}$	$\frac{OB}{BD}$
Fig. 3.	$-\frac{BD}{OB}$	$-\frac{OD}{OB}$	$\frac{BD}{OD}$	$\frac{OD}{BD}$	$-\frac{OB}{OD}$	$-\frac{OB}{BD}$
Fig. 4.	$-\frac{BD}{OB}$	$\frac{OD}{OB}$	$-\frac{BD}{OD}$	$-\frac{OD}{BD}$	$\frac{OB}{OD}$	$-\frac{OB}{BD}$

But since the right triangles OBD, OEA, and OCF are similar, and $OA = OC = 1$, we have

$$\frac{BD}{OD} = \frac{AE}{OA} = AE, \qquad \frac{OB}{OD} = \frac{OE}{OA} = OE,$$

$$\frac{OD}{BD} = \frac{CF}{OC} = CF, \qquad \frac{OB}{BD} = \frac{OF}{OC} = OF.$$

Whence, since $OB = 1$, the functions of AOB are:

	Sin.	*Cos.*	*Tan.*	*Cot.*	*Sec.*	*Csc.*
Fig. 1.	BD	OD	AE	CF	OE	OF
Fig. 2.	BD	$-OD$	$-AE$	$-CF$	$-OE$	OF
Fig. 3.	$-BD$	$-OD$	AE	CF	$-OE$	$-OF$
Fig. 4.	$-BD$	OD	$-AE$	$-CF$	OE	$-OF$

That is, *if the radius of the circle is* 1,

The *sine* is the perpendicular drawn to XX' from the intersection of the circle with the terminal line.

The *cosine* is the line drawn from the centre to the foot of the sine.

The *tangent* is that portion of the geometrical tangent to the circle at its intersection with OX included between OX and the terminal line, produced if necessary.

The *cotangent* is that portion of the geometrical tangent to the circle at its intersection with OY included between OY and the terminal line, produced if necessary.

The *secant* is that portion of the terminal line, or terminal line produced, included between the centre and the tangent.

The *cosecant* is that portion of the terminal line, or terminal line produced, included between the centre and the cotangent.

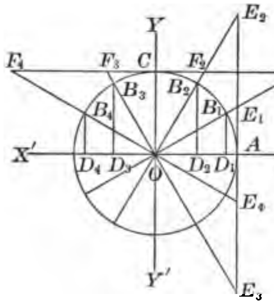
And with regard to algebraic signs,

Sines and tangents measured *above* XX' are *positive*, and *below*, *negative;* cosines and cotangents measured to the *right* of YY' are *positive*, and to the *left*, *negative;* secants and cosecants measured on the terminal line itself are *positive*, and on the terminal line *produced*, *negative.*

The above are called the *line values* of the trigonometric functions.

They simply *represent* the values of the functions when the radius is 1; that is, the *numerical value* of the sine of an angle is the same as the *number* which expresses the length of the perpendicular drawn to XX' from the intersection of the circle and terminal line.

61. *To trace the changes in the six principal trigonometric functions of an angle as the angle increases from* 0° *to* 360°.



Let the terminal line start from the position OA, and revolve about the point O as a pivot, in a direction contrary to the motion of the hands of a clock.

Then since the sine of the angle commences with the value 0, and assumes in succession the values B_1D_1, B_2D_2, OC, B_3D_3, B_4D_4, etc. (§ 60), it is evident that, as the angle increases from 0° to 90°, the sine increases from 0 to 1; from 90° to 180°, it decreases from 1 to 0; from 180° to 270°, it decreases (algebraically) from 0 to -1; and from 270° to 360°, it increases from -1 to 0.

Since the cosine commences with the value OA, and assumes in succession the values OD_1, OD_2, 0, $-OD_3$, $-OD_4$, etc., from 0° to 90°, it decreases from 1 to 0; from 90° to 180°, it decreases from 0 to -1; from 180° to 270°, it increases from -1 to 0; and from 270° to 360°, it increases from 0 to 1.

Since the tangent commences with the value 0, and assumes in succession the values AE_1, AE_2, ∞, $-AE_3$, $-AE_4$, etc., from 0° to 90°, it increases from 0 to ∞; from 90° to 180°, it increases from $-\infty$ to 0; from 180° to 270°, it increases from 0 to ∞; and from 270° to 360°, it increases from $-\infty$ to 0.

Since the cotangent commences at ∞, and assumes in succession the values CF_1, CF_2, 0, $-CF_3$, $-CF_4$, etc., from 0° to 90°, it decreases from ∞ to 0; from 90° to 180°, it decreases from 0 to $-\infty$; from 180° to 270°, it decreases from ∞ to 0; and from 270° to 360°, it decreases from 0 to $-\infty$.

Since the secant commences with the value OA, and assumes in succession the values OE_1, OE_2, ∞, $-OE_3$, $-OE_4$, etc., from 0° to 90°, it increases from 1 to ∞; from 90° to 180°, it increases from $-\infty$ to -1; from 180° to 270°, it decreases from -1 to $-\infty$; and from 270° to 360°, it decreases from ∞ to 1.

Since the cosecant commences at ∞, and assumes in succession the values OF_1, OF_2, OC, OF_3, OF_4, etc., from 0° to 90°, it decreases from ∞ to 1; from 90° to 180°, it increases from 1 to ∞; from 180° to 270°, it increases from $-\infty$ to -1; and from 270° to 360°, it decreases from -1 to $-\infty$.

Note. Wherever the symbol ∞ occurs in the above discussion, it must be interpreted as explained in the Note to § 25.

62. Trigonometric Equations.

1. Find the value of A when $\cos A = \frac{1}{2}$.

We know that one value of A is 60° (§ 8).

And since $\cos(-60°) = \cos 60°$ (§ 29), another value of A is $-60°$.

Again, by the principle of § 21, any multiple of 360° may be added to, or subtracted from, an angle, without altering its functions.

Hence, other values of A are

$$360°+60°,\ 720°+60°,\ -360°+60°,\ 360°-60°,\ 720°-60°,\ -360°-60°, \text{ etc.}$$

It is evident from the above that the number of possible values of A is indefinitely great; and that each is in the form

$$n \times 360° + 60°, \text{ or } n \times 360° - 60°;$$

where n is 0, or any positive or negative integer.

Using the circular notation, we have

$$A = n \times 2\pi \pm \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}.$$

2. Find the value of A when $\tan A = \frac{1}{3}\sqrt{3}$.

We know that one value of A is 30° (§ 8); another is $180°+30°$ (§ 27).

Adding to, and subtracting from, these angles multiples of 360°, other values of A are

$$360° + 30°,\ 540° + 30°,\ -360° + 30°,\ -180° + 30°, \text{ etc.}$$

It is evident from the above that all the values of A are given by the expression

$$n \times 180° + 30°;$$

where n is 0, or any positive or negative integer.

Or, $$A = n\pi + \frac{\pi}{6}.$$

3. Find the value of A when $\sin A = \frac{1}{2}\sqrt{2}$.

One value of A is 45° (§ 7).

And since $\sin(180° - 45°) = \sin 45°$ (§ 33), another value of A is $180° - 45°$.

Adding to, and subtracting from, these angles multiples of 360°, other values of A are

$$360° + 45°,\ 540° - 45°,\ -360° + 45°,\ -180° - 45°, \text{ etc.}$$

It is evident from the above that all the values of A are given by the expression

$$n \times 180° + (-1)^n 45°;$$

where n is 0, or any positive or negative integer.

Or, $$A = n\pi + (-1)^n \frac{\pi}{4}.$$

It is evident that, to find the value of A in any equation of the above forms, we find *any one of the values of* A, and substitute it for A in the following expressions:

If $\sin A$ is given, $n\pi + (-1)^n A$.

If $\cos A$ is given, $2n\pi \pm A$.

If $\tan A$ is given, $n\pi + A$.

The rule for equations giving the value of $\cot A$ is the same as for $\tan A$; for $\sec A$, the same as for $\cos A$; and for $\csc A$, the same as for $\sin A$.

EXAMPLES.

In each of the following find the value of A:

4. $\tan A = \sqrt{3}$.
6. $\sin A = \frac{1}{2}$.
8. $\cot A = -1$.
10. $\cot A = 0$.
5. $\cos A = -\frac{1}{2}\sqrt{3}$.
7. $\sec A = \sqrt{2}$.
9. $\csc A = -\frac{2}{3}\sqrt{3}$.
11. $\sec A = -1$.

63. **1.** Solve the equation $\cos 2A = \cos A$.

By (**28**), $2\cos^2 A - 1 = \cos A$, or $2\cos^2 A - \cos A = 1$.

Solving this equation, $\cos A = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} = 1$ or $-\frac{1}{2}$.

If $\cos A = 1$, one value of A is 0° (§ 22), and $A = 2n\pi$ (§ 62).

If $\cos A = -\frac{1}{2}$, one value of A is 120° (§ 27), and $A = 2n\pi \pm \frac{2\pi}{3}$.

2. Solve the equation $\tan 2x = 6\tan x$.

By (**29**), $$\frac{2\tan x}{1 - \tan^2 x} = 6\tan x. \qquad \text{(A)}$$

One solution is evidently $\tan x = 0$.

In this case, one value of x is 0°, and $x = n\pi$ (§ 62).

Dividing (A) by $2 \tan x$, we have

$$\frac{1}{1-\tan^2 x} = 3, \text{ or } 1 = 3 - 3\tan^2 x, \text{ or } 3\tan^2 x = 2.$$

Whence, $\tan^2 x = \frac{2}{3}$, or $\tan x = \pm\sqrt{\frac{2}{3}} = \pm\frac{1}{3}\sqrt{6}$.

Therefore, $x = \tan^{-1}(\pm\frac{1}{3}\sqrt{6}) = \pm\tan^{-1}(\frac{1}{3}\sqrt{6})$.

EXAMPLES.

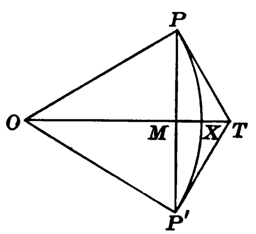
In each of the following find the value of x:

3. $\sin x = \sin 2x$.
4. $\sin 2x + \cos x = 0$.
5. $\cos x + \cos 3x = 0$.
6. $\tan 3x + \tan x = 0$.
7. $\cot 2x + \cot x = 0$.
8. $\tan(45° - x) + \cot(45° - x) = 4$.
9. $\tan 3x = 5\tan x$.
10. $\cos x \cot x = 1$.

64. Limiting Values of $\frac{\sin x}{x}$ and $\frac{\tan x}{x}$.

To find the limiting values of the fractions $\frac{\sin x}{x}$ *and* $\frac{\tan x}{x}$ *when* x *is indefinitely decreased.*

Note. We suppose x to be expressed in *circular measure* (§ 52).



Let $OPXP'$ be a sector of a circle.

Draw PT and $P'T$ tangent to the arc at P and P', and join OT and PP'.

By Geometry, $PT = P'T$.

Then OT is perpendicular to PP' at its middle point M, and bisects the arc PP' at X.

Let $\angle XOP = \angle XOP' = x$.

By Geometry, arc $PP' >$ chord PP', and $< PTP'$.

Whence, arc $PX > PM$, and $< PT$.

Therefore, $\frac{\text{arc } PX}{OP} > \frac{PM}{OP}$, and $< \frac{PT}{OP}$.

Or by § 52, circ. meas. $x > \sin x$, and $< \tan x$.

Representing the circular measure of x by x simply, and dividing through by $\sin x$, we have

$$\frac{x}{\sin x} > 1, \text{ and } < \frac{\tan x}{\sin x} \text{ or } \frac{1}{\cos x}.$$

Whence, $$\frac{\sin x}{x} < 1, \text{ and } > \cos x.$$

But when x is indefinitely decreased, $\cos x$ approaches the limit 1 (§ 22).

Hence, $\frac{\sin x}{x}$ approaches the limit 1 when x is indefinitely decreased.

Again, $$\frac{\tan x}{x} = \frac{\sin x}{x \cos x} = \frac{\sin x}{x} \times \frac{1}{\cos x}.$$

But $\frac{\sin x}{x}$ and $\frac{1}{\cos x}$ approach the limit 1 when x is indefinitely decreased.

Hence, $\frac{\tan x}{x}$ approaches the limit 1 when x is indefinitely decreased.

V. LOGARITHMS.

65. Every positive number may be expressed, exactly or approximately, as a power of 10.

Thus, $100 = 10^2$; $13 = 10^{1.113943\cdots}$; etc.

When thus expressed, the corresponding exponent is called its **Logarithm to the Base 10.**

Thus, 2 is the logarithm of 100 to the base 10; a relation which is written $\log_{10} 100 = 2$, or simply $\log 100 = 2$.

66. Logarithms of numbers to the base 10 are called *Common Logarithms*, and, collectively, form the *Common System.*

They are the only ones used for numerical computations.

Any positive number, except unity, may be taken as the base of a system of logarithms; thus, if $a^x = m$, where a and m are positive numbers, then $x = \log_a m$.

Note. A negative number is not considered as having a logarithm.

67. We have by Algebra,

$$10^0 = 1, \qquad 10^{-1} = \frac{1}{10} = .1,$$

$$10^1 = 10, \qquad 10^{-2} = \frac{1}{10^2} = .01,$$

$$10^2 = 100, \qquad 10^{-3} = \frac{1}{10^3} = .001, \text{ etc.}$$

Whence by the definition of § 65,

$$\log 1 = 0, \qquad \log .1 = -1 = 9 - 10,$$

$$\log 10 = 1, \qquad \log .01 = -2 = 8 - 10,$$

$$\log 100 = 2, \qquad \log .001 = -3 = 7 - 10, \text{ etc.}$$

Note. The second form for log .1, log .01, etc., is preferable in practice. If no base is expressed, the base 10 is understood.

68. It is evident from § 67 that the logarithm of a number greater than 1 is positive, and the logarithm of a number between 0 and 1 negative.

69. If a number is not an exact power of 10, its common logarithm can only be expressed approximately.

The integral part of the logarithm is called the *characteristic*, and the decimal part the *mantissa.*

For example, $\log 13 = 1.113943.$

In this case, the characteristic is 1, and the mantissa .113943.

For reasons which will appear hereafter, only the mantissa of the logarithm is given in a table of logarithms of numbers; the characteristic must be found by aid of the rules of §§ 70 and 71.

70. It is evident from § 67 that the logarithm of a number between

1 and 10 is equal to $0 +$ a decimal;
10 and 100 is equal to $1 +$ a decimal;
100 and 1000 is equal to $2 +$ a decimal; etc.

Therefore, the characteristic of the logarithm of a number with *one* figure to the left of the decimal point, is 0; with *two* figures to the left of the decimal point, is 1; with *three* figures to the left of the decimal point, is 2; etc.

Hence, *the characteristic of the logarithm of a number greater than* 1 *is* 1 *less than the number of places to the left of the decimal point.*

For example, the characteristic of log 906328.51 is 5.

71. In like manner, the logarithm of a number between

1 and .1 is equal to $9 +$ a decimal $- 10$;
.1 and .01 is equal to $8 +$ a decimal $- 10$;
.01 and .001 is equal to $7 +$ a decimal $- 10$; etc.

Therefore, the characteristic of the logarithm of a decimal with *no* ciphers between its decimal point and first significant figure, is 9, with -10 after the mantissa; of a decimal with *one* cipher between its point and first significant figure is 8, with -10 after the mantissa; of a decimal with *two* ciphers between its point and first significant figure is 7, with -10 after the mantissa; etc.

Hence, *to find the characteristic of the logarithm of a number less than* 1, *subtract the number of ciphers between the decimal point and first significant figure from* 9, *writing* -10 *after the mantissa.*

For example, the characteristic of log .007023 is 7, with -10 written after the mantissa.

Note. Some writers combine the two portions of the characteristic, and write the result as a *negative characteristic* before the mantissa.

Thus, instead of $7.603658 - 10$, the student will frequently find $\bar{3}.603658$, a minus sign being written over the characteristic to denote that it alone is negative, the mantissa being always positive.

PROPERTIES OF LOGARITHMS.

72. *In any system, the logarithm of* 1 *is* 0.

For by Algebra, $a^0 = 1$; whence by § 66, $\log_a 1 = 0$.

73. *In any system, the logarithm of the base is* 1.

For $a^1 = a$; whence, $\log_a a = 1$.

74. *In any system whose base is greater than* 1, *the logarithm of* 0 *is* $-\infty$.

For if a is greater than 1, $a^{-\infty} = \frac{1}{a^\infty} = \frac{1}{\infty} = 0$.

Whence by § 66, $\log_a 0 = -\infty$.

Note. No literal meaning can be attached to such a result as $\log_a 0 = -\infty$; it must be interpreted as follows:

If, in any system whose base is greater than unity, a number approaches the limit 0, its logarithm is negative, and increases without limit in absolute value.

75. *In any system, the logarithm of a product is equal to the sum of the logarithms of its factors.*

Assume the equations

$$\left.\begin{array}{l} a^x = m \\ a^y = n \end{array}\right\}; \text{ whence by § 66, } \left\{\begin{array}{l} x = \log_a m, \\ y = \log_a n. \end{array}\right.$$

Multiplying the assumed equations,

$$a^x \times a^y = mn, \text{ or } a^{x+y} = mn.$$

Whence, $\qquad \log_a mn = x + y = \log_a m + \log_a n.$

In like manner, the theorem may be proved for the product of three or more factors.

76. By aid of § 75, the logarithm of a composite number may be found when the logarithms of its factors are known.

1. Given $\log 2 = .3010$ and $\log 3 = .4771$; find $\log 72$.

$$\begin{aligned} \log 72 &= \log(2 \times 2 \times 2 \times 3 \times 3) \\ &= \log 2 + \log 2 + \log 2 + \log 3 + \log 3 \text{ (§ 75)} \\ &= 3 \times \log 2 + 2 \times \log 3 = .9030 + .9542 = 1.8572. \end{aligned}$$

EXAMPLES.

Given $\log 2 = .3010$, $\log 3 = .4771$, $\log 5 = .6990$, $\log 7 = .8451$, find:

2. $\log 35$.	**6.** $\log 126$.	**10.** $\log 324$.	**14.** $\log 2625$.
3. $\log 50$.	**7.** $\log 196$.	**11.** $\log 378$.	**15.** $\log 6048$.
4. $\log 42$.	**8.** $\log 245$.	**12.** $\log 875$.	**16.** $\log 12005$.
5. $\log 75$.	**9.** $\log 210$.	**13.** $\log 686$.	**17.** $\log 15876$.

77. *In any system, the logarithm of a fraction is equal to the logarithm of the numerator minus the logarithm of the denominator.*

Assume the equations

$$\left.\begin{aligned} a^x &= m \\ a^y &= n \end{aligned}\right\}; \text{ whence, } \left\{\begin{aligned} x &= \log_a m, \\ y &= \log_a n. \end{aligned}\right.$$

Dividing the assumed equations,

$$\frac{a^x}{a^y} = \frac{m}{n}, \text{ or } a^{x-y} = \frac{m}{n}.$$

Whence,
$$\log_a \frac{m}{n} = x - y = \log_a m - \log_a n.$$

78. 1. Given $\log 2 = .3010$; find $\log 5$.

$$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 \text{ (§ 77)} = 1 - .3010 = .6990.$$

EXAMPLES.

Given $\log 2 = .3010$, $\log 3 = .4771$, $\log 7 = .8451$, find:

2. $\log \frac{10}{3}$.	**5.** $\log 14\frac{2}{7}$.	**8.** $\log \frac{48}{25}$.	**11.** $\log 28\frac{4}{5}$.
3. $\log \frac{7}{4}$.	**6.** $\log \frac{49}{27}$.	**9.** $\log 6\frac{2}{9}$.	**12.** $\log \frac{200}{9}$.
4. $\log 45$.	**7.** $\log 225$.	**10.** $\log 135$.	**13.** $\log 110\frac{1}{4}$.

79. *In any system, the logarithm of any power of a quantity is equal to the logarithm of the quantity multiplied by the exponent of the power.*

Assume the equation $a^x = m$; whence, $x = \log_a m$.

Raising both members of the assumed equation to the pth power,

$$a^{px} = m^p; \text{ whence, } \log_a m^p = px = p \log_a m.$$

80. *In any system, the logarithm of any root of a quantity is equal to the logarithm of the quantity divided by the index of the root.*

For, $$\log_a \sqrt[r]{m} = \log_a (m^{\frac{1}{r}}) = \frac{1}{r} \log_a m \text{ (§ 79).}$$

81. 1. Given $\log 2 = .3010$; find $\log 2^{\frac{5}{3}}$.

$$\log 2^{\frac{5}{3}} = \frac{5}{3} \times \log 2 = \frac{5}{3} \times .3010 = .5017.$$

Note. To multiply a logarithm by a fraction, multiply first by the numerator, and divide the result by the denominator.

2. Given $\log 3 = .4771$; find $\log \sqrt[8]{3}$.

$$\log \sqrt[8]{3} = \frac{\log 3}{8} = \frac{.4771}{8} = .0596.$$

EXAMPLES.

Given $\log 2 = .3010$, $\log 3 = .4771$, $\log 7 = .8451$, find:

3. $\log 3^7$.	**6.** $\log 28^6$.	**9.** $\log \sqrt[7]{2}$.	**12.** $\log \sqrt[5]{525}$.
4. $\log 5^{\frac{5}{2}}$.	**7.** $\log 18^{\frac{5}{6}}$.	**10.** $\log \sqrt[6]{5}$.	**13.** $\log \sqrt[4]{294}$.
5. $\log 7^{\frac{3}{4}}$.	**8.** $\log 96^{\frac{2}{7}}$.	**11.** $\log \sqrt[9]{7}$.	**14.** $\log \sqrt[8]{216}$.

15. Find $\log (2^{\frac{1}{3}} \times 3^{\frac{5}{4}})$.

By § 75, $\log (2^{\frac{1}{3}} \times 3^{\frac{5}{4}}) = \log 2^{\frac{1}{3}} + \log 3^{\frac{5}{4}} = \frac{1}{3} \log 2 + \frac{5}{4} \log 3$
$= .1003 + .5964 = .6967.$

Find the values of the following:

16. $\log \sqrt[11]{\frac{7}{2}}$.	**18.** $\log (2^{\frac{2}{3}} \times 10^{\frac{1}{2}})$.	**20.** $\log \frac{\sqrt[4]{5}}{\sqrt[5]{3}}$.	**22.** $\log \frac{3^{\frac{5}{3}}}{\sqrt{24}}$.
17. $\log \left(\frac{7}{5}\right)^{\frac{1}{4}}$.	**19.** $\log 7\sqrt[10]{2}$.	**21.** $\log \frac{3^{\frac{4}{3}}}{7^{\frac{1}{4}}}$.	**23.** $\log \frac{\sqrt[3]{63}}{5^{\frac{4}{5}}}$.

82. *To prove the relation*

$$\log_b m = \frac{\log_a m}{\log_a b}.$$

Assume the equations

$$\left.\begin{aligned} a^x &= m \\ b^y &= m \end{aligned}\right\}; \text{ whence, } \left\{\begin{aligned} x &= \log_a m, \\ y &= \log_b m. \end{aligned}\right.$$

From the assumed equations, $a^x = b^y$.

Taking the yth root of both members, $a^{\frac{x}{y}} = b$.

Therefore, $$\log_a b = \frac{x}{y}, \text{ or } y = \frac{x}{\log_a b}.$$

That is, $$\log_b m = \frac{\log_a m}{\log_a b}.$$

83. *To prove the relation*

$$\log_b a \times \log_a b = 1.$$

Putting $m = a$ in the result of § 82, we have

$$\log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b} \text{ (§ 73).}$$

Whence, $$\log_b a \times \log_a b = 1.$$

84. *In the common system, the mantissæ of the logarithms of numbers having the same sequence of figures are equal.*

Suppose, for example, that $\log 3.053 = .484727$.

Then, $$\log 305.3 = \log (100 \times 3.053) = \log 100 + \log 3.053$$
$$= 2 + .484727 = 2.484727;$$
$$\log .03053 = \log (.01 \times 3.053) = \log .01 + \log 3.053$$
$$= 8 - 10 + .484727 = 8.484727 - 10; \text{ etc.}$$

It is evident from the above that, if a number be multiplied or divided by any integral power of 10, producing another number with the same sequence of figures, the mantissæ of their logarithms will be equal.

The reason will now be seen for the statement made in § 69, that only the mantissæ are given in a table of logarithms of numbers.

For, to find the logarithm of any number, we have only to take from the table the mantissa corresponding to its sequence of figures, and the characteristic may then be prefixed in accordance with the rules of §§ 70 or 71.

Thus, if $\log 3.053 = .484727$, then

$$\log 30.53 = 1.484727, \qquad \log .3053 = 9.484727 - 10,$$
$$\log 305.3 = 2.484727, \qquad \log .03053 = 8.484727 - 10,$$
$$\log 3053. = 3.484727, \qquad \log .003053 = 7.484727 - 10, \text{ etc.}$$

This property is only enjoyed by the common system of logarithms, and constitutes its superiority over others for the purposes of numerical computation.

85. 1. Given $\log 2 = .3010$, $\log 3 = .4771$; find $\log .00432$.

We have, $\log 432 = \log(2^4 \times 3^3) = 4 \log 2 + 3 \log 3 = 2.6353$.

Then by § 84, the *mantissa* of the result is .6353.

Whence by § 71, $\log .00432 = 7.6353 - 10$.

EXAMPLES.

Given $\log 2 = .3010$, $\log 3 = .4771$, $\log 7 = .8451$, find:

2. $\log 3.6$.	**6.** $\log .00343$.	**10.** $\log .1944$.
3. $\log 11.2$.	**7.** $\log 2880$.	**11.** $\log 202.5$.
4. $\log .84$.	**8.** $\log .0392$.	**12.** $\log \sqrt[7]{6.4}$.
5. $\log .098$.	**9.** $\log .000405$.	**13.** $\log (14.7)^{\frac{2}{3}}$.

USE OF THE TABLE OF LOGARITHMS OF NUMBERS.

(For directions as to the use of the Table of Logarithms of Numbers, see pages iii to v of the Introduction to the Author's Six Place Logarithmic Tables.)

EXAMPLES.

86. Find the logarithms of the following numbers:

1. .053.	**5.** 336.908.	**9.** .001030746.
2. 51.8.	**6.** .000602851.	**10.** .00000876092.
3. .2956.	**7.** 65000.63.	**11.** 730407.8.
4. 1.0274.	**8.** 9122.55.	**12.** .0000436927.

Find the numbers corresponding to the following logarithms:

13. 1.880814.	**17.** 8.044891 − 10.	**21.** 3.990191.
14. 9.470410 − 10.	**18.** 2.270293.	**22.** 5.670180.
15. 0.820204.	**19.** 7.350064 − 10.	**23.** 6.535003 − 10.
16. 4.745126.	**20.** 5.000027 − 10.	**24.** 4.115658 − 10.

APPLICATIONS.

87. The approximate value of an arithmetical quantity, in which the operations indicated involve only multiplication, division, involution, or evolution, may be conveniently found by logarithms.

The utility of the process consists in the fact that addition takes the place of multiplication, subtraction of division, multiplication of involution, and division of evolution.

Note. In computations with six-place logarithms, the results cannot usually be depended upon to more than *six* significant figures.

88. 1. Find the value of $.0631 \times 7.208 \times .51272$.

By § 75, $\log(.0631 \times 7.208 \times .51272) = \log .0631 + \log 7.208 + \log .51272$.

$$\begin{aligned} \log .0631 &= 8.800029 - 10 \\ \log 7.208 &= 0.857815 \\ \log .51272 &= \underline{9.709880 - 10} \end{aligned}$$

Adding, $\log$ of result $= 19.367724 - 20 = 9.367724 - 10$. (See Note 1.)

Number corresponding to $9.367724 - 10 = .233197$.

Note 1. If the sum is a negative logarithm, it should be written in such a form that the negative portion of the characteristic may be -10.

Thus, $19.367724 - 20$ is written in the form $9.367724 - 10$.

2. Find the value of $\dfrac{336.852}{7980.04}$.

By § 77, $$\log\frac{336.852}{7980.04} = \log 336.852 - \log 7980.04.$$

$$\begin{aligned} \log 336.852 &= 12.527439 - 10 \quad \text{(See Note 2.)} \\ \log 7980.04 &= \underline{3.902005} \end{aligned}$$

Subtracting, $\log$ of result $= 8.625434 - 10$

Number corresponding $= .0422118$.

Note 2. To subtract a greater logarithm from a less, or to subtract a negative logarithm from a positive, increase the characteristic of the minuend by 10, writing -10 after the mantissa to compensate.

Thus, to subtract 3.902005 from 2.527439, write the minuend in the form $12.527439 - 10$; subtracting 3.902005 from this, the result is $8.625434 - 10$.

3. Find the value of $(.0980937)^5$.

By § 79, $\log(.0980937)^5 = 5 \times \log .0980937$.

$$\begin{aligned} \log .0980937 &= 8.991641 - 10 \\ &\underline{5} \\ &44.958205 - 50 = 4.958205 - 10. \quad \text{(See Note 1.)} \end{aligned}$$

Number corresponding $= .0000090825$.

4. Find the value of $\sqrt[3]{.035063}$.

By § 80, $\log \sqrt[3]{.035063} = \frac{1}{3} \log .035063$.

$$\begin{aligned} \log .035063 &= 8.544849 - 10 \\ &\;3\underline{)28.544849 - 30} \quad \text{(See Note 3.)} \\ &9.514950 - 10 \end{aligned}$$

Number corresponding $= .327303$.

Note 3. To divide a negative logarithm, write it in such a form that the negative portion of the characteristic may be exactly divisible by the divisor, with -10 as the quotient.

Thus, to divide $8.544849 - 10$ by 3, we write the logarithm in the form $28.544849 - 30$; dividing this by 3, the quotient is $9.514950 - 10$.

89. Arithmetical Complement.

The *Arithmetical Complement* of the logarithm of a number, or, briefly, the *Cologarithm* of the number, is the logarithm of the reciprocal of that number.

Thus,
$$\text{colog}\,409 = \log\frac{1}{409} = \log 1 - \log 409.$$
$$\begin{aligned} \log 1 &= 10. \qquad -10 \text{ (Note 2, § 88.)} \\ \log 409 &= \underline{2.611723} \end{aligned}$$
Then,
$$\text{colog}\,409 = 7.388277 - 10.$$
Again,
$$\text{colog}\,.067 = \log\frac{1}{.067} = \log 1 - \log .067.$$
$$\begin{aligned} \log 1 &= 10. \qquad -10 \\ \log .067 &= \underline{8.826075 - 10} \end{aligned}$$
Then,
$$\text{colog}\,.067 = 1.173925.$$

It follows from the above that *the cologarithm of a number may be found by subtracting its logarithm from* $10 - 10$.

Note. The cologarithm may be obtained by subtracting the last *significant* figure of the logarithm from 10, and each of the others from 9, -10 being written after the result in the case of a positive logarithm.

90. *Example.* Find the value of $\dfrac{51.384}{8.709 \times .0946}$

$$\begin{aligned} \log\frac{51.384}{8.709 \times .0946} &= \log\left(51.384 \times \frac{1}{8.709} \times \frac{1}{.0946}\right) \\ &= \log 51.384 + \log\frac{1}{8.709} + \log\frac{1}{.0946} \\ &= \log 51.384 + \text{colog}\,8.709 + \text{colog}\,.0946. \end{aligned}$$

$$\begin{aligned} \log 51.384 &= 1.710828 \\ \text{colog}\,8.709 &= 9.060032 - 10 \\ \text{colog}\,.0946 &= \underline{1.024109} \\ & 1.794969 = \log 62.369. \end{aligned}$$

It is evident from the above example that the logarithm of a fraction whose terms are composed of factors may be found by the following rule:

Add together the logarithms of the factors of the numerator, and the cologarithms of the factors of the denominator.

Note. The value of the above fraction may be found without using cologarithms, by the following formula:

$$\log\frac{51.384}{8.709\times .0946}=\log 51.384-\log(8.709\times .0946)$$
$$=\log 51.384-(\log 8.709+\log .0946).$$

The advantage in the use of cologarithms is that the written work of computation is exhibited in a more compact form.

EXAMPLES.

Note. A *negative* quantity has no common logarithm (§ 66, Note).

If such quantities occur in computation, they should be treated as if they were positive, and the *sign* of the result determined irrespective of the logarithmic work.

Thus, in Ex. 2, § 91, the value of $84.759\times(-2280.76)$ is obtained by finding the value of 84.759×2280.76, and putting a negative sign before the result. See also Ex. 29.

91. Find by logarithms the values of the following:

1. $3.1425\times 603.93.$

2. $84.759\times(-2280.76).$

3. $(-4.39182)\times(-.0703968).$

4. $.936537\times .00117854.$

5. $\dfrac{4867.2}{765.16}.$

6. $\dfrac{1.05478}{34.9564}.$

7. $\dfrac{2.7085}{.0868097}.$

8. $\dfrac{-.000680239}{.00512643}.$

9. $\dfrac{3.89612\times .6946}{4694.9\times .00454}.$

10. $\dfrac{715\times(-.024158)}{(-.5157)\times 1420.63}.$

11. $\dfrac{(-.870284)\times 3.73}{(-.06585)\times(-42.317)}.$

12. $\dfrac{.082136\times(-73.39)}{.838\times 2808.72}.$

13. $(7.7954)^4.$

14. $(.83287)^7.$

15. $(-25.1437)^3.$

16. $(.01)^{\frac{3}{4}}.$

17. $(-964.38)^{\frac{4}{5}}.$

18. $(.0951293)^{\frac{5}{2}}.$

19. $(.000105936)^{\frac{5}{3}}.$

20. $\sqrt{5}.$

21. $\sqrt[5]{2}.$

22. $\sqrt[9]{-6}.$

23. $\sqrt[8]{100}.$

24. $\sqrt[4]{.19946}.$

25. $\sqrt[6]{.0725628}.$

26. $\sqrt[3]{.002613874}.$

27. $\sqrt[7]{-.000951735}.$

28. Find the value of $\dfrac{2\sqrt[3]{5}}{3^{\frac{5}{6}}}.$

By § 90,

$$\log\frac{2\sqrt[3]{5}}{3^{\frac{5}{6}}}=\log 2+\log\sqrt[3]{5}+\operatorname{colog}3^{\frac{5}{6}}=\log 2+\tfrac{1}{3}\log 5+\tfrac{5}{6}\operatorname{colog}3.$$

$$\begin{array}{lrl}
 & \log 2 = & .301030 \\
\log 5 = .698970; & \text{divide by } 3 = & .232990 \\
\operatorname{colog} 3 = 9.522879-10; & \text{multiply by } \tfrac{5}{6} = & \underline{9.602399-10} \\
 & & .136419=\log 1.36905.
\end{array}$$

29. Find the value of $\sqrt[3]{\dfrac{-.032956}{7.96183}}$.

$$\log\sqrt[3]{\frac{.032956}{7.96183}}=\tfrac{1}{3}\log\frac{.032956}{7.96183}=\tfrac{1}{3}(\log .032956-\log 7.96183).$$

$$\begin{aligned}\log .032956 &= 8.517934-10\\ \log 7.96183 &= 0.901013\end{aligned}$$

$$3\overline{)27.616921-30}$$

$$9.205640-10=\log .160561.$$

Result, $-.160561$.

Find the values of the following:

30. $4^{\frac{4}{3}}\times 7^{\frac{2}{3}}$.

31. $\dfrac{3^{\frac{5}{4}}}{8^{\frac{2}{3}}}$.

32. $\sqrt[10]{\dfrac{79}{46}}$.

33. $\dfrac{(.001)^{\frac{3}{2}}}{\sqrt[5]{7}}$.

34. $\dfrac{\sqrt{.08}}{(-10)^{\frac{8}{5}}}$.

35. $\left(-\dfrac{4400}{6927.7}\right)^{\frac{4}{5}}$.

36. $\sqrt{\dfrac{276.85}{940}}$.

37. $\dfrac{5^{\frac{7}{4}}}{\sqrt[3]{-.1}}$.

38. $\dfrac{-\sqrt[4]{1000}}{(-.6)^{\frac{4}{3}}}$.

39. $\sqrt[6]{\dfrac{3}{5}}\div\sqrt[5]{\dfrac{7}{8}}$.

40. $\sqrt[3]{3}\times\sqrt[5]{5}\times\sqrt[7]{7}$.

41. $\left(\dfrac{76.1\times .05929}{1.3073}\right)^{\frac{3}{4}}$.

42. $\sqrt[3]{-\dfrac{75.438}{31.4\times .4146}}$.

43. $\dfrac{\sqrt[4]{.000965782}}{\sqrt[3]{.00497836}}$.

44. $\dfrac{-(.256929)^{\frac{6}{5}}}{(-.834574)^{\frac{7}{3}}}$.

45. $(25.4673)^{10}\times(-.052)^{12}$.

46. $\sqrt[8]{5106.526\times .000031093}$.

47. $(837.48\times .00943246)^{\frac{2}{7}}$.

48. $(4.867184)^{\frac{7}{2}}\times(.175437)^{\frac{1}{5}}$

49. $\dfrac{\sqrt{3.9285}\times\sqrt[4]{65.4775}}{\sqrt[6]{721.329}}$.

50. $\dfrac{(.5732)^{\frac{3}{5}}}{8693.84\times\sqrt[4]{.033074}}$.

51. $\dfrac{(-.00019162)^{\frac{2}{3}}\times\sqrt{68.18}}{-.2755653}$.

52. $\dfrac{\sqrt[4]{.052866}}{\sqrt{.374}\times\sqrt[9]{.007835912}}$.

EXPONENTIAL EQUATIONS.

92. An **Exponential Equation** is an equation of the form $a^x=b$.

To solve an equation of this form, take the logarithms of both members.

1. Given $31^x=23$; find the value of x.

Taking the logarithms of both members,

$$\log(31^x)=\log 23.$$

Whence by § 79, $x\log 31=\log 23.$

Then, $x=\dfrac{\log 23}{\log 31}=\dfrac{1.361728}{1.491362}=.9130+.$

2. Given $.2^x = 3$; find the value of x.

Taking the logarithms of both members,

$$x \log .2 = \log 3.$$

Whence, $x = \dfrac{\log 3}{\log .2} = \dfrac{.477121}{9.301030 - 10} = \dfrac{.477121}{-.698970} = -.6826 +.$

EXAMPLES.

Solve the following equations:

3. $332.9^x = 5.178.$

4. $.4162^x = 6.724.$

5. $.0158^x = .0082958.$

6. $5.3364^x = .744.$

7. $a^x = b^{2x}c^5.$

8. $m^2a^{\frac{3}{x}} = n^4.$

9. $6^{2x-8} = .0277778.$

10. $.7^{x^2+4x} = .16807.$

93. 1. Find the logarithm of .3 to the base 7.

By § 82, $\log_7 .3 = \dfrac{\log_{10} .3}{\log_{10} 7} = \dfrac{9.477121 - 10}{.845098} = \dfrac{-.522879}{.845098} = -.6187 +.$

EXAMPLES.

Find the values of the following:

2. $\log_2 13.$

3. $\log_5 .9.$

4. $\log_{.74} 6.2.$

5. $\log_{.48} .087.$

6. $\log_{9.1} .362.$

7. $\log_{65} 4.3.$

Examples like the above may be solved by inspection if the number can be expressed as an exact power of the base.

8. Find the logarithm of 128 to the base 16.

Let $\log_{16} 128 = x$; then by § 66, $16^x = 128.$

That is, $(2^4)^x = 2^7$, or $2^{4x} = 2^7$.

Whence by inspection, $4x = 7$; and $x = \log_{16} 128 = \dfrac{7}{4}.$

9. Find the logarithm of 81 to the base 3.

10. Find the logarithm of 32 to the base 8.

11. Find the logarithm of $\frac{1}{3}$ to the base 27.

12. Find the logarithm of $\frac{1}{64}$ to the base $\frac{1}{32}$.

EXAMPLES IN THE USE OF TRIGONOMETRIC TABLES.

(For directions, see pages v to xi of the Introduction to the Author's Six Place Logarithmic Tables.)

94. Table of Logarithmic Sines, Cosines, etc.

Find the values of the following:

1. log sin $12° 48' 52''$.
2. log tan $67° 13' 27''$.
3. log cos $31° 5' 43''$.
4. log cot $53° 42' 9''$.
5. log cos $79° 54' 35''$.
6. log tan $8° 17' 21''$.
7. log cot $26° 30' 14''$.
8. log sec $45° 26' 38''$.
9. log csc $84° 9' 56''$.

Find the angles corresponding in the following:

10. log sin $= 9.934232 - 10$.
11. log cos $= 9.923569 - 10$.
12. log tan $= 0.806571$.
13. log cot $= 0.282956$.
14. log tan $= 9.184367 - 10$.
15. log cot $= 9.404692 - 10$.
16. log sec $= 0.188783$.
17. log csc $= 0.400314$.

95. Table of Natural Sines, Cosines, etc.

Find the values of the following:

1. sin $43° 17' 35''$.
2. cot $75° 50' 19''$.
3. cos $86° 21' 46''$.
4. tan $34° 48' 23''$.
5. sin $67° 9' 54''$.
6. cos $29° 35' 8''$.

Find the angles corresponding in the following:

7. tan $= 1.2622$. 8. cos $= .96376$. 9. sin $= .91527$. 10. cot $= 1.7927$

96. Auxiliary Table for Small Angles.

Find the values of the following:

1. log sin $1° 14' 53''$.
2. log tan $3° 42' 8''$.
3. log cot $2° 26' 35''$.

Find the angles corresponding in the following:

4. log sin $= 8.233459 - 10$.
5. log tan $= 7.859872 - 10$.
6. log cot $= 1.546267$.

VI. SOLUTION OF RIGHT TRIANGLES.

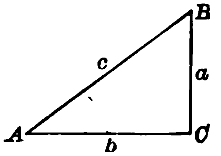
97. The *elements* of a triangle are its three sides and its three angles.

We know by Geometry that a triangle is, in general, completely determined when three of its elements are known, provided one of them is a side.

The *solution* of a triangle is the process of computing the unknown from the given elements.

98. To solve a *right triangle*, two elements must be given in addition to the right angle, one of which must be a side.

The various cases which can occur may all be solved by aid of the following formulæ:



$$\sin A = \frac{a}{c}. \qquad \cos A = \frac{b}{c}. \qquad \tan A = \frac{a}{b}.$$

$$\sin B = \frac{b}{c}. \qquad \cos B = \frac{a}{c}. \qquad \tan B = \frac{b}{a}.$$

99. CASE I. *When the given elements are a side and an angle.*

The proper formula for computing either of the remaining sides may be found by the following rule:

Take that function of the angle which involves the given side and the required side.

1. Given $c = 68$, $B = 21° 42' 39''$; find a and b.

In this case the formulæ to be used are

$$\cos B = \frac{a}{c}, \text{ and } \sin B = \frac{b}{c}.$$

Whence, $$a = c \cos B, \text{ and } b = c \sin B. \qquad \text{(A)}$$

Solution by Natural Functions.

$$a = 68 \times \cos 21° 42' 39'' = 68 \times .92906 = 63.176.$$

$$b = 68 \times \sin 21° 42' 39'' = 68 \times .36993 = 25.155.$$

Solution by Logarithms.

Taking the logarithms of both members, in formulæ (A),

$$\log a = \log c + \log\cos B, \text{ and } \log b = \log c + \log\sin B.$$

$$\begin{array}{rl} \log c = & 1.832509 \\ \log\cos B = & \underline{9.968045 - 10} \\ \log a = & 1.800554 \\ a = & 63.1762. \end{array} \qquad \begin{array}{rl} \log c = & 1.832509 \\ \log\sin B = & \underline{9.568111 - 10} \\ \log b = & 1.400620 \\ b = & 25.1547. \end{array}$$

2. Given $a = .235867$, $A = 67° 9' 23''$; find b and c.

In this case, $\tan A = \frac{a}{b}$, and $\sin A = \frac{a}{c}$.

Whence, $b = \frac{a}{\tan A}$, and $c = \frac{a}{\sin A}$.

By logarithms, $\log b = \log a - \log\tan A$, and $\log c = \log a - \log\sin A$.

$$\begin{array}{rl} \log a = & 9.372667 - 10 \\ \log\tan A = & \underline{0.375452 \qquad\quad} \\ \log b = & 8.997215 - 10 \\ b = & .0993607. \end{array} \qquad \begin{array}{rl} \log a = & 9.372667 - 10 \\ \log\sin A = & \underline{9.964527 - 10} \\ \log c = & 9.408140 - 10 \\ c = & .255941. \end{array}$$

100. Case II. *When both the given elements are sides.*

First calculate one of the angles by aid of either formula involving the given elements, and then compute the remaining side by the rule of Case I.

Example. Given $b = .15124$, $c = .30807$; find A and a.

We first find A by the formula $\cos A = \frac{b}{c}$, and then find a by the formula $\sin A = \frac{a}{c}$, or $a = c\sin A$.

By logarithms, $\log\cos A = \log b - \log c$, and $\log a = \log c + \log\sin A$.

$$\begin{array}{rl} \log b = & 9.179667 - 10 \\ \log c = & \underline{9.488650 - 10} \\ \log\cos A = & 9.691017 - 10 \\ A = & 60° 35' 54.4''. \end{array} \qquad \begin{array}{rl} \log c = & 9.488650 - 10 \\ \log\sin A = & \underline{9.940118 - 10} \\ \log a = & 9.428768 - 10 \\ a = & .268391. \end{array}$$

101. In the Trigonometric solution of any example under Case II., it is necessary to first find one of the angles, and the remaining side may then be calculated.

It is possible, however, to compute the third side directly, without first finding the angle, by Geometry.

Thus, in the example of § 100, we have, by Geometry, $a^2 + b^2 = c^2$.

Whence, $a = \sqrt{c^2 - b^2} = \sqrt{(c+b)(c-b)}$.

By logarithms, $\log a = \frac{1}{2}[\log(c+b) + \log(c-b)]$.

$$c + b = .45931;\ \log = 9.662106 - 10$$
$$c - b = .15683;\ \log = 9.195429 - 10$$
$$2\overline{)18.857535 - 20}$$
$$\log a = 9.428768 - 10$$
$$a = .268391, \text{ as before.}$$

If the given sides are a and b, the expression for c is $\sqrt{a^2 + b^2}$, which is not adapted to logarithmic computation.

In such a case, it is usually shorter to proceed as in § 100.

EXAMPLES.

Note. In those examples of the following set in which the given sides are numbers of not more than three significant figures, and the operations indicated involve only multiplication, it is usually shorter to employ Natural Functions.

In such a case, the results cannot be depended upon to more than *five* significant figures; while in the solutions by logarithms, they can be depended upon to *six* significant figures.

102. Solve the following right triangles:

1. Given $A = 15°$, $c = 7$.

2. Given $B = 67°$, $a = 5$.

3. Given $B = 50°$, $b = 20$.

4. Given $a = .35$, $c = .62$.

5. Given $a = 273$, $b = 418$.

6. Given $A = 38°$, $a = 8.09$.

7. Given $B = 75°$, $c = .014$.

8. Given $b = 58.6$, $c = 76.3$.

9. Given $A = 9°$, $b = 937$.

10. Given $a = 3.414$, $b = 2.875$.

11. Given $A = 84° 16'$, $a = .0033503$.

12. Given $A = 46° 23'$, $c = 5278.6$.

13. Given $a = 529.3$, $c = 902.7$.

14. Given $B = 23° 9'$, $b = 75.48$.

15. Given $A = 72° 52'$, $b = 6306$.

16. Given $B = 18° 38'$, $c = 2.5432$.

17. Given $a = .0001689$, $b = .0004761$.

18. Given $A = 31° 45'$, $a = 48.0408$.

19. Given $b = 617.57$, $c = 729.59$.

20. Given $B = 82° 6' 18''$, $a = 89.32$.

21. Given $A = 55° 43' 29''$, $c = 41518$.

22. Given $B = 31° 47' 7''$, $a = 7.23246$.

23. Given $a = 99.464$, $c = 156.819$.

24. Given $A = 43° 21' 36''$, $b = .00261751$.

25. Given $B = 79° 14' 31''$, $b = 84218.5$.

26. Given $B = 67° 39' 53''$, $c = 9537514$.

27. Given $b = 5789.72$, $c = 24916.45$.

28. Given $A = 26° 12' 24''$, $c = 469422.7$.

29. Given $B = 14° 55' 42''$, $b = .1353371$.

30. Given $a = 672.3853$, $b = 384.5038$.

Solve the following isosceles triangles, in which A and B are the equal angles, and a, b, and c the sides opposite the angles A, B, and C, respectively:

31. Given $A = 68° 57'$, $b = 350.94$.

32. Given $B = 27° 8'$, $c = 3.0892$.

33. Given $C = 84° 47'$, $b = 91032.7$.

34. Given $a = 79.2434$, $c = 106.6362$.

35. Given $A = 35° 19' 47''$, $c = .56235$.

36. Given $C = 151° 28' 52''$, $c = 9547.12$.

37. A regular pentagon is inscribed in a circle whose diameter is 35. Find the length of its side.

38. At a distance of 105 ft. from the base of a tower, the angle of elevation of its top is observed to be 38° 25′. Find its height.

39. What is the angle of elevation of the sun when a tower whose height is 103.74 ft. casts a shadow 167.38 ft. in length?

40. If the diameter of a circle is 32689, find the angle at the centre subtended by an arc whose chord is 10273.

41. If the diameter of the earth is 7912 miles, what is the distance of the remotest point of the surface visible from the summit of a mountain $1\frac{1}{4}$ miles in height?

42. Find the length of the diagonal of a regular pentagon whose side is 6.3257.

43. Find the angle of elevation of a mountain-slope which rises 238 ft. in a horizontal distance of one-eighth of a mile.

44. From the top of a lighthouse, 146 ft. above the sea, the angle of depression of a buoy is observed to be 21° 46′. Find the horizontal distance of the buoy.

45. If a pole casts a shadow which is two-thirds its own length, what is the angle of elevation of the sun?

46. A vessel is sailing due east at the rate of 7.8 miles an hour. A headland is observed to bear due north at 10.37 A.M., and 33° west of north at 12.43 P.M. Find the distance of the headland from each point of observation.

47. If a chord whose length is 41.368 subtends an arc of 145° 37′, what is the radius of the circle?

48. The length of the side of a regular octagon is 12. Find the radii of the inscribed and circumscribed circles.

49. How far from the foot of a flagpole 110 ft. high must an observer stand, so that the angle of elevation of the top of the pole may be 12°?

50. If the diagonal of a regular pentagon is 32.835, what is the radius of the circumscribed circle?

51. From the top of a tower, the angle of depression of the extremity of a horizontal base line, 1250 ft. in length measured from the foot of the tower, is observed to be 18° 36′ 29″. Find the height of the tower.

52. If the radius of a circle is 723.294, what is the length of a chord which subtends an arc of 35° 13′?

53. A regular hexagon is circumscribed about a circle whose diameter is 18. Find the length of its side.

54. From the top of a lighthouse 200 ft. above the sea, the angles of depression of two boats in line with the lighthouse are observed to be 14° and 32°, respectively. Find the distance between the boats.

55. A vessel is sailing due east at a uniform rate of speed. At 7 A.M., a lighthouse is observed bearing due north, 10.326 miles distant; and at 7.30 A.M. it bears 18° 13′ west of north. Find the rate of sailing of the vessel, and the bearing of the lighthouse at 10 A.M.

103. Care must be taken to use the Auxiliary Table for Small Angles in finding the logarithmic functions of angles between 0° and 5°, or between 85° and 90°, or the angles corresponding in the same cases.

This provides for every case which can arise in solving right triangles, except in looking out the angle corresponding to a logarithmic sine when between 85° and 90°, or a logarithmic cosine when between 0° and 5°.

We will now derive a formula for right triangles by aid of which, when b and c are given, the angle A may be determined with accuracy if it is between 85° and 90°.

By § 98, $\cos A = \frac{b}{c}$.

Then by (**31**), $2\sin^2\frac{1}{2}A = 1 - \cos A = 1 - \frac{b}{c} = \frac{c-b}{c}$.

Therefore, $\sin\frac{1}{2}A = \sqrt{\frac{c-b}{2c}}$.

In like manner, $\sin\frac{1}{2}B = \sqrt{\frac{c-a}{2c}}$.

These formulæ involve the *half-angles;* hence, if the angle itself is between 85° and 90°, its half is between 42° 30′ and 45°, and the correction in seconds may in that case be found from the table with sufficient precision.

An angle between 0° and 5° may always be avoided in solving a right triangle by working with the other acute angle.

104. 1. Given $b = 1.08249$, $c = 1.08261$; find the angles.

Here A is near to 0°, and B is near to 90°, as may be determined by inspection.

We then proceed to find B by the formula of § 103.

For this purpose, we must first find a, which may be done as in § 101.

$$\begin{aligned} c + b &= 2.1651;\ \log = 0.335478 \\ c - b &= .00012;\ \log = 6.079181 - 10 \\ & \qquad 2\overline{)16.414659 - 20} \\ \log a &= 8.207330 - 10 \end{aligned}$$

Whence $a = .0161187$.

Now to find B, we use the formula $\sin\frac{1}{2}B = \sqrt{\frac{c-a}{2c}}$.

By logarithms, $\log\sin\frac{1}{2}B = \frac{1}{2}[\log(c-a) - \log 2c]$.

$$\begin{aligned} c - a &= 1.0664913;\ \log = 0.027957 \\ 2c &= 2.16522;\ \log = 0.335502 \\ & \qquad 2\overline{)19.692455 - 20} \\ \log\sin\tfrac{1}{2}B &= 9.846228 - 10 \end{aligned}$$

Whence, $\frac{1}{2}B = 44°\ 34'\ 24.7''$.

Then, $B = 89°\ 8'\ 49.4''$, and $A = 90° - B = 0°\ 51'\ 10.6''$.

If b is small compared with c, then A is near to 90°, and should be calculated directly by aid of the formula of § 103.

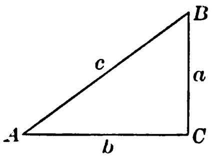
EXAMPLES.

In each of the following right triangles find the angles:

2. Given $a = .0128$, $c = 152.337$.
3. Given $b = 5.81006$, $c = 5.81039$.
4. Given $c = 11527.2$, $b = 1.32$.
5. Given $a = .77$, $c = 98276.4$.
6. Given $a = 42.0098$, $c = 42.0103$.

FORMULÆ FOR THE AREA OF A RIGHT TRIANGLE.

105. Case I. *Given the hypotenuse and an acute angle.*



Denoting the area by K, we have by Geometry,

$$2K = ab.$$

But by § 4, $a = c \sin A$, and $b = c \cos A$.

Whence, $2K = c^2 \sin A \cos A = \frac{1}{2} c^2 \sin 2A$, by (**25**).

Then, $$4K = c^2 \sin 2A. \qquad (\mathbf{38})$$

In like manner, $$4K = c^2 \sin 2B. \qquad (\mathbf{39})$$

Case II. *Given an angle and its opposite side.*

By § 4, $b = a \cot A$.

Whence, $$2K = a \times a \cot A = a^2 \cot A. \qquad (\mathbf{40})$$

In like manner, $$2K = b^2 \cot B. \qquad (\mathbf{41})$$

Case III. *Given an angle and its adjacent side.*

By § 4, $b = a \tan B$.

Whence, $$2K = a \times a \tan B = a^2 \tan B. \qquad (\mathbf{42})$$

In like manner, $$2K = b^2 \tan A. \qquad (\mathbf{43})$$

CASE IV. *Given the hypotenuse and another side.*

By Geometry, $b^2 = c^2 - a^2.$

Whence, $2K = ab = a\sqrt{c^2 - a^2} = a\sqrt{(c+a)(c-a)}.$ (44)

In like manner, $2K = b\sqrt{(c+b)(c-b)}.$ (45)

CASE V. *Given the two sides about the right angle.*

In this case, $2K = ab.$ (46)

EXAMPLES.

106. 1. Given $c = 10.3572$, $B = 74° 57' 14''$; find the area.

By (39), $4K = c^2 \sin 2B.$

Whence, $\log(4K) = 2\log c + \log\sin 2B.$

$$\begin{aligned} \log c = 1.015242;\ \text{multiply by } 2 &= 2.030484 \\ 2B = 149° 54' 28'';\quad \log\sin &= \underline{9.700178 - 10} \\ \log(4K) &= 1.730662 \\ 4K &= 53.7851 \end{aligned}$$

Dividing by 4, $K = 13.4463.$

Note. To find $\log\sin 149° 54' 28''$, take either $\log\cos 59° 54' 28''$, or $\log\sin 30° 5' 32''$. (See Introduction to Tables, page viii.)

Find the areas of the following right angles:

2. Given $A = 19° 36'$, $a = 2.2178$.
3. Given $B = 24° 7' 48''$, $a = .8213$.
4. Given $a = 149.417$, $b = 76.292$.
5. Given $b = .305694$, $c = .660156$.
6. Given $A = 30° 56' 19''$, $c = 192.035$.
7. Given $A = 78° 42' 53''$, $b = .0520281$.
8. Given $a = .932368$, $c = 4.786723$.
9. Given $B = 72° 18' 27''$, $c = 27.28338$.
10. Given $B = 49° 25' 34''$, $b = .3375494$.

VII. GENERAL PROPERTIES OF TRIANGLES.

107. *In any triangle, the sides are proportional to the sines of their opposite angles.*

I. To prove $$a : b = \sin A : \sin B. \tag{47}$$

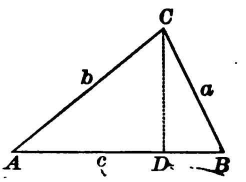


FIG. 1.

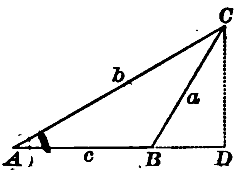


FIG. 2.

There will be two cases, according as the angles A and B are both acute (Fig. 1), or one of them obtuse (Fig. 2).

In each case, draw CD perpendicular to AB.

Then in each figure, $CD = b \sin A$ (§ 4).

Also in Fig. 1, $CD = a \sin B$.

And in Fig. 2,
$$CD = a \sin CBD$$
$$= a \sin (180° - B) = a \sin B \text{ (§ 33)}.$$

Then in either case, $b \sin A = a \sin B$.

Whence by the theory of proportion,

$$a : b = \sin A : \sin B.$$

In like manner, $$b : c = \sin B : \sin C, \tag{48}$$

and $$c : a = \sin C : \sin A. \tag{49}$$

108. *In any triangle, the sum of any two sides is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.*

By (**47**), $a : b = \sin A : \sin B$.

Whence by composition and division,

$$a + b : a - b = \sin A + \sin B : \sin A - \sin B.$$

Or, $$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B}.$$

But, $$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}, \text{ by } (\mathbf{21}).$$

Whence, $$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}. \qquad (50)$$

In like manner, $$\frac{b+c}{b-c}=\frac{\tan\frac{1}{2}(B+C)}{\tan\frac{1}{2}(B-C)}, \qquad (51)$$

and $$\frac{c+a}{c-a}=\frac{\tan\frac{1}{2}(C+A)}{\tan\frac{1}{2}(C-A)}. \qquad (52)$$

109. *In any triangle, the square of any side is equal to the sum of the squares of the other two sides, minus twice their product into the cosine of their included angle.*

I. To prove $$a^2=b^2+c^2-2\,bc\cos A. \qquad (53)$$

CASE I. *When the included angle A is acute.* (Figures of § 107.)

There will be two cases, according as the angle B is acute (Fig. 1), or obtuse (Fig. 2).

Then in Fig. 1, $BD=c-AD$, and in Fig. 2, $BD=AD-c$.

Squaring, we have in either case,

$$\overline{BD}^2=\overline{AD}^2+c^2-2\,c\times AD.$$

Adding $\overline{CD}^2$ to both members,

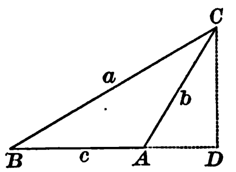
$$\overline{BD}^2+\overline{CD}^2=\overline{AD}^2+\overline{CD}^2+c^2-2\,c\times AD.$$

But $$\overline{BD}^2+\overline{CD}^2=a^2,\text{ and }\overline{AD}^2+\overline{CD}^2=b^2.$$

Also, by § 4, $$AD=b\cos A.$$

Whence, $$a^2=b^2+c^2-2\,bc\cos A.$$

CASE II. *When the included angle A is obtuse.*



Draw CD perpendicular to AB.

We have $$BD=AD+c.$$

Squaring, and adding $\overline{CD}^2$ to both members,

$$\overline{BD}^2+\overline{CD}^2=\overline{AD}^2+\overline{CD}^2+c^2+2\,c\times AD.$$

But $\overline{BD}^2 + \overline{CD}^2 = a^2$, and $\overline{AD}^2 + \overline{CD}^2 = b^2$.

And by § 4, $AD = b \cos CAD = b \cos (180° - A) = - b \cos A$ (§ 33).

Whence, $$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

In like manner, $$b^2 = c^2 + a^2 - 2\,ca \cos B, \quad (54)$$

and $$c^2 = a^2 + b^2 - 2\,ab \cos C. \quad (55)$$

110. *To express the cosines of the angles of a triangle in terms of the sides of the triangle.*

By (**53**), $$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

Transposing, $$2\,bc \cos A = b^2 + c^2 - a^2.$$

Whence, $$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}. \quad (56)$$

In like manner, $$\cos B = \frac{c^2 + a^2 - b^2}{2\,ca}, \quad (57)$$

and $$\cos C = \frac{a^2 + b^2 - c^2}{2\,ab}. \quad (58)$$

111. *To express the sines, cosines, and tangents of the half-angles of a triangle in terms of the sides of the triangle.*

By (**56**), $$1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{a^2 - b^2 + 2\,bc - c^2}{2\,bc}.$$

Whence, by (**31**), $$2 \sin^2 \tfrac{1}{2} A = \frac{a^2 - (b - c)^2}{2\,bc}.$$

Or, $$\sin^2 \tfrac{1}{2} A = \frac{(a - b + c)(a + b - c)}{4\,bc}.$$

Denoting the sum of the sides, $a + b + c$, by $2\,s$, we have

$$a - b + c = (a + b + c) - 2\,b = 2\,s - 2\,b = 2\,(s - b),$$

and $$a + b - c = (a + b + c) - 2\,c = 2\,s - 2\,c = 2\,(s - c).$$

Whence, $$\sin^2 \tfrac{1}{2} A = \frac{4\,(s - b)(s - c)}{4\,bc}.$$

Or, $$\sin \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}}. \quad (59)$$

In like manner, $$\sin\tfrac{1}{2}B=\sqrt{\frac{(s-c)(s-a)}{ca}},\tag{60}$$

and $$\sin\tfrac{1}{2}C=\sqrt{\frac{(s-a)(s-b)}{ab}}.\tag{61}$$

Again, by (56), $$1+\cos A=1+\frac{b^2+c^2-a^2}{2bc}=\frac{b^2+2bc+c^2-a^2}{2bc}.$$

Whence, by (32), $$2\cos^2\tfrac{1}{2}A=\frac{(b+c)^2-a^2}{2bc}.$$

Or, $$\cos^2\tfrac{1}{2}A=\frac{(b+c+a)(b+c-a)}{4bc}.$$

But, $b+c+a=2s$;

and $b+c-a=(b+c+a)-2a=2(s-a)$.

Whence, $$\cos^2\tfrac{1}{2}A=\frac{4s(s-a)}{4bc}.$$

Or, $$\cos\tfrac{1}{2}A=\sqrt{\frac{s(s-a)}{bc}}.\tag{62}$$

In like manner, $$\cos\tfrac{1}{2}B=\sqrt{\frac{s(s-b)}{ca}},\tag{63}$$

and $$\cos\tfrac{1}{2}C=\sqrt{\frac{s(s-c)}{ab}}.\tag{64}$$

Dividing (59) by (62), we have, by (4),

$$\tan\tfrac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{bc}{s(s-a)}}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.\tag{65}$$

In like manner, $$\tan\tfrac{1}{2}B=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}},\tag{66}$$

and $$\tan\tfrac{1}{2}C=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}.\tag{67}$$

Note. Since each angle of a triangle is less than 180°, its half is less than 90°; hence, the *positive sign* must be taken before the radical in each formula of § 111.

FORMULÆ FOR THE AREA OF AN OBLIQUE TRIANGLE.

112. Case I. *Given two sides and their included angle.*

I. When the given parts are b, c, and A.

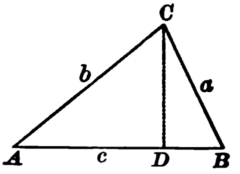


FIG. 1.

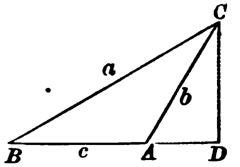


FIG. 2.

There will be two cases, according as A is acute (Fig. 1), or obtuse (Fig. 2).

In each case, draw CD perpendicular to AB.

Then denoting the area by K, we have by Geometry,

$$2K = c \times CD.$$

But in Fig. 1, $CD = b \sin A$ (§ 4).

And in Fig. 2, $CD = b \sin CAD$

$$= b \sin(180° - A) = b \sin A \quad (§\ 33).$$

Then in either case, $2K = bc \sin A.$ **(68)**

In like manner, $2K = ca \sin B,$ **(69)**

and $2K = ab \sin C.$ **(70)**

CASE II. *Given one side and all the angles.*

I. When the given parts are a, A, B, and C.

By **(70)**, $2K = ab \sin C.$

But by **(47)**, $\dfrac{b}{a} = \dfrac{\sin B}{\sin A}$, or $b = \dfrac{a \sin B}{\sin A}.$

Whence, $2K = a \times \dfrac{a \sin B}{\sin A} \times \sin C = \dfrac{a^2 \sin B \sin C}{\sin A}.$ **(71)**

In like manner, $2K = \dfrac{b^2 \sin C \sin A}{\sin B},$ **(72)**

and $2K = \dfrac{c^2 \sin A \sin B}{\sin C}.$ **(73)**

CASE III. *Given the three sides.*

By **(68)**, $2K = bc \sin A = 2bc \sin \frac{1}{2} A \cos \frac{1}{2} A$, by **(25)**.

Dividing by 2, and substituting the values of $\sin \frac{1}{2} A$ and $\cos \frac{1}{2} A$ from **(59)** and **(62)**, we have,

$$K = bc\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-a)}{bc}} = \sqrt{s(s-a)(s-b)(s-c)}. \quad \textbf{(74)}$$

VIII. SOLUTION OF OBLIQUE TRIANGLES.

113. In the solution of oblique triangles, we may distinguish four cases.

114. CASE I. *Given a side and any two angles.*

The third angle may be found by Geometry, and then by aid of § 107 the remaining sides may be calculated.

The triangle is always possible for any values of the given elements, provided the sum of the given angles is less than 180°.

1. Given $b = 20.24$, $A = 103° \, 36'$, $B = 19° \, 21'$; find C, a, and c.

We have, $C = 180° - (A + B) = 180° - 122° \, 57' = 57° \, 3'$.

By (**47**), $$\frac{a}{b} = \frac{\sin A}{\sin B}, \text{ and } \frac{c}{b} = \frac{\sin C}{\sin B}.$$

Then, $$a = b \sin A \csc B, \text{ and } c = b \sin C \csc B.$$

Whence, $$\log a = \log b + \log \sin A + \log \csc B,$$

and $$\log c = \log b + \log \sin C + \log \csc B.$$

$$\begin{aligned} \log b &= 1.306211 \\ \log \sin A &= 9.987649 - 10 \\ \log \csc B &= 0.479729 \\ \hline \log a &= 1.773589 \\ a &= 59.3730. \end{aligned} \qquad \begin{aligned} \log b &= 1.306211 \\ \log \sin C &= 9.923837 - 10 \\ \log \csc B &= 0.479729 \\ \hline \log c &= 1.709777 \\ c &= 51.2598. \end{aligned}$$

Note. To find $\log \sin 103° \, 36'$, take either $\log \cos 13° \, 36'$, or $\log \sin 76° \, 24'$. To find the log cosecant of an angle, subtract the log sine from $10 - 10$. (See Introduction to Tables, page viii.)

EXAMPLES.

Solve the following triangles:

2. Given $a = 180$, $A = 38°$, $B = 75° \, 43'$.
3. Given $b = .82$, $B = 51° \, 42' \, 37''$, $C = 109° \, 17' \, 23''$.
4. Given $c = 24.637$, $A = 83° \, 39'$, $B = 38° \, 56'$.
5. Given $b = .06708$, $A = 26° \, 10' \, 45''$, $C = 44° \, 35' \, 12''$.
6. Given $a = 5.0454$, $B = 98° \, 8' \, 26''$, $C = 21° \, 51' \, 34''$.
7. Given $c = 4592.36$, $A = 74° \, 27'$, $C = 61°$.
8. Given $c = .93109$, $A = 15° \, 34' \, 9''$, $C = 123° \, 29' \, 46''$.
9. Given $b = 3.67683$, $A = 67° \, 21' \, 54''$, $B = 57° \, 48' \, 8''$.
10. Given $a = 71396.72$, $B = 42° \, 55' \, 13''$, $C = 16° \, 4' \, 57''$.

115. Case II. *Given two sides and their included angle.*

Since one angle is known, the sum of the other two angles may be found; and then their difference may be calculated by aid of § 108.

Knowing the sum and difference of the angles, the angles themselves may be found; and then the remaining side may be computed as in Case I.

The triangle is possible for any values of the data.

1. Given $a = 82$, $c = 167$, $B = 98° 14'$; find A, C, and b.

By Geometry, $\quad C + A = 180° - B = 81° 46'.$

By (**52**), $$\frac{c+a}{c-a} = \frac{\tan\frac{1}{2}(C+A)}{\tan\frac{1}{2}(C-A)}.$$

Or, $$\tan\tfrac{1}{2}(C-A) = \frac{c-a}{c+a}\tan\tfrac{1}{2}(C+A).$$

Then, $\log\tan\frac{1}{2}(C-A) = \log(c-a) + \operatorname{colog}(c+a) + \log\tan\frac{1}{2}(C+A)$.

$$\begin{array}{rlrl} c - a &= 85 & \log &= 1.929419 \\ c + a &= 249 & \operatorname{colog} &= 7.603801 - 10 \\ \frac{1}{2}(C+A) &= 40° 53' & \log\tan &= \underline{9.937377 - 10} \\ & & \log\tan\frac{1}{2}(C-A) &= 9.470597 - 10 \\ & & \frac{1}{2}(C-A) &= 16° 27' 49.8''. \end{array}$$

Therefore, $\quad C = \frac{1}{2}(C+A) + \frac{1}{2}(C-A) = 57° 20' 49.8'',$

and $\quad A = \frac{1}{2}(C+A) - \frac{1}{2}(C-A) = 24° 25' 10.2''.$

To find the remaining side, we have by (**47**),

$$b = \frac{a\sin B}{\sin A} = a\sin B\csc A.$$

Whence, $\quad \log b = \log a + \log\sin B + \log\csc A.$

$$\begin{aligned} \log a &= 1.913814 \\ \log\sin B &= 9.995501 - 10 \\ \log\csc A &= \underline{0.383615} \\ \log b &= 2.292930 \\ b &= 196.305. \end{aligned}$$

EXAMPLES.

Solve the following triangles:

2. Given $a = 67$, $\quad c = 33$, $\quad B = 36°$.
3. Given $a = 886$, $\quad b = 747$, $\quad C = 71° 54'$.
4. Given $b = 4.102$, $\quad c = 4.549$, $\quad A = 62° 9' 38$.

5. Given $a = .5953,\quad b = .9639,\quad C = 134°.$

6. Given $b = 1292.1,\quad c = 286.3,\quad A = 27° 13'.$

7. Given $a = 7.48,\quad c = 12.409,\quad B = 83° 26' 52''.$

8. Given $a = 93.273,\quad b = 81.512,\quad C = 58°.$

9. Given $b = .0261579,\quad c = .0608657,\quad A = 115° 42'.$

10. Given $a = 35384.82,\quad c = 57946.34,\quad B = 19° 37' 25''.$

116. Case III. *Given the three sides.*

The angles might be calculated by the formulæ of § 110; but as these are not adapted to logarithmic computation, it is usually more convenient to use the formulæ of § 111.

Each of the three angles should be computed trigonometrically, for we then have a check on the work, since their sum should be 180°.

If all the angles are to be computed, the *tangent* formulæ are the most convenient, since only four different logarithms are required. If but one angle is required, the *cosine* formula will be found to involve the least work.

The triangle is possible for any values of the data, provided no side is greater than the sum of the other two.

If all the angles are required, and the tangent formulæ are used, it is convenient to modify them as follows; by (65),

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s-a)(s-b)(s-c)}{s(s-a)^2}} = \frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Denoting $\sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$ by r, we have

$$\tan \tfrac{1}{2} A = \frac{r}{s-a}.$$

In like manner, $\tan \frac{1}{2} B = \dfrac{r}{s-b}$, and $\tan \frac{1}{2} C = \dfrac{r}{s-c}$.

1. Given $a = 2.51,\ b = 2.79,\ c = 2.33$; find A, B, and C.

Here, $$2s = a + b + c = 7.63.$$

Whence, $s = 3.815,\ s - a = 1.305,\ s - b = 1.025,\ s - c = 1.485.$

We have, $\log r = \frac{1}{2}[\log (s-a) + \log (s-b) + \log (s-c) + \operatorname{colog} s].$

Also, $$\log \tan \tfrac{1}{2} A = \log r - \log (s-a),$$

$$\log \tan \tfrac{1}{2} B = \log r - \log (s-b),$$

and $$\log \tan \tfrac{1}{2} C = \log r - \log (s-c).$$

$$\begin{aligned}
\log (s-a) &= 0.115611\\
\log (s-b) &= 0.010724\\
\log (s-c) &= 0.171726\\
\text{colog}\, s &= 9.418505-10\\
&2\overline{)19.716566-20}\\
\log r &= 9.858283-10\\
\log (s-a) &= \underline{0.115611}\\
\log \tan \tfrac{1}{2}A &= 9.742672-10\\
\tfrac{1}{2}A &= 28^\circ 56' 22.7''.\\
A &= 57^\circ 52' 45.4''.
\end{aligned}$$

$$\begin{aligned}
\log r &= 9.858283-10\\
\log (s-b) &= \underline{0.010724}\\
\log \tan \tfrac{1}{2}B &= 9.847559-10\\
\tfrac{1}{2}B &= 35^\circ 8' 40.9''.\\
B &= 70^\circ 17' 21.8''.
\end{aligned}$$

$$\begin{aligned}
\log r &= 9.858283-10\\
\log (s-c) &= \underline{0.171726}\\
\log \tan \tfrac{1}{2}C &= 9.686557-10\\
\tfrac{1}{2}C &= 25^\circ 54' 56.2''.\\
C &= 51^\circ 49' 52.4''.
\end{aligned}$$

Check, $A+B+C = 179^\circ 59' 59.6''$.

2. Given $a=7$, $b=11$, $c=9.6$; find B.

By (**63**),
$$\cos \tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ca}}.$$

Whence, $\log \cos \frac{1}{2}B = \frac{1}{2}[\log s + \log (s-b) + \text{colog}\, c + \text{colog}\, a]$.

Here, $2s = a+b+c = 27.6$; whence, $s = 13.8$, and $s-b = 2.8$.

$$\begin{aligned}
\log s &= 1.139879\\
\log (s-b) &= 0.447158\\
\text{colog}\, c &= 9.017729-10\\
\text{colog}\, a &= 9.154902-10\\
&2\overline{)19.759668-20}\\
\log \cos \tfrac{1}{2}B &= 9.879834-10
\end{aligned}$$

$\frac{1}{2}B = 40^\circ 41' 11.5''$, and $B = 81^\circ 22' 23.0''$.

EXAMPLES.

Solve the following triangles:

3. Given $a=2$, $b=3$, $c=4$.

4. Given $a=5$, $b=7$, $c=6$.

5. Given $a=10$, $b=9$, $c=8$.

6. Given $a=5.6$, $b=4.3$, $c=4.9$.

7. Given $a=.85$, $b=.92$, $c=.78$.

8. Given $a=61.3$, $b=84.7$, $c=47.6$.

9. Given $a=705$, $b=562$, $c=639$; find A.

10. Given $a=.0291$, $b=.0184$, $c=.0358$; find B.

11. Given $a=3019$, $b=6731$, $c=4228$; find C.

117. Case IV. *Given two sides and the angle opposite to one of them.*

It was stated in § 97 that a triangle is, in general, completely determined when three of its elements are known, provided one of them is a side. The only exceptions occur in Case IV.

To illustrate, let us consider the following example:

Given $a = 52.1$, $b = 61.2$, $A = 31°26'$; find B, C, and c.

By (47), $$\frac{\sin B}{\sin A} = \frac{b}{a}, \text{ or } \sin B = \frac{b \sin A}{a}.$$

Whence, $$\log \sin B = \log b + \operatorname{colog} a + \log \sin A.$$

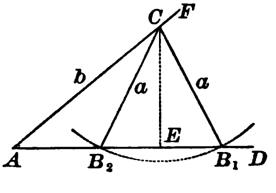
$$\begin{aligned} \log b &= 1.786751 \\ \operatorname{colog} a &= 8.283162 - 10 \\ \log \sin A &= \underline{9.717259 - 10} \\ \log \sin B &= 9.787172 - 10 \\ B &= 37°\,46'\,37.9'', \text{ from the table.} \end{aligned}$$

But in determining the angle corresponding, attention must be paid to the fact that an angle and its supplement have the same sine (§ 33).

Hence, another value of B will be $180° - 37°\,46'\,37.9''$, or $142°\,13'\,22.1''$; and calling these values B_1 and B_2, we have

$$B_1 = 37°\,46'\,37.9'', \text{ and } B_2 = 142°\,13'\,22.1''.$$

Note. The reason for this ambiguity is at once apparent when we attempt to construct the triangle from the data.



We first lay off the angle DAF equal to $31°\,26'$, and on AF take $AC = 61.2$. With C as a centre, and a radius equal to 52.1, describe an arc cutting AD at B_1 and B_2. Then either of the triangles AB_1C or AB_2C satisfies the given conditions.

The two values of B which were obtained are the values of the angles AB_1C and AB_2C, respectively; and it is evident geometrically that these angles are supplementary.

To complete the solution, denote the angles ACB_1 and ACB_2 by C_1 and C_2, and the sides AB_1 and AB_2 by c_1 and c_2.

Then, $C_1 = 180° - (A + B_1) = 180° - 69°\,12'\,37.9'' = 110°\,47'\,22.1''$,

and $C_2 = 180° - (A + B_2) = 180° - 173°\,39'\,22.1'' = 6°\,20'\,37.9''$.

Again, by (49), $\frac{c_1}{a} = \frac{\sin C_1}{\sin A}$, and $\frac{c_2}{a} = \frac{\sin C_2}{\sin A}$.

Whence, $c_1 = a \sin C_1 \csc A$, and $c_2 = a \sin C_2 \csc A$.

$$\begin{array}{rl} \log a &= 1.716838 \\ \log \sin C_1 &= 9.970761 - 10 \\ \log \csc A &= 0.282741 \\ \hline \log c_1 &= 1.970340 \\ \therefore c_1 &= 93.3985. \end{array} \qquad \begin{array}{rl} \log a &= 1.716838 \\ \log \sin C_2 &= 9.043343 - 10 \\ \log \csc A &= 0.282741 \\ \hline \log c_2 &= 1.042922 \\ c_2 &= 11.0388. \end{array}$$

118. Whenever an angle of an oblique triangle is determined from its *sine*, both the acute and obtuse values must be retained as solutions, unless one of them can be shown by other considerations to be inadmissible; and hence there may sometimes be two solutions, sometimes one, and sometimes none, in an example under Case IV.

1. Let the data be a, b, and A, and suppose $b < a$.

By Geometry, B must be $< A$; hence, only the *acute* value of B can be taken; in this case there is but *one* solution.

2. Let the data be a, b, and A, and suppose $b > a$.

Since B must be $> A$, the triangle is impossible unless A is acute.

Again, since $\frac{\sin B}{\sin A} = \frac{b}{a}$, and b is $> a$, $\sin B$ is $> \sin A$.

Hence, both the acute and obtuse values of B are $> A$, and there are *two* solutions, except in the following cases:

If $\log \sin B = 0$, then $\sin B = 1$ (§ 72), and $B = 90°$, and the triangle is a *right* triangle; if $\log \sin B$ is *positive*, then $\sin B$ is > 1, and the triangle is impossible.

119. The results of § 118 may be stated as follows:

If, of the given sides, that adjacent to the given angle is the *less*, there is but *one* solution, corresponding to the *acute* value of the opposite angle.

If the side adjacent to the given angle is the *greater*, there are *two* solutions, unless the log sine of the opposite angle is 0 or positive; in which cases there are *one* solution (a *right* triangle), and *no* solution, respectively.

120. We will illustrate the above points by examples:

1. Given $a = 7.42$, $b = 3.39$, $A = 105° 13'$; find B.

Since b is $< a$, there is but *one* solution, corresponding to the *acute* value of B.

By (47), $\sin B = \dfrac{b \sin A}{a}$.

Whence, $\log \sin B = \log b + \text{colog}\, a + \log \sin A$.

$$\begin{aligned} \log b &= 0.530200 \\ \text{colog}\, a &= 9.129596 - 10 \\ \log \sin A &= \underline{9.984500 - 10} \\ \log \sin B &= 9.644296 - 10 \\ B &= 26^\circ\, 9'\, 30.5''. \end{aligned}$$

2. Given $b = 3$, $c = 2$, $C = 100^\circ$; find B.

Since b is $> c$, and C is obtuse, the triangle is impossible.

3. Given $a = 22.7643$, $c = 50$, $A = 27^\circ\, 5'$; find C.

We have, $\sin C = \dfrac{c \sin A}{a}$.

$$\begin{aligned} \log c &= 1.698970 \\ \text{colog}\, a &= 8.642746 - 10 \\ \log \sin A &= \underline{9.658284 - 10} \\ \log \sin C &= 0.000000 \end{aligned}$$

Therefore, $\sin C = 1$, and $C = 90^\circ$.

Here there is but one solution; a *right* triangle.

4. Given $a = .83$, $b = .715$, $B = 61^\circ\, 47'$; find A.

We have, $\sin A = \dfrac{a \sin B}{b}$.

$$\begin{aligned} \log a &= 9.919078 - 10 \\ \text{colog}\, b &= 0.145694 \\ \log \sin B &= \underline{9.945058 - 10} \\ \log \sin A &= 0.009830 \end{aligned}$$

Since $\log \sin A$ is positive, the triangle is impossible.

EXAMPLES.

121. Solve the following triangles:

1. Given $a = 5.98$, $b = 3.59$, $A = 63^\circ\, 50'$.
2. Given $b = 74.1$, $c = 64.2$, $C = 27^\circ\, 18'$.
3. Given $b = .2337$, $c = .0982$, $B = 108^\circ$.
4. Given $a = 4.254$, $c = 4.536$, $C = 37^\circ\, 9'$.
5. Given $a = .2789$, $b = .2271$, $B = 65^\circ\, 38'$.

6. Given $a = 60.935$, $c = 76.097$, $A = 133° 41'$.
7. Given $b = 74.8067$, $c = 98.7385$, $C = 81° 47'$.
8. Given $a = 9.51987$, $c = 11$, $A = 59° 56'$.
9. Given $b = 4.521$, $c = 5.03$, $B = 40° 32' 7''$.
10. Given $a = 186.82$, $b = 394.2$, $B = 114° 29' 51''$.
11. Given $b = 5143.4$, $c = 4795.56$, $C = 72° 53' 38''$.
12. Given $a = .860619$, $c = .635761$, $A = 19° 12' 43''$.
13. Given $a = 139.27$, $b = 195.9716$, $A = 45° 17' 20''$.
14. Given $a = .32163$, $c = .27083$, $C = 52° 24' 16''$.
15. Given $b = 91139.04$, $c = 80640.37$, $B = 126° 5' 34''$.

AREA OF AN OBLIQUE TRIANGLE.

122. 1. Given $a = 18.063$, $A = 96° 30' 15''$, $B = 35° 0' 13''$; find K.

By (71), $$2K = \frac{a^2 \sin B \sin C}{\sin A} = a^2 \sin B \sin C \csc A.$$

Whence, $\log(2K) = 2\log a + \log \sin B + \log \sin C + \log \csc A$.

From the data, $C = 180° - (A + B) = 48° 29' 32''$.

$$\begin{aligned}
\log a = 1.256790;\ \text{multiply by } 2 &= 2.513580 \\
\log \sin B &= 9.758630 - 10 \\
\log \sin C &= 9.874404 - 10 \\
\log \csc A &= 0.002804 \\
\hline
\log(2K) &= 2.149418 \\
2K &= 141.065. \\
K &= 70.533.
\end{aligned}$$

EXAMPLES.

Find the areas of the following triangles:

2. Given $a = 38.09$, $c = 11.2$, $B = 67° 55'$.
3. Given $a = 5$, $b = 8$, $c = 6$.
4. Given $b = 6.074$, $A = 70° 39'$, $B = 56° 23'$.
5. Given $b = 761.86$, $c = 526.02$, $A = 124° 6' 13''$.
6. Given $a = 97$, $b = 83$, $c = 71$.
7. Given $a = 1.9375$, $A = 43° 18'$, $B = 29° 47' 36''$.

8. Given $b = .439592$, $A = 62° 40' 8''$, $C = 54° 32' 25''$.

9. Given $a = 39.5$, $b = 44.8$, $c = 52.3$.

10. Given $a = .804639$, $c = .357173$, $B = 18° 11' 49''$.

11. Given $c = 95.86157$, $B = 115° 24' 52''$, $C = 32° 57' 21''$.

12. Given $a = .02409481$, $b = .02763834$, $C = 81° 9' 34''$.

13. Given $a = 7.825$, $b = 6.592$, $c = 9.643$.

MISCELLANEOUS EXAMPLES.

123. 1. From a point in the same horizontal plane with the base of a tower, the angle of elevation of its top is 52° 39′, and from a point 100 ft. further away it is 35° 16′. Find the height of the tower, and its distance from each point of observation.

2. One side of a parallelogram is 56, and the angles between this side and the diagonals are 31° 14′ and 45° 37′. Find all the sides of the parallelogram.

3. In a field *ABCD*, the sides *AB*, *BC*, *CD*, and *DA* are 155, 236, 252, and 105 rods, respectively, and the diagonal *AC* is 311 rods. Find the area of the field.

4. The area of a triangle is 1356, and two of its sides are 53 and 69. Find the angle between them.

5. From the top of a bluff, the angles of depression of two posts in the plain below, in line with the observer and 1000 ft. apart, are found to be 27° 40′ and 9° 33′, respectively. Find the height of the bluff above the plain.

6. The parallel sides of a trapezoid are 86 and 138, and the angles at the extremities of the latter are 53° 49′ and 67° 55′. Find the non-parallel sides.

7. Two trains start at the same time from the same point, and move along straight railways, which intersect at an angle of 74° 30′, at the rates of 30 and 45 miles an hour, respectively. How far apart are they at the end of 45 minutes?

8. Two sides of a triangle are .5623 and .4977, and the difference of the angles opposite these sides is 15° 48′ 32″. Solve the triangle.

9. Two yachts start at the same time from the same point, and sail, one due north at the rate of 10.44 miles an hour, and the other due north-east at the rate of 7.71 miles an hour. What is the bearing of the first yacht from the second at the end of half an hour?

10. A vessel is sailing due south-west at the rate of 8 miles an hour. At 10.30 A.M., a lighthouse is observed to bear 30° west of north, and at 12.15 P.M., it is observed to bear 15° east of north. Find the distance of the lighthouse from each position of the vessel.

11. Two sides of a parallelogram are 65 and 133, and one of the diagonals is 159. Find the angles of the parallelogram, and the other diagonal.

12. To find the distance of an inaccessible object A from a position B, I measure a line BC, 208.3 ft. in length. The angles ABC and ACB are measured, and found to be 126° 35′ and 31° 48′, respectively. Find the distance AB.

13. The diagonals of a parallelogram are 81 and 106, and the angle between them is 29° 18′. Find the sides and angles of the parallelogram.

14. A flagpole 40 ft. high stands on the top of a tower. From a position near the base of the tower, the angles of elevation of the top and bottom of the pole are 38° 53′ and 20° 18′, respectively. Find the distance and height of the tower.

15. AD and BC are the parallel sides of a trapezoid $ABCD$; the sides AB and BC are 7.8 and 9.4, respectively, and the angles B and C are 113° 47′ and 125° 34′, respectively. Find AD and CD.

16. A surveyor observes that his position A is exactly in line with two inaccessible objects B and C. He measures a line AD 500 ft. long, making the angle $BAD = 60°$, and at D observes the angles ADB and BDC to be 40° and 60°, respectively. Find the distance BC.

17. A side of a parallelogram is 48, a diagonal is 73, and the angle between the diagonals, opposite the given side, is 98° 6′. Find the other diagonal and the other side.

18. To find the distance between two buoys A and B, I measure a base-line CD on the shore, 150 ft. long. At the point C the angles ACD and BCD are measured, and found to be 95° and 70°, respectively; and at D the angles BDC and ADC are found to be 83° and 30°, respectively. Find the distance between the buoys.

19. The sides AB, BC, and CD, of a quadrilateral $ABCD$ are 38, 55, and 42, respectively, and the angles B and C are 132° 56′ and 98° 29′, respectively. Find the side AD, and the angles A and D.

20. The sides AB, BC, and DA of a field $ABCD$ are 37, 63, and 20 rods, respectively, and the diagonals AC and BD are 75 and 42 rods, respectively. Find the area of the field.

IX. CUBIC EQUATIONS.

124. We know, by Algebra, that a cubic equation can always be transformed into another in which the term containing the square of the unknown quantity shall be wanting.

Thus, if the equation is $x^3 + px^2 + qx + r = 0$, putting $x = y - \frac{p}{3}$, we have

$$y^3 - py^2 + \frac{p^2y}{3} - \frac{p^3}{27} + py^2 - \frac{2\,p^2y}{3} + \frac{p^3}{9} + qy - \frac{pq}{3} + r = 0;$$

or, $$y^3 + y\left(q - \frac{p^2}{3}\right) + \frac{2\,p^3}{27} - \frac{pq}{3} + r = 0;$$

which is in the required form.

125. Cardan's Method enables us to solve any cubic equation of the form $x^3 + ax + b = 0$, except in the case where a is negative, and $\frac{a^3}{27}$ numerically $> \frac{b^2}{4}$.

In this case, it is possible to find the roots by Trigonometry.

126. Trigonometric Solution of Cubic Equations.

To solve the equation

$$x^3 - ax - b = 0,$$

where a is positive, and $\frac{a^3}{27} > \frac{b^2}{4}$.

Putting $x = 2\,m \cos A$, the equation becomes

$$8\,m^3 \cos^3 A - 2\,am \cos A - b = 0, \text{ or } 4 \cos^3 A - \frac{a}{m^2} \cos A - \frac{b}{2\,m^3} = 0.$$

But by (**36**), $$4 \cos^3 A = \cos 3\,A + 3 \cos A.$$

Whence, $$\cos 3\,A + 3 \cos A - \frac{a}{m^2} \cos A - \frac{b}{2\,m^3} = 0.$$

Or, $$\cos 3\,A + \left(3 - \frac{a}{m^2}\right) \cos A = \frac{b}{2\,m^3}. \qquad \text{(A)}$$

We may take m so that $3 - \frac{a}{m^2} = 0$; then, $3\,m^2 = a$, and $m = \sqrt{\frac{a}{3}}$. (B)

Then (A) becomes $$\cos 3\,A = \frac{b}{2\,m^3}.$$

Substituting in this the value of m from (B), we have

$$\cos 3A = \frac{b}{2}\sqrt{\frac{27}{a^3}}. \qquad \text{(C)}$$

Since, by hypothesis, $\frac{b^2}{4} < \frac{a^3}{27}$, we have $\frac{b^2}{4} \times \frac{27}{a^3} < 1$.

Taking the square root of each member of the inequality, $\frac{b}{2}\sqrt{\frac{27}{a^3}} < 1$.

Hence, the value of $3A$ in (C) is possible, since its cosine is < 1.

Let z be the least positive angle whose cosine is equal to $\frac{b}{2}\sqrt{\frac{27}{a^3}}$.

Then, one value of $3A$ is z; and all its values are given by the expression $2n\pi \pm z$ (§ 62), where n is 0 or any positive or negative integer.

Whence, $\qquad \cos A = \cos \frac{1}{3}(2n\pi \pm z)$.

Now let $n = 3q + n'$, where q is 0 or any positive or negative integer, and $n' = 0$ or ± 1; then,

$$\cos A = \cos\frac{(6q + 2n')\pi \pm z}{3} = \cos\left[2q\pi + \frac{2n'\pi \pm z}{3}\right] = \cos\frac{2n'\pi \pm z}{3};$$

for by § 21, any multiple of 360° may be added to, or subtracted from, an angle, without altering its functions.

Putting $n' = 0$, 1, and -1, we have

$$\cos A = \cos\left(\pm\frac{z}{3}\right), \cos\frac{2\pi \pm z}{3}, \text{ or } \cos\frac{-2\pi \pm z}{3} = \cos\frac{z}{3} \text{ or } \cos\frac{2\pi \pm z}{3};$$

for by § 29, the cosine of the angle $(-A)$ is equal to the cosine of A.

But $x = 2m\cos A$; and hence the three values of x are

$$2\sqrt{\frac{a}{3}}\cos\frac{z}{3},\quad 2\sqrt{\frac{a}{3}}\cos\left(\frac{2\pi}{3} - \frac{z}{3}\right), \text{ and } 2\sqrt{\frac{a}{3}}\cos\left(\frac{2\pi}{3} + \frac{z}{3}\right);$$

where z is given by the equation $\cos z = \frac{b}{2}\sqrt{\frac{27}{a^3}}$.

EXAMPLES.

1. Solve the equation $x^3 - 4x + 2 = 0$.

Here $a = 4$, $b = -2$; then, $\cos z = -\sqrt{\frac{27}{64}}$, or $\cos(\pi - z) = \sqrt{\frac{27}{64}}$ (§ 33).

By logarithms, $\log\cos(\pi - z) = \frac{1}{2}(\log 27 - \log 64)$.

$$\begin{aligned} \log 27 &= 1.431364 \\ \log 64 &= 1.806180 \\ 2&\overline{)19.625184 - 20} \\ \log\cos(\pi - z) &= 9.812592 - 10 \end{aligned}$$

Then, $\pi - z = 49° \, 29' \, 40.5''$, and $z = 130° \, 30' \, 19.5''$.

Whence, $\frac{z}{3} = 43° \, 30' \, 6.5''$, and $2\sqrt{\frac{a}{3}} = 2\sqrt{\frac{4}{3}} = \sqrt{\frac{16}{3}}$.

Then the three values of x are

$$\sqrt{\frac{16}{3}} \cos 43° \, 30' \, 6.5'',$$

$$\sqrt{\frac{16}{3}} \cos (120° - 43° \, 30' \, 6.5'') = \sqrt{\frac{16}{3}} \cos 76° \, 29' \, 53.5'',$$

and

$$\sqrt{\frac{16}{3}} \cos (120° + 43° \, 30' \, 6.5'') = \sqrt{\frac{16}{3}} \cos (90° + 73° \, 30' \, 6.5'')$$

$$= -\sqrt{\frac{16}{3}} \sin 73° \, 30' \, 6.5'' \text{ (§ 30)}.$$

Now,

$$\log \sqrt{\frac{16}{3}} = \tfrac{1}{2}(\log 16 - \log 3) = \tfrac{1}{2}(1.204120 - .477121) = .363500. \quad (1)$$

Also, $\log \cos 43° \, 30' \, 6.5'' = 9.860549 - 10,$ (2)

$\log \cos 76° \, 29' \, 53.5'' = 9.368242 - 10,$ (3)

and $\log \sin 73° \, 30' \, 6.5'' = 9.981741 - 10.$ (4)

Adding (2), (3), and (4) in succession to (1), the logarithms of the absolute values of x are

$$0.224049, \quad 9.731742 - 10, \quad \text{and} \quad 0.345241.$$

The numbers corresponding to these logarithms are

$$1.67513, \quad .53919, \quad \text{and} \quad 2.21432.$$

Whence, $x = 1.67513$, $.53919$, or -2.21432.

Solve the following equations:

2. $x^3 - 4x - 1 = 0.$

3. $x^3 - 6x + 3 = 0.$

4. $x^3 + 6x^2 - x - 1 = 0.$

5. $x^3 - 3x^2 - 2x + 1 = 0.$

SPHERICAL TRIGONOMETRY.

X. GEOMETRICAL PRINCIPLES.

127. If a triedral angle be formed with its vertex at the centre of a sphere, it intercepts on the surface a *spherical triangle.*

The triangle is bounded by three arcs of great circles, called its *sides,* which measure the face angles of the triedral angle.

The *angles* of the spherical triangle are the spherical angles formed by the adjacent sides; and, by Geometry, each is equal to the angle between two straight lines drawn, one in the plane of each of its sides, and perpendicular to the intersection of these planes at the same point.

128. The sides of a spherical triangle are usually expressed in degrees.

129. *Spherical Trigonometry* treats of the trigonometric relations between the sides and angles of a spherical triangle.

The face and diedral angles of the triedral angle are not altered by varying the radius of the sphere; and hence the relations between the sides and angles of a spherical triangle are independent of the length of the radius.

130. We shall limit ourselves in the present work to such triangles as are considered in Geometry, where each angle is less than two right angles, and each side less than the semi-circumference of a great circle; that is, where each element is less than 180°.

131. The proofs of the following properties of spherical triangles may be found in any treatise on Solid Geometry:

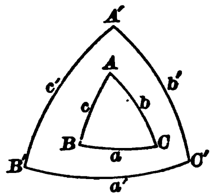
1. The sum of any two sides of a spherical triangle is greater than the third side.

2. In any spherical triangle, the greater side lies opposite the greater angle; and, conversely, the greater angle lies opposite the greater side.

3. The sum of the sides of a spherical triangle is less than 360°.

4. The sum of the angles of a spherical triangle is greater than 180°, and less than 540°.

5. If $A'B'C'$ is the polar triangle of ABC, that is, if A, B, and C are the poles of the sides $B'C'$, $C'A'$, and $A'B'$, respectively, then, conversely, ABC is the polar triangle of $A'B'C'$.



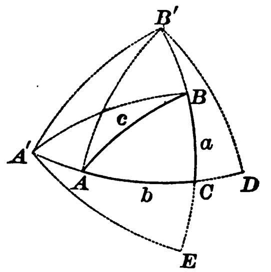
6. In two polar triangles, each angle of one is measured by the supplement of the side lying opposite the homologous angle of the other; that is

$$a' = 180° - A. \qquad b' = 180° - B. \qquad c' = 180° - C.$$

$$A' = 180° - a. \qquad B' = 180° - b. \qquad C' = 180° - c.$$

132. A spherical triangle is called *tri-rectangular* when it has three right angles; each side is a quadrant, and each vertex is the pole of the opposite side.

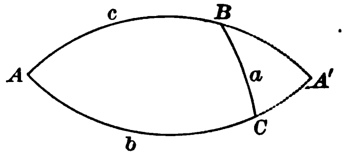
133. I. Let C be the right angle of the right spherical triangle ABC, and suppose $a < 90°$ and $b < 90°$.



Complete the tri-rectangular triangle $A'B'C$; also, since B' is the pole of AC, and A' of BC, construct the tri-rectangular triangles $AB'D$ and $A'BE$.

Then since B lies within the triangle $AB'D$, AB or c is $< 90°$.
Since BC is $< B'C$, the angle A is $< B'AD$, or $< 90°$.
Since AC is $< A'C$, the angle B is $< A'BE$, or $< 90°$.

II. Suppose $a < 90°$ and $b > 90°$.



Complete the lune $ABA'C$.

Then in the right triangle $A'BC$, $A'C = 180° - b$.

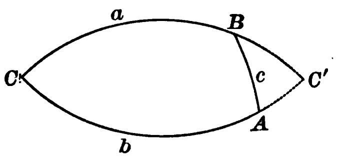
That is, the sides a and $A'C$ of the triangle $A'BC$ are each $< 90°$; and by I., $A'B$ and the angles A' and $A'BC$ are each $< 90°$.

But, $c = 180° - A'B$, $A = A'$, and $B = 180° - A'BC$.

Whence, c is $> 90°$, $A < 90°$, and $B > 90°$.

Similarly, if a is $> 90°$ and $b < 90°$, then c is $> 90°$, $A > 90°$, and $B < 90°$.

III. Suppose $a > 90°$ and $b > 90°$.



Complete the lune $ACBC'$.

Then in the right triangle ABC', $AC' = 180° - b$, and $BC' = 180° - a$.

That is, the sides AC' and BC' of the triangle ABC' are each $< 90°$; and by I., AB and the angles BAC' and ABC' are each $< 90°$.

But, $A = 180° - BAC'$, and $B = 180° - ABC'$.

Whence, c is $< 90°$, $A > 90°$, and $B > 90°$.

Hence, in any right spherical triangle:

1. *If the sides about the right angle are in the same quadrant, the hypotenuse is* $< 90°$; *if they are in different quadrants, the hypotenuse is* $> 90°$.

2. *An angle is in the same quadrant as its opposite side.*

134. In the figure of § 131, we have, by § 131, 1, $a' < b' + c'$.

Putting for a', b', and c' the values given in § 131, 6, we have

$$180° - A < 180° - B + 180° - C, \text{ or } B + C - A < 180°.$$

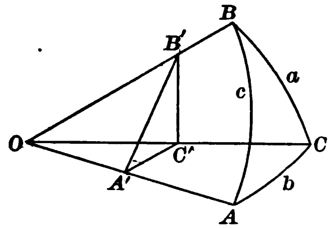
Again, by § 130, $B + C + 180° > A$; whence, $B + C - A > -180°$.

Therefore, $B + C - A$ is between $180°$ and $-180°$.

Similarly, $C + A - B$ and $A + B - C$ are between $180°$ and $-180°$.

XI. RIGHT SPHERICAL TRIANGLES.

135. Let C be the right angle of the right spherical triangle ABC.



Let O be the centre of the sphere, and draw OA, OB, and OC.

At any point A' of OA draw $A'B'$ and $A'C'$ perpendicular to OA, meeting OB and OC at B' and C', and draw $B'C'$.

Then OA is perpendicular to the plane $A'B'C'$.

Hence, each of the planes $A'B'C'$ and OBC is perpendicular to the plane OAC, and their intersection $B'C'$ is perpendicular to OAC.

Therefore, $B'C'$ is perpendicular to $A'C'$ and OC'.

In the right triangle $OA'B'$, we have

$$\cos c = \cos A'OB' = \frac{OA'}{OB'} = \frac{OC'}{OB'} \times \frac{OA'}{OC'}.$$

But in the right triangles $OB'C'$ and $OC'A'$,

$$\frac{OC'}{OB'} = \cos a, \text{ and } \frac{OA'}{OC'} = \cos b.$$

Whence, $$\cos c = \cos a \cos b. \qquad (75)$$

Again, $$\sin A = \sin B'A'C' = \frac{B'C'}{A'B'} = \frac{\frac{B'C'}{OB'}}{\frac{A'B'}{OB'}} = \frac{\sin a}{\sin c}. \qquad (76)$$

And, $$\cos A = \cos B'A'C' = \frac{A'C'}{A'B'} = \frac{\frac{A'C'}{OA'}}{\frac{A'B'}{OA'}} = \frac{\tan b}{\tan c}. \qquad (77)$$

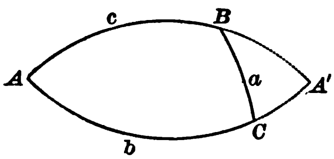
In like manner, $$\sin B = \frac{\sin b}{\sin c}, \qquad (78)$$

and $$\cos B = \frac{\tan a}{\tan c}. \qquad (79)$$

136. The proofs of § 135 cannot be regarded as general, for in the construction of the figure we have assumed a and b, and therefore c and A (§ 133), to be less than 90°.

To prove formulæ (**75**) to (**79**) universally, we must consider two additional cases:

CASE I. *When one of the sides a and b is $< 90°$, and the other $> 90°$.*



In the right spherical triangle ABC, let a be $< 90°$ and $b > 90°$.

Complete the lune $ABA'C$; then, in the spherical triangle $A'BC$,

$$A'B = 180° - c,\quad A'C = 180° - b,\quad A' = A,\quad \text{and}\quad A'BC = 180° - B.$$

But by § 133, c is $> 90°$, $A < 90°$, and $B > 90°$.

Hence, each element, except the right angle, of the right spherical triangle $A'BC$ is $< 90°$; and we have by § 135,

$$\cos A'B = \cos a \cos A'C,$$

$$\sin A' = \frac{\sin a}{\sin A'B}, \qquad \sin A'BC = \frac{\sin A'C}{\sin A'B},$$

$$\cos A' = \frac{\tan A'C}{\tan A'B}, \qquad \cos A'BC = \frac{\tan a}{\tan A'B}.$$

Putting for $A'B$, $A'C$, A' and $A'BC$ their values, we have

$$\cos(180° - c) = \cos a \cos(180° - b),$$

$$\sin A = \frac{\sin a}{\sin(180° - c)}, \qquad \sin(180° - B) = \frac{\sin(180° - b)}{\sin(180° - c)},$$

$$\cos A = \frac{\tan(180° - b)}{\tan(180° - c)}, \qquad \cos(180° - B) = \frac{\tan a}{\tan(180° - c)}.$$

Whence, by § 33, $\qquad -\cos c = \cos a(-\cos b),$

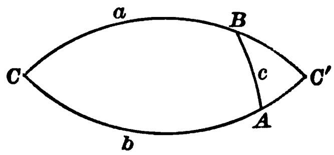
$$\sin A = \frac{\sin a}{\sin c}, \qquad \sin B = \frac{\sin b}{\sin c},$$

$$\cos A = \frac{-\tan b}{-\tan c}, \qquad -\cos B = \frac{\tan a}{-\tan c};$$

and we obtain formulæ (**75**) to (**79**) as before.

In like manner, the formulæ may be proved to hold when a is $> 90°$ and $b < 90°$.

CASE II. *When both a and b are $> 90°$.*



In the right spherical triangle ABC, let a and b be $> 90°$.

Complete the lune $ACBC'$.

By § 133, c is $< 90°$, $A > 90°$, and $B > 90°$.

Hence, each element, except the right angle, of the right spherical triangle ABC' is $< 90°$; and we have by § 135,

$$\cos c = \cos AC' \cos BC',$$

$$\sin BAC' = \frac{\sin BC'}{\sin c}, \qquad \sin ABC' = \frac{\sin AC'}{\sin c},$$

$$\cos BAC' = \frac{\tan AC'}{\tan c}, \qquad \cos ABC' = \frac{\tan BC'}{\tan c}.$$

Putting for AC', BC', BAC', and ABC' their values, we have

$$\cos c = \cos(180° - a)\cos(180° - b),$$

$$\sin(180° - A) = \frac{\sin(180° - a)}{\sin c}, \qquad \sin(180° - B) = \frac{\sin(180° - b)}{\sin c},$$

$$\cos(180° - A) = \frac{\tan(180° - b)}{\tan c}, \qquad \cos(180° - B) = \frac{\tan(180° - a)}{\tan c}.$$

Whence, by § 33, $\quad \cos c = (-\cos a)(-\cos b),$

$$\sin A = \frac{\sin a}{\sin c}, \qquad \sin B = \frac{\sin b}{\sin c},$$

$$-\cos A = \frac{-\tan b}{\tan c}, \qquad -\cos B = \frac{-\tan a}{\tan c};$$

and we obtain formulæ (**75**) to (**79**), as before.

137. From (**76**) and (**77**), we obtain

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin a}{\sin c} \times \frac{\tan c}{\tan b} = \frac{\sin a}{\cos c \tan b}.$$

Whence by (**75**), $\quad \tan A = \dfrac{\sin a}{\cos a \cos b \tan b} = \dfrac{\tan a}{\sin b}. \qquad$ (**80**)

In like manner, $\quad \tan B = \dfrac{\tan b}{\sin a}. \qquad$ (**81**)

138. By (4), $\sin a = \cos a \tan a$; then (76) may be written

$$\sin A = \frac{\cos a \tan a}{\cos c \tan c} = \frac{\dfrac{\tan a}{\tan c}}{\dfrac{\cos c}{\cos a}}.$$

Whence by (75) and (79),

$$\sin A = \frac{\cos B}{\cos b}. \tag{82}$$

In like manner,

$$\sin B = \frac{\cos A}{\cos a}. \tag{83}$$

139. From (75), (82), and (83), we have

$$\cos c = \cos a \cos b = \frac{\cos A}{\sin B} \times \frac{\cos B}{\sin A} = \cot A \cot B. \tag{84}$$

140. The formulæ of §§ 135 to 139 are collected below for convenience of reference:

$$\cos c = \cos a \cos b.$$

$$\sin A = \frac{\sin a}{\sin c}. \qquad \sin B = \frac{\sin b}{\sin c}.$$

$$\cos A = \frac{\tan b}{\tan c}. \qquad \cos B = \frac{\tan a}{\tan c}.$$

$$\tan A = \frac{\tan a}{\sin b}. \qquad \tan B = \frac{\tan b}{\sin a}.$$

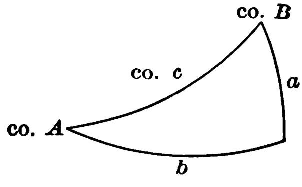
$$\sin A = \frac{\cos B}{\cos b}. \qquad \sin B = \frac{\cos A}{\cos a}.$$

$$\cos c = \cot A \cot B.$$

The student should compare the formulæ for the sines, cosines, and tangents of A and B with the corresponding formulæ in §§ 2 and 5.

141. Napier's Rules of Circular Parts.

These are two rules which include all the formulæ of § 140.



In any right spherical triangle, the elements a and b, and the complements of the elements A, B, and c (written in abbreviated form, co. A, co. B, and co. c), are called the *circular parts*.

If we suppose them arranged in the order in which the letters occur in the triangle, any one of the five may be taken and called the *middle part;* the two immediately adjacent are called the *adjacent parts,* and the remaining two the *opposite parts.*

Then Napier's rules are:

I. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

II. *The sine of the middle part is equal to the product of the cosines of the opposite parts.*

142. Napier's rules may be proved by taking each circular part in succession as the middle part, and showing that the results agree with the formulæ of § 140.

1. If a be taken as the middle part, b and co. B are the adjacent parts, and co. c and co. A the opposite parts.

Then the rules give

$$\sin a = \tan b \tan (\text{co. } B), \text{ and } \sin a = \cos (\text{co. } c) \cos (\text{co. } A).$$

Or by § 32, $\sin a = \tan b \cot B$, and $\sin a = \sin c \sin A$;

which are equivalent to (**81**) and (**76**).

2. If b be taken as the middle part, a and co. A are the adjacent parts, and co. c and co. B the opposite parts.

Then, $\sin b = \tan a \tan (\text{co. } A) = \tan a \cot A,$

and $\sin b = \cos (\text{co. } c) \cos (\text{co. } B) = \sin c \sin B;$

which are equivalent to (**80**) and (**78**).

3. If co. c be taken as the middle part, co. A and co. B are the adjacent parts, and a and b the opposite parts.

Then, $\sin (\text{co. } c) = \tan (\text{co. } A) \tan (\text{co. } B)$, and $\sin (\text{co. } c) = \cos a \cos b$.

Or, $\cos c = \cot A \cot B$, and $\cos c = \cos a \cos b$;

which agree with (**84**) and (**75**).

4. If co. A be taken as the middle part, b and co. c are the adjacent parts, and a and co. B the opposite parts.

Then, $\sin (\text{co. } A) = \tan b \tan (\text{co. } c)$, and $\sin (\text{co. } A) = \cos a \cos (\text{co. } B)$.

Or, $\cos A = \tan b \cot c$, and $\cos A = \cos a \sin B$;

which are equivalent to (**77**) and (**83**).

5. If co. B be taken as the middle part, a and co. c are the adjacent parts, and b and co. A the opposite parts.

Then, $\sin(\text{co. } B) = \tan a \tan(\text{co. } c)$, and $\sin(\text{co. } B) = \cos b \cos(\text{co. } A)$.

Or, $\cos B = \tan a \cot c$, and $\cos B = \cos b \sin A$;

which are equivalent to (**79**) and (**82**).

Writers on Trigonometry differ as to the practical value of Napier's rules; but in the opinion of the highest authorities, it seems to be regarded as preferable to attempt to remember the formulæ by comparing them with the analogous formulæ for plane right triangles, as stated in § 140.

SOLUTION OF RIGHT SPHERICAL TRIANGLES.

143. To solve a right spherical triangle, two elements must be given in addition to the right angle.

There may be six cases:

1. *Given the hypotenuse and an adjacent angle.*
2. *Given an angle and its opposite side.*
3. *Given an angle and its adjacent side.*
4. *Given the hypotenuse and another side.*
5. *Given the two sides a and b.*
6. *Given the two angles A and B.*

144. Either of the above cases may be solved by aid of § 140.

The formula for computing either of the remaining elements when any two are given may be found by the following rule:

Take that formula which involves the given parts and the required part.

If *all* the remaining elements are required, the following rule may be found convenient in selecting the formulæ:

Take the three formulæ which involve the given parts.

145. It is convenient in the solution to have a check on the logarithmic work, which may be done in every case without the necessity of looking out any new logarithms.

Examples of this will be found in § 148.

The check formula for any particular case may be selected from the set in § 140 by the following rule:

Take that formula which involves the three required parts.

Note. If Napier's rules are used, the following rule will indicate which of the circular parts corresponding to the given elements and any required element is to be regarded as the middle part.

If these three circular parts are adjacent, take the middle one as the middle part, and the others are then adjacent parts.

If they are not adjacent, take the part which is not adjacent to either of the others as the middle part, and the others are then opposite parts.

For the check formula, proceed as above with the circular parts corresponding to the three required elements.

Thus, if c and A are the given elements,

1. To find a, consider the circular parts a, co. c, and co. A; of these, a is the middle part, and co. c and co. A are opposite parts. Then, by Napier's rules,

$$\sin a = \cos(\text{co. } c)\cos(\text{co. } A) = \sin c \sin A.$$

2. To find b, the circular parts are b, co. c, and co. A; in this case co. A is the middle part, and b and co. c are adjacent parts. Then,

$$\sin(\text{co. } A) = \tan b \tan(\text{co. } c), \text{ or } \cos A = \tan b \cot c.$$

3. To find B, the circular parts are co. B, co. c, and co. A; co. c is the middle part, and co. A and co. B are adjacent parts. Then,

$$\sin(\text{co. } c) = \tan(\text{co. } A)\tan(\text{co. } B), \text{ or } \cos c = \cot A \cot B.$$

4. For the check formula, the circular parts are a, b, and co. B; a is the middle part, and b and co. B are adjacent parts. Then,

$$\sin a = \tan b \tan(\text{co. } B) = \tan b \cot B.$$

146. In solving spherical triangles, careful attention must be given to the *algebraic signs* of the functions; the cosines, tangents, and cotangents of angles between 90° and 180° being taken *negative* (§ 20).

It is convenient to place the sign of each function just above or below it, as shown in the examples of § 148; the sign of the function in the first member being then determined in accordance with the principle that like signs produce +, and unlike signs produce −.

Note. In the examples after the first of § 148, the signs are omitted in every case where both factors of the second member are +.

147. In finding the angles corresponding, if the function is a cosine, tangent, or cotangent, its sign determines whether the angle is acute or obtuse; that is, if it is +, the angle is acute; and if it is −, the angle is obtuse, and the *supplement* of the acute angle obtained from the tables must be taken (§ 33).

If the function is a sine, since the sine of an angle is equal to the sine of its supplement (§ 33), both the acute angle obtained from the tables and its supplement must be retained as solutions, unless the ambiguity can be removed by the principles of § 133.

EXAMPLES.

148. **1.** Given $B = 33° 50'$, $a = 108°$; find A, b, and c.

By the rule of § 144, the formulæ from § 140 are,

$$\sin B = \frac{\cos A}{\cos a},\quad \tan B = \frac{\tan b}{\sin a},\quad \text{and}\quad \cos B = \frac{\tan a}{\tan c}.$$

$$\text{Or,}\quad \overset{-}{\cos A} = \overset{-}{\cos a}\,\overset{+}{\sin B},\quad \overset{+}{\tan b} = \overset{+}{\sin a}\,\overset{-}{\tan B},\quad \text{and}\quad \overset{-}{\tan c} = \frac{\overset{+}{\tan a}}{\underset{+}{\cos B}}.$$

Hence,

$$\log\cos A = \log\cos a + \log\sin B.$$
$$\log\tan b = \log\sin a + \log\tan B.$$
$$\log\tan c = \log\tan a - \log\cos B.$$

Since $\cos A$ and $\tan c$ are negative, the *supplements* of the acute angles obtained from the tables must be taken (§ 147).

Note 1. When the supplement of the angle obtained from the tables is to be taken, it is convenient to write 180° minus the element in the first member, as shown below in the cases of A and c.

By the rule of § 145, the check formula for this case is

$$\cos A = \frac{\tan b}{\tan c},\quad \text{or}\quad \log\cos A = \log\tan b - \log\tan c.$$

The values of $\log\tan b$ and $\log\tan c$ may be taken from the first part of the work, and their difference should be equal to the result previously found for $\log\cos A$.

$$\begin{aligned} \log\cos a &= 9.489982 - 10 \\ \log\sin B &= 9.745683 - 10 \\ \hline \log\cos A &= 9.235665 - 10 \\ 180° - A &= 80° 5' 33.8''. \\ A &= 99° 54' 26.2''. \end{aligned}$$

$$\begin{aligned} \log\sin a &= 9.978206 - 10 \\ \log\tan B &= 9.826259 - 10 \\ \hline \log\tan b &= 9.804465 - 10 \\ b &= 32° 30' 59.8''. \end{aligned}$$

$$\begin{aligned} \log\tan a &= 0.488224 \\ \log\cos B &= 9.919424 - 10 \\ \hline \log\tan c &= 0.568800 \\ 180° - c &= 74° 53' 45.0''. \\ c &= 105° 6' 15.0''. \end{aligned}$$

Check.

$$\begin{aligned} \log\tan b &= 9.804465 - 10 \\ \log\tan c &= 0.568800 \\ \hline \log\cos A &= 9.235665 - 10 \end{aligned}$$

2. Given $c = 70° 30'$, $A = 100°$; find a, b, and B.

In this case the three formulæ are

$$\sin A = \frac{\sin a}{\sin c},\quad \cos A = \frac{\tan b}{\tan c},\quad \text{and}\quad \cos c = \cot A \cot B.$$

$$\text{Or,}\quad \sin a = \sin c \sin A,\quad \overset{-}{\tan b} = \overset{+}{\tan c}\,\overset{-}{\cos A},\quad \text{and}\quad \overset{-}{\cot B} = \overset{+}{\cos c}\,\overset{-}{\tan A}.$$

Here the side a is determined from its sine; but the ambiguity is removed by the principles of § 133; for a and A must be in the same quadrant. Therefore, a is obtuse; and the supplement of the angle obtained from the tables must be taken.

By § 145, the check formula is

$$\tan B = \frac{\tan b}{\sin a}, \text{ or } \sin a = \tan b \cot B.$$

Note 2. The check formula should always be expressed in terms of the functions used in determining the required parts; thus, in the case above, the check formula is transformed so as to involve $\cot B$ instead of $\tan B$.

$$\begin{aligned}
\log \sin c &= 9.974347 - 10 \\
\log \sin A &= 9.993351 - 10 \\
\hline
\log \sin a &= 9.967698 - 10 \\
180° - a &= 68° 10' 28.2''. \\
a &= 111° 49' 31.8''.
\end{aligned}
\qquad
\begin{aligned}
\log \cos c &= 9.523495 - 10 \\
\log \tan A &= 0.753681 \\
\hline
\log \cot B &= 0.277176 \\
180° - B &= 27° 50' 39.8''. \\
B &= 152° 9' 20.2''.
\end{aligned}$$

$$\begin{aligned}
\log \tan c &= 0.450851 \\
\log \cos A &= 9.239670 - 10 \\
\hline
\log \tan b &= 9.690521 - 10 \\
180° - b &= 26° 7' 18.4''. \\
b &= 153° 52' 41.6''.
\end{aligned}$$

Check.

$$\begin{aligned}
\log \tan b &= 9.690521 - 10 \\
\log \cot B &= 0.277176 \\
\hline
\log \sin a &= 9.967697 - 10
\end{aligned}$$

Note 3. We observe here a difference of .000001 in the two values of $\log \sin a$. This does not necessarily indicate an error in the work, for such a small difference might easily be due to the fact that the logarithms are only *approximately* correct to the sixth decimal place.

3. Given $a = 132° 6'$, $b = 77° 51'$; find A, B, and c.

In this case the three formulæ are

$$\overset{-}{\tan} A = \frac{\overset{-}{\tan a}}{\underset{+}{\sin b}}, \quad \tan B = \frac{\tan b}{\sin a}, \text{ and } \overset{-}{\cos c} = \overset{-}{\cos a} \overset{+}{\cos b}.$$

The check formula is

$$\cos c = \cot A \cot B, \text{ or } \cos c \tan A \tan B = 1.$$

That is, $\quad \log \cos c + \log \tan A + \log \tan B = \log 1 = 0.$

$$\begin{aligned}
\log \tan a &= 0.044039 \\
\log \sin b &= 9.990161 - 10 \\
\hline
\log \tan A &= 0.053878 \\
180° - A &= 48° 32' 41.8''. \\
A &= 131° 27' 18.2''.
\end{aligned}
\qquad
\begin{aligned}
\log \cos a &= 9.826351 - 10 \\
\log \cos b &= 9.323194 - 10 \\
\hline
\log \cos c &= 9.149545 - 10 \\
180° - c &= 81° 53' 17.4''. \\
c &= 98° 6' 42.6''.
\end{aligned}$$

$$\begin{aligned} \log\tan b &= 0.666967 \\ \log\sin a &= 9.870390 - 10 \\ \hline \log\tan B &= 0.796577 \\ B &= 80^\circ\, 55'\, 26.6''. \end{aligned}$$

Check.

$$\begin{aligned} \log\cos c &= 9.149545 - 10 \\ \log\tan A &= 0.053878 \\ \log\tan B &= 0.796577 \\ \hline \log 1 &= 0.000000 \end{aligned}$$

4. Given $A = 105^\circ\, 59'$, $a = 128^\circ\, 33'$; find b, B, and c.

The formulæ are

$$\overset{+}{\sin}\, b = \frac{\overset{-}{\tan a}}{\underset{-}{\tan A}},\quad \overset{+}{\sin}\, B = \frac{\overset{-}{\cos A}}{\underset{-}{\cos a}},\quad \text{and}\quad \sin c = \frac{\sin a}{\sin A}.$$

The check formula is $\sin B = \dfrac{\sin b}{\sin c}$.

In this case, each of the required parts is determined from its sine; and as the ambiguity cannot be removed by § 133, both the acute angle obtained from the tables and its supplement must be retained in each case.

$$\begin{aligned} \log\tan a &= 0.098617 \\ \log\tan A &= 0.542981 \\ \hline \log\sin b &= 9.555636 - 10 \\ b &= 21^\circ\, 3'\, 58.7'' \\ &\text{or } 158^\circ\, 56'\, 1.3''. \end{aligned}$$

$$\begin{aligned} \log\sin a &= 9.893243 - 10 \\ \log\sin A &= 9.982878 - 10 \\ \hline \log\sin c &= 9.910365 - 10 \\ c &= 54^\circ\, 26'\, 26.7'' \\ &\text{or } 125^\circ\, 33'\, 33.3''. \end{aligned}$$

$$\begin{aligned} \log\cos A &= 9.439897 - 10 \\ \log\cos a &= 9.794626 - 10 \\ \hline \log\sin B &= 9.645271 - 10 \\ B &= 26^\circ\, 13'\, 18.2'' \\ &\text{or } 153^\circ\, 46'\, 41.8''. \end{aligned}$$

Check.

$$\begin{aligned} \log\sin b &= 9.555636 - 10 \\ \log\sin c &= 9.910365 - 10 \\ \hline \log\sin B &= 9.645271 - 10 \end{aligned}$$

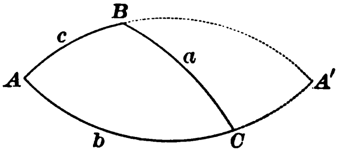
It does not follow, however, that these values can be combined promiscuously; for by § 133, since a is $> 90^\circ$, with the value of b less than 90° must be taken the value of c greater than 90°, and the value of B less than 90°; while with the value of b greater than 90° must be taken the value of c less than 90°, and the value of B greater than 90°.

Thus the only solutions of the example are:

1. $b = 21^\circ\, 3'\, 58.7''$, $c = 125^\circ\, 33'\, 33.3''$, $B = 26^\circ\, 13'\, 18.2''$.

2. $b = 158^\circ\, 56'\, 1.3''$, $c = 54^\circ\, 26'\, 26.7''$, $B = 153^\circ\, 46'\, 41.8''$.

Note 4. The figure shows geometrically why there are two solutions in this case.



For if AB and AC be produced to A', forming the lune $ABA'C$, the triangle $A'BC$ has the side a and the angle A' equal, respectively, to the side a and the angle A of the triangle ABC, and both triangles are right-angled at C.

It is evident that the sides $A'B$ and $A'C$ and the angle $A'BC$ are the supplements of the sides c and b and the angle ABC, respectively.

Solve the following right spherical triangles:

5. Given $c = 49°$, $a = 27°$.

6. Given $A = 38°$, $B = 63°$.

7. Given $A = 31°$, $a = 23°$.

8. Given $B = 153°$, $a = 35°$.

9. Given $a = 15°$, $b = 106°$.

10. Given $c = 139°$, $A = 165°$.

11. Given $B = 82° 25'$, $b = 68° 35'$.

12. Given $c = 75° 37'$, $B = 29° 4'$.

13. Given $c = 118° 49'$, $b = 44° 23'$.

14. Given $a = 171° 6'$, $b = 161° 58'$.

15. Given $B = 100° 40'$, $a = 170° 38'$.

16. Given $A = 102° 57'$, $B = 143° 46'$.

17. Given $a = 10° 28'$, $b = 7° 10'$.

18. Given $A = 54° 11'$, $b = 83° 29'$.

19. Given $A = 50° 43'$, $B = 122° 18'$.

20. Given $c = 59° 3'$, $A = 147° 32'$.

21. Given $B = 103° 30'$, $b = 132° 54'$.

22. Given $A = 95° 15'$, $b = 166° 7'$.

23. Given $c = 78° 52'$, $a = 114° 26'$.

24. Given $c = 127° 9'$, $B = 80° 51'$.

25. Given $A = 98° 34'$, $a = 113° 12'$.

26. Given $c = 136° 21'$, $b = 157° 41'$.

149. Quadrantal Triangles.

A spherical triangle is called *quadrantal* when it has one side equal to a quadrant.

By § 131, 6, the polar triangle of a quadrantal triangle is a *right* spherical triangle.

Hence, to solve a quadrantal triangle, we have only to solve its polar triangle, and take the *supplements* of the results.

1. Given $c = 90°$, $a = 67° 38'$, $b = 48° 50'$; find A, B, and C.

Denoting the polar triangle by $A'B'C'$, we have by § 131, 6:

$$C' = 90°,\ A' = 112° 22',\ B' = 131° 10';\ \text{to find } a',\ b',\ \text{and } c'.$$

By § 144, the formulæ for the solution are

$$\overset{-}{\cos}\, a' = \frac{\overset{-}{\cos A'}}{\underset{+}{\sin B'}},\quad \overset{-}{\cos}\, b' = \frac{\overset{-}{\cos B'}}{\underset{+}{\sin A'}},\quad \text{and}\quad \overset{+}{\cos}\, c' = \overset{-}{\cot} A' \overset{-}{\cot} B'.$$

The check formula is $\cos c' = \cos a' \cos b'$.

$$\begin{aligned}
\log \cos A' &= 9.580392 - 10\\
\log \sin B' &= \underline{9.876678 - 10}\\
\log \cos a' &= 9.703714 - 10\\
180° - a' &= 59° 38' 9.7''.\\
\log \cos B' &= 9.818392 - 10\\
\log \sin A' &= \underline{9.966033 - 10}\\
\log \cos b' &= 9.852359 - 10\\
180° - b' &= 44° 37' 5.8''.
\end{aligned}
\qquad
\begin{aligned}
\log \cot A' &= 9.614359 - 10\\
\log \cot B' &= \underline{9.941713 - 10}\\
\log \cos c' &= 9.556072 - 10\\
c' &= 68° 54' 41.5''.\\
&\textit{Check.}\\
\log \cos a' &= 9.703714 - 10\\
\log \cos b' &= \underline{9.852359 - 10}\\
\log \cos c' &= 9.556073 - 10
\end{aligned}$$

Then in the given quadrantal triangle, we have

$$\begin{aligned}
A &= 180° - a' = 59° 38' 9.7'',\\
B &= 180° - b' = 44° 37' 5.8'',\\
C &= 180° - c' = 111° 5' 18.5''.
\end{aligned}$$

EXAMPLES.

Solve the following quadrantal triangles:

2. Given $A = 122°$, $b = 154°$.

3. Given $A = 45° 52'$, $B = 139° 24'$.

4. Given $a = 30° 19'$, $C = 42° 31'$.

5. Given $B = 51° 35'$, $C = 116° 13'$.

6. Given $A = 105° 8'$, $a = 104° 56'$.

7. Given $a = 67° 27'$, $b = 81° 40'$.

150. Isosceles Spherical Triangles.

We know, by Geometry, that if an arc of a great circle be drawn from the vertex of an isosceles spherical triangle to the middle point of the base, it is perpendicular to the base, bisects the vertical angle, and divides the triangle into two symmetrical right spherical triangles.

By solving one of these, we can find the required parts of the given triangle.

1. Given $a = 115°$, $b = 115°$, $C = 71° 48'$; find A, B, and c.

Denoting the elements of one of the right triangles by A', B', C', a', b', and c', where C' is the right angle, we have

$$c' = a = 115°, \text{ and } A' = \tfrac{1}{2} C = 35° 54'.$$

We have then to find the parts a' and B' in this triangle.

By § 140, $\quad \sin A' = \dfrac{\sin a'}{\sin c'}$, and $\cos c' = \cot A' \cot B'$.

$$\overset{-}{\cos c'} = \overset{-}{\cot B'} \ldots \overset{+}{\tan A'}$$

Or, $\quad \sin a' = \sin c' \sin A'$, and $\cot B' = \cos c' \tan A'$.

$$\begin{array}{ll} \log \sin c' = 9.957276 - 10 & \log \cos c' = 9.625948 - 10 \\ \log \sin A' = \underline{9.768173 - 10} & \log \tan A' = \underline{9.859666 - 10} \\ \log \sin a' = 9.725449 - 10 & \log \cot B' = 9.485614 - 10 \\ a' = 32° 6' 8.6''. & 180° - B' = 72° 59' 23.5''. \\ & B' = 107° 0' 36.5''. \end{array}$$

Then in the given isosceles triangle,

$$A = B = B' = 107° 0' 36.5'', \text{ and } c = 2a' = 64° 12' 17.2''.$$

EXAMPLES.

Solve the following isosceles spherical triangles:

2. Given $A = 27° 12'$, $B = 27° 12'$, $c = 135° 20'$.
3. Given $a = 152° 6'$, $b = 152° 6'$, $C = 67° 46'$.
4. Given $a = 112° 25'$, $b = 112° 25'$, $c = 123° 48'$.
5. Given $A = 159°$, $B = 159°$, $a = 137° 39'$.

XII. OBLIQUE SPHERICAL TRIANGLES.

GENERAL PROPERTIES OF SPHERICAL TRIANGLES.

151. *In any spherical triangle, the sines of the sides are proportional to the sines of their opposite angles.*

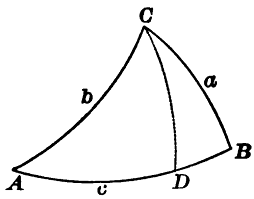


FIG. 1.

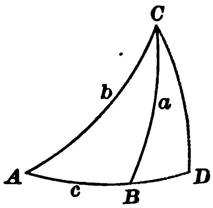


FIG. 2.

Let ABC be any spherical triangle, and draw the arc CD perpendicular to AB.

There will be two cases according as CD falls upon AB (Fig. 1), or upon AB produced (Fig. 2).

In the right triangle ACD, in either figure, we have

$$\sin A = \frac{\sin CD}{\sin b}, \text{ by } (\mathbf{76}).$$

Also, in Fig. 1, $$\sin B = \frac{\sin CD}{\sin a}.$$

And in Fig. 2, $$\sin B = \sin(180° - CBD)$$

$$= \sin CBD \ (\S\ 33) = \frac{\sin CD}{\sin a}.$$

Dividing these equations, we have in either case

$$\frac{\sin A}{\sin B} = \frac{\dfrac{\sin CD}{\sin b}}{\dfrac{\sin CD}{\sin a}} = \frac{\sin a}{\sin b}. \qquad (\mathbf{85})$$

In like manner, $$\frac{\sin B}{\sin C} = \frac{\sin b}{\sin c}, \qquad (\mathbf{86})$$

and $$\frac{\sin A}{\sin C} = \frac{\sin a}{\sin c}. \qquad (\mathbf{87})$$

152. *In any spherical triangle, the cosine of any side is equal to the product of the cosines of the other two sides, plus the continued product of their sines and the cosine of their included angle.*

In the right triangle BCD, in Fig. 1, § 151, we have, by (**75**),

$$\cos a = \cos BD \cos CD = \cos (c - AD) \cos CD.$$

And in Fig. 2,

$$\cos a = \cos BD \cos CD = \cos (AD - c) \cos CD.$$

Whence, in either case, by (**12**),

$$\cos a = \cos c \cos AD \cos CD + \sin c \sin AD \cos CD.$$

But in the right triangle ACD,

$$\cos AD \cos CD = \cos b, \text{ by (75).}$$

And, $$\sin AD \cos CD = \sin AD \frac{\cos b}{\cos AD} = \cos b \tan AD$$

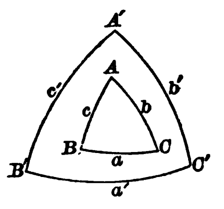
$$= \sin b \frac{\tan AD}{\tan b} = \sin b \cos A, \text{ by (77).}$$

Whence, $$\cos a = \cos b \cos c + \sin b \sin c \cos A. \qquad (88)$$

In like manner, $$\cos b = \cos c \cos a + \sin c \sin a \cos B, \qquad (89)$$

and $$\cos c = \cos a \cos b + \sin a \sin b \cos C. \qquad (90)$$

153. Let ABC and $A'B'C'$ be a pair of polar triangles.



Applying formula (**88**) to the triangle $A'B'C'$, we obtain

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

Putting for a', b', c', and A' the values given in § 131, 6, we have

$$\cos (180° - A) = \cos (180° - B) \cos (180° - C)$$
$$+ \sin (180° - B) \sin (180° - C) \cos (180° - a).$$

Whence, $$-\cos A = (-\cos B)(-\cos C) + \sin B \sin C (-\cos a) \text{ (§ 33).}$$

That is, $$\cos A = -\cos B \cos C + \sin B \sin C \cos a. \qquad (91)$$

Similarly, $$\cos B = -\cos C \cos A + \sin C \sin A \cos b, \qquad (92)$$

and $$\cos C = -\cos A \cos B + \sin A \sin B \cos c. \qquad (93)$$

The above proof illustrates a very important application of the theory of polar triangles to Spherical Trigonometry.

If any relation has been found between the elements of a spherical triangle, an analogous relation may be derived from it, in which each side or angle is replaced by the opposite angle or side, with suitable modifications in the algebraic signs.

154. *To express the sines, cosines, and tangents of the half-angles of a spherical triangle in terms of the sides of the triangle.*

From (**88**), § 152 $\sin b \sin c \cos A = \cos a - \cos b \cos c.$

Whence,
$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}. \tag{A}$$

Subtracting both members from 1, we have

$$1 - \cos A = 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c} = \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c}.$$

Whence, by (**31**), $2 \sin^2 \frac{1}{2} A = \dfrac{\cos (b - c) - \cos a}{\sin b \sin c}.$

But by (**20**), $\cos y - \cos x = 2 \sin \frac{1}{2} (x + y) \sin \frac{1}{2} (x - y).$ (B)

Whence, $2 \sin^2 \frac{1}{2} A = \dfrac{2 \sin \frac{1}{2} [a + (b - c)] \sin \frac{1}{2} [a - (b - c)]}{\sin b \sin c},$

or,
$$\sin^2 \tfrac{1}{2} A = \frac{\sin \frac{1}{2} (a + b - c) \sin \frac{1}{2} (a - b + c)}{\sin b \sin c}.$$

Denoting the sum of the sides, $a + b + c$, by $2s$, we have

$$a + b - c = (a + b + c) - 2c = 2s - 2c = 2(s - c),$$

and
$$a - b + c = (a + b + c) - 2b = 2s - 2b = 2(s - b).$$

Whence,
$$\sin^2 \tfrac{1}{2} A = \frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}.$$

Or,
$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}}. \tag{94}$$

In like manner,
$$\sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s - c) \sin (s - a)}{\sin c \sin a}}, \tag{95}$$

and
$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin (s - a) \sin (s - b)}{\sin a \sin b}}. \tag{96}$$

Again, adding both members of (A) to 1, we have

$$1 + \cos A = 1 + \frac{\cos a - \cos b \cos c}{\sin b \sin c} = \frac{\cos a - (\cos b \cos c - \sin b \sin c)}{\sin b \sin c}.$$

Whence, by (**32**), $2 \cos^2 \frac{1}{2} A = \dfrac{\cos a - \cos (b + c)}{\sin b \sin c}.$

Then by (B), $2\cos^2\frac{1}{2}A = \frac{2\sin\frac{1}{2}(b+c+a)\sin\frac{1}{2}(b+c-a)}{\sin b \sin c}.$

Putting $a+b+c=2s$, whence $b+c-a=2(s-a)$, we have

$$\cos^2\tfrac{1}{2}A = \frac{\sin s \sin(s-a)}{\sin b \sin c}.$$

Or, $$\cos\tfrac{1}{2}A = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}. \qquad (97)$$

In like manner, $$\cos\tfrac{1}{2}B = \sqrt{\frac{\sin s \sin(s-b)}{\sin c \sin a}}, \qquad (98)$$

and $$\cos\tfrac{1}{2}C = \sqrt{\frac{\sin s \sin(s-c)}{\sin a \sin b}}. \qquad (99)$$

Dividing (94) by (97), we have

$$\tan\tfrac{1}{2}A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}\sqrt{\frac{\sin b \sin c}{\sin s \sin(s-a)}}$$

$$= \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}}. \qquad (100)$$

In like manner, $$\tan\tfrac{1}{2}B = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s \sin(s-b)}}, \qquad (101)$$

and $$\tan\tfrac{1}{2}C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s \sin(s-c)}}. \qquad (102)$$

155. *To express the sines, cosines, and tangents of the half-sides of a spherical triangle in terms of the angles of the triangle.*

From (91), § 153, $\sin B \sin C \cos a = \cos A + \cos B \cos C$.

Whence, $$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}. \qquad \text{(A)}$$

Then, $$1 - \cos a = 1 - \frac{\cos A + \cos B \cos C}{\sin B \sin C}.$$

Or, $$2\sin^2\tfrac{1}{2}a = \frac{-(\cos B \cos C - \sin B \sin C) - \cos A}{\sin B \sin C}$$

$$= -\frac{\cos(B+C) + \cos A}{\sin B \sin C}.$$

Then by (19), $$2\sin^2\tfrac{1}{2}a = -\frac{2\cos\frac{1}{2}(B+C+A)\cos\frac{1}{2}(B+C-A)}{\sin B \sin C}.$$

Denoting the sum of the angles, $A+B+C$, by $2S$, we have

$$B+C-A=2(S-A).$$

Whence, $$\sin^2 \tfrac{1}{2} a = -\frac{\cos S \cos (S-A)}{\sin B \sin C}.$$

Or, $$\sin \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos (S-A)}{\sin B \sin C}}. \tag{103}$$

In like mannner, $$\sin \tfrac{1}{2} b = \sqrt{-\frac{\cos S \cos (S-B)}{\sin C \sin A}}, \tag{104}$$

and $$\sin \tfrac{1}{2} c = \sqrt{-\frac{\cos S \cos (S-C)}{\sin A \sin B}}. \tag{105}$$

Again, adding both members of (A) to 1, we have

$$1+\cos a = 1 + \frac{\cos A + \cos B \cos C}{\sin B \sin C} = \frac{\cos A + \cos B \cos C + \sin B \sin C}{\sin B \sin C}.$$

Then, $$2\cos^2 \tfrac{1}{2} a = \frac{\cos A + \cos (B-C)}{\sin B \sin C}$$

$$= \frac{2 \cos \frac{1}{2}[A+B-C] \cos \frac{1}{2}[A-(B-C)]}{\sin B \sin C}.$$

Or, $$\cos^2 \tfrac{1}{2} a = \frac{\cos \frac{1}{2}(A+B-C) \cos \frac{1}{2}(A-B+C)}{\sin B \sin C}.$$

But $A+B-C=2(S-C)$, and $A-B+C=2(S-B)$.

Whence, $$\cos^2 \tfrac{1}{2} a = \frac{\cos (S-B) \cos (S-C)}{\sin B \sin C}.$$

Or, $$\cos \tfrac{1}{2} a = \sqrt{\frac{\cos (S-B) \cos (S-C)}{\sin B \sin C}}. \tag{106}$$

In like manner, $$\cos \tfrac{1}{2} b = \sqrt{\frac{\cos (S-C) \cos (S-A)}{\sin C \sin A}}, \tag{107}$$

and $$\cos \tfrac{1}{2} c = \sqrt{\frac{\cos (S-A) \cos (S-B)}{\sin A \sin B}}. \tag{108}$$

Dividing (**103**) by (**106**), we have

$$\tan \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)}}. \tag{109}$$

In like manner, $$\tan \tfrac{1}{2} b = \sqrt{-\frac{\cos S \cos (S-B)}{\cos (S-C) \cos (S-A)}}, \tag{110}$$

and $$\tan \tfrac{1}{2} c = \sqrt{-\frac{\cos S \cos (S-C)}{\cos (S-A) \cos (S-B)}}. \tag{111}$$

NAPIER'S ANALOGIES.

156. Dividing (**100**) by (**101**), we have

$$\frac{\tan\frac{1}{2}A}{\tan\frac{1}{2}B}=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}\sqrt{\frac{\sin s\sin(s-b)}{\sin(s-c)\sin(s-a)}}.$$

Or,
$$\frac{\sin\frac{1}{2}A\cos\frac{1}{2}B}{\cos\frac{1}{2}A\sin\frac{1}{2}B}=\sqrt{\frac{\sin^2(s-b)}{\sin^2(s-a)}}=\frac{\sin(s-b)}{\sin(s-a)}.$$

Whence by composition and division,

$$\frac{\sin\frac{1}{2}A\cos\frac{1}{2}B+\cos\frac{1}{2}A\sin\frac{1}{2}B}{\sin\frac{1}{2}A\cos\frac{1}{2}B-\cos\frac{1}{2}A\sin\frac{1}{2}B}=\frac{\sin(s-b)+\sin(s-a)}{\sin(s-b)-\sin(s-a)}.$$

Then by (**9**), (**11**), and (**21**),

$$\frac{\sin(\frac{1}{2}A+\frac{1}{2}B)}{\sin(\frac{1}{2}A-\frac{1}{2}B)}=\frac{\tan\frac{1}{2}[s-b+s-a]}{\tan\frac{1}{2}[s-b-(s-a)]}.$$

But
$$s-b+s-a=2s-a-b=c.$$

Whence,
$$\frac{\sin\frac{1}{2}(A+B)}{\sin\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)}. \qquad (\mathbf{112})$$

157. Multiplying (**100**) by (**101**), we have

$$\tan\tfrac{1}{2}A\tan\tfrac{1}{2}B=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}\sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s\sin(s-b)}}.$$

Or,
$$\frac{\sin\frac{1}{2}A\sin\frac{1}{2}B}{\cos\frac{1}{2}A\cos\frac{1}{2}B}=\sqrt{\frac{\sin^2(s-c)}{\sin^2 s}}=\frac{\sin(s-c)}{\sin s}.$$

Whence by composition and division,

$$\frac{\cos\frac{1}{2}A\cos\frac{1}{2}B-\sin\frac{1}{2}A\sin\frac{1}{2}B}{\cos\frac{1}{2}A\cos\frac{1}{2}B+\sin\frac{1}{2}A\sin\frac{1}{2}B}=\frac{\sin s-\sin(s-c)}{\sin s+\sin(s-c)}.$$

Or, by (**21**),
$$\frac{\cos(\frac{1}{2}A+\frac{1}{2}B)}{\cos(\frac{1}{2}A-\frac{1}{2}B)}=\frac{\tan\frac{1}{2}[s-(s-c)]}{\tan\frac{1}{2}[s+s-c]}.$$

But
$$s+s-c=2s-c=a+b.$$

Whence,
$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)}. \qquad (\mathbf{113})$$

158. Applying formula (**112**) to the triangle $A'B'C'$, in the figure of § 153, we obtain

$$\frac{\sin\frac{1}{2}(A'+B')}{\sin\frac{1}{2}(A'-B')}=\frac{\tan\frac{1}{2}c'}{\tan\frac{1}{2}(a'-b')}.$$

But, $\frac{1}{2}(A'+B') = \frac{1}{2}(180° - a + 180° - b) = 180° - \frac{1}{2}(a+b)$;

$$\frac{1}{2}(A'-B') = \frac{1}{2}(180° - a - 180° + b) = -\frac{1}{2}(a-b);$$

$$\frac{1}{2}c' = \frac{1}{2}(180° - C) = 90° - \frac{1}{2}C;$$

and $\frac{1}{2}(a'-b') = \frac{1}{2}(180° - A - 180° + B) = -\frac{1}{2}(A-B)$.

Whence, $$\frac{\sin[180° - \frac{1}{2}(a+b)]}{\sin[-\frac{1}{2}(a-b)]} = \frac{\tan(90° - \frac{1}{2}C)}{\tan[-\frac{1}{2}(A-B)]}.$$

Therefore, by §§ 29, 32, and 33,

$$\frac{\sin\frac{1}{2}(a+b)}{-\sin\frac{1}{2}(a-b)} = \frac{\cot\frac{1}{2}C}{-\tan\frac{1}{2}(A-B)}.$$

Or, $$\frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)} = \frac{\cot\frac{1}{2}C}{\tan\frac{1}{2}(A-B)}. \quad \textbf{(114)}$$

In like manner, from (**113**), we obtain

$$\frac{\cos\frac{1}{2}(A'+B')}{\cos\frac{1}{2}(A'-B')} = \frac{\tan\frac{1}{2}c'}{\tan\frac{1}{2}(a'+b')}.$$

But, $\frac{1}{2}(a'+b') = \frac{1}{2}(180° - A + 180° - B) = 180° - \frac{1}{2}(A+B)$.

Whence, $$\frac{\cos[180° - \frac{1}{2}(a+b)]}{\cos[-\frac{1}{2}(a-b)]} = \frac{\tan(90° - \frac{1}{2}C)}{\tan[180° - \frac{1}{2}(A+B)]}.$$

Therefore, by §§ 29, 32, and 33,

$$\frac{-\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)} = \frac{\cot\frac{1}{2}C}{-\tan\frac{1}{2}(A+B)}.$$

Or, $$\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)} = \frac{\cot\frac{1}{2}C}{\tan\frac{1}{2}(A+B)}. \quad \textbf{(115)}$$

159. The formulæ exemplified in §§ 156, 157, and 158 are known as *Napier's Analogies.* In each case there may be other forms according as other elements are used.

SOLUTION OF OBLIQUE SPHERICAL TRIANGLES.

160. In the solution of oblique spherical triangles, we may distinguish six cases:

1. *Given a side and the adjacent angles.*
2. *Given two sides and their included angle.*
3. *Given the three sides.*
4. *Given the three angles.*
5. *Given two sides and the angle opposite to one of them.*
6. *Given two angles and the side opposite to one of them.*

By application of the principles of § 131, 6, the solution of an example under Case 2, 4, or 6, may be made to depend upon the solution of an example under Case 1, 3, or 5, respectively; and *vice versa.*

Hence, it is not essential to consider more than *three* cases in the solution of oblique spherical triangles.

The student must carefully bear in mind the remarks made in §§ 146 and 147.

161. CASE I. *Given a side and the adjacent angles.*

1. Given $A = 70°$, $B = 132°$, $c = 116°$; find a, b, and C.

By Napier's Analogies (§§ 156, 157), we have

$$\frac{\sin \frac{1}{2}(B+A)}{\sin \frac{1}{2}(B-A)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(b-a)}, \text{ and } \frac{\cos \frac{1}{2}(B+A)}{\cos \frac{1}{2}(B-A)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(b+a)}.$$

Whence, $\tan \frac{1}{2}(b-a) = \sin \frac{1}{2}(B-A) \csc \frac{1}{2}(B+A) \tan \frac{1}{2}c$,

and $\overset{-}{\tan} \frac{1}{2}(b+a) = \overset{+}{\cos} \frac{1}{2}(B-A) \overset{-}{\sec} \frac{1}{2}(B+A) \overset{+}{\tan} \frac{1}{2}c$.

From the data, $\frac{1}{2}(B-A) = 31°$, $\frac{1}{2}(B+A) = 101°$, $\frac{1}{2}c = 58°$.

$$\begin{aligned} \log \sin \tfrac{1}{2}(B-A) &= 9.711839 - 10 \\ \log \csc \tfrac{1}{2}(B+A) &= 0.008053 \\ \log \tan \tfrac{1}{2}c &= 0.204211 \\ \hline \log \tan \tfrac{1}{2}(b-a) &= 9.924103 - 10 \\ \tfrac{1}{2}(b-a) &= 40°\,1'\,7.7''. \end{aligned} \qquad \begin{aligned} \log \cos \tfrac{1}{2}(B-A) &= 9.933066 - 10 \\ \log \sec \tfrac{1}{2}(B+A) &= 0.719401 \\ \log \tan \tfrac{1}{2}c &= 0.204211 \\ \hline \log \tan \tfrac{1}{2}(b+a) &= 0.856678 \\ 180° - \tfrac{1}{2}(b+a) &= 82°\,4'\,51.8''. \\ \tfrac{1}{2}(b+a) &= 97°\,55'\,8.2''. \end{aligned}$$

Then, $a = \frac{1}{2}(b+a) - \frac{1}{2}(b-a) = 57°\,54'\,0.5''$,

and $b = \frac{1}{2}(b+a) + \frac{1}{2}(b-a) = 137°\,56'\,15.9''$.

To find C, we have by § 158,

$$\cot \tfrac{1}{2}C = \frac{\sin \frac{1}{2}(b+a)}{\sin \frac{1}{2}(b-a)} \tan \tfrac{1}{2}(B-A) = \sin \tfrac{1}{2}(b+a) \csc \tfrac{1}{2}(b-a) \tan \tfrac{1}{2}(B-A).$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(b+a) &= 9.995839 - 10 \\ \log \csc \tfrac{1}{2}(b-a) &= 0.191763 \\ \log \tan \tfrac{1}{2}(B-A) &= 9.778774 - 10 \\ \hline \log \cot \tfrac{1}{2}C &= 9.966376 - 10 \\ \tfrac{1}{2}C &= 47°\,12'\,56.7'', \text{ and } C = 94°\,25'\,53.4''. \end{aligned}$$

Note 1. The value of C may also be found by the formula

$$\cot \tfrac{1}{2}C = \frac{\cos \frac{1}{2}(b+a)}{\cos \frac{1}{2}(b-a)} \tan \tfrac{1}{2}(B+A) \quad (§\ 158).$$

Note 2. The triangle is possible for any values of the given elements.

EXAMPLES.

Solve the following spherical triangles:

2. Given $A = 78°$, $B = 41°$, $c = 108°$.

3. Given $B = 115°$, $C = 50°$, $a = 70° \, 20'$.

4. Given $A = 31° \, 40'$, $C = 122° \, 20'$, $b = 40° \, 40'$.

5. Given $A = 108° \, 12'$, $B = 145° \, 46'$, $c = 126° \, 32'$.

162. Case II. *Given two sides and their included angle.*

1. Given $b = 138°$, $c = 116°$, $A = 70°$; find B, C, and a.

By Napier's Analogies (§ 158), we have

$$\frac{\sin \frac{1}{2}(b+c)}{\sin \frac{1}{2}(b-c)} = \frac{\cot \frac{1}{2} A}{\tan \frac{1}{2}(B-C)}, \text{ and } \frac{\cos \frac{1}{2}(b+c)}{\cos \frac{1}{2}(b-c)} = \frac{\cot \frac{1}{2} A}{\tan \frac{1}{2}(B+C)}.$$

Whence, $\tan \frac{1}{2}(B - C) = \sin \frac{1}{2}(b - c) \csc \frac{1}{2}(b + c) \cot \frac{1}{2} A,$

and $$\tan \tfrac{1}{2}(\overset{-}{B + C}) = \cos \tfrac{1}{2}(\overset{+}{b - c}) \sec \tfrac{1}{2}(\overset{-}{b + c}) \cot \tfrac{1}{2} \overset{+}{A}.$$

From the data, $\frac{1}{2}(b - c) = 11°$, $\frac{1}{2}(b + c) = 127°$, $\frac{1}{2} A = 35°$.

$$\begin{aligned} \log \sin \tfrac{1}{2}(b-c) &= 9.280599 - 10 \\ \log \csc \tfrac{1}{2}(b+c) &= 0.097651 \\ \log \cot \tfrac{1}{2} A &= \underline{0.154773} \\ \log \tan \tfrac{1}{2}(B-C) &= 9.533023 - 10 \\ \tfrac{1}{2}(B-C) &= 18° \, 50' \, 24.7''. \end{aligned} \qquad \begin{aligned} \log \cos \tfrac{1}{2}(b-c) &= 9.991947 - 10 \\ \log \sec \tfrac{1}{2}(b+c) &= 0.220537 \\ \log \cot \tfrac{1}{2} A &= \underline{0.154773} \\ \log \tan \tfrac{1}{2}(B+C) &= 0.367257 \\ 180° - \tfrac{1}{2}(B+C) &= 66° \, 46' \, 1.2''. \\ \tfrac{1}{2}(B+C) &= 113° \, 13' \, 58.8''. \end{aligned}$$

Then, $$B = \tfrac{1}{2}(B + C) + \tfrac{1}{2}(B - C) = 132° \, 4' \, 23.5'',$$

and $$C = \tfrac{1}{2}(B + C) - \tfrac{1}{2}(B - C) = 94° \, 23' \, 34.1''.$$

To find a, we have by § 156,

$$\tan \tfrac{1}{2} a = \frac{\sin \frac{1}{2}(B+C)}{\sin \frac{1}{2}(B-C)} \tan \tfrac{1}{2}(b-c) = \sin \tfrac{1}{2}(B+C) \csc \tfrac{1}{2}(B-C) \tan \tfrac{1}{2}(b-c).$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(B+C) &= 9.963272 - 10 \\ \log \csc \tfrac{1}{2}(B-C) &= 0.490892 \\ \log \tan \tfrac{1}{2}(b-c) &= \underline{9.288652 - 10} \\ \log \tan \tfrac{1}{2} a &= 9.742816 - 10 \end{aligned}$$

$$\tfrac{1}{2} a = 28° \, 56' \, 51.6'', \text{ and } a = 57° \, 53' \, 43.2''.$$

Note. The triangle is possible for any values of the given elements.

EXAMPLES.

Solve the following spherical triangles:

2. Given $a = 72°$, $b = 47°$, $C = 33°$.

3. Given $a = 98°$, $c = 60°$, $B = 110°$.

4. Given $b = 70° 40'$, $c = 120° 20'$, $A = 50°$.

5. Given $a = 125° 10'$, $b = 153° 50'$, $C = 140° 20'$.

163. Case III. *Given the three sides.*

The angles may be calculated by the formulæ of § 154.

If all the angles are to be computed, the *tangent* formulæ are the most convenient, since only four different logarithms are required. If but one angle is required, the *cosine* formula will be found to involve the least work.

The triangle is possible for any values of the data, provided that no side is greater than the sum of the other two, and that the sum of the sides is less than 360° (§ 131, 1 and 3).

If all the angles are required, and the tangent formulæ are used, it is convenient to modify them as follows.

By (**100**),
$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s \sin^2(s-a)}}$$
$$= \frac{1}{\sin(s-a)}\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.$$

Denoting $\sqrt{\dfrac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}$ by k, we have
$$\tan \tfrac{1}{2} A = \frac{k}{\sin(s-a)}.$$

In like manner, $\tan \tfrac{1}{2} B = \dfrac{k}{\sin(s-b)}$, and $\tan \tfrac{1}{2} C = \dfrac{k}{\sin(s-c)}$.

1. Given $a = 57°$, $b = 137°$, $c = 116°$; find A, B, and C.

Here, $2s = a + b + c = 310°$.

Whence, $s = 155°$, $s - a = 98°$, $s - b = 18°$, $s - c = 39°$.

$$\begin{aligned}
\log \sin(s-a) &= 9.995753 - 10\\
\log \sin(s-b) &= 9.489982 - 10\\
\log \sin(s-c) &= 9.798872 - 10\\
\log \csc s &= 0.374052\\
2)&\overline{19.658659 - 20}\\
\log k &= 9.829330 - 10\\
\log \sin(s-a) &= 9.995753 - 10\\
\log \tan \tfrac{1}{2} A &= 9.833577 - 10\\
\tfrac{1}{2} A &= 34° 16' 52.5''.\\
A &= 68° 33' 45.0''.
\end{aligned}$$

$$\begin{aligned}
\log k &= 9.829330 - 10\\
\log \sin(s-b) &= 9.489982 - 10\\
\log \tan \tfrac{1}{2} B &= 0.339348\\
\tfrac{1}{2} B &= 65° 24' 10.4''.\\
B &= 130° 48' 20.8''\\
\log k &= 9.829330 - 10\\
\log \sin(s-c) &= 9.798872 - 10\\
\log \tan \tfrac{1}{2} C &= 0.030458\\
\tfrac{1}{2} C &= 47° 0' 27.0''.\\
C &= 94° 0' 54.0''.
\end{aligned}$$

EXAMPLES.

Solve the following spherical triangles:

2. Given $a = 38°$, $b = 42°$, $c = 51°$.

3. Given $a = 101°$, $b = 49°$, $c = 60°$.

4. Given $a = 126°$, $b = 152°$, $c = 75°$.

5. Given $a = 62° 20'$, $b = 54° 10'$, $c = 97° 50'$; find A.

164. Case IV. *Given the three angles.*

The sides may be calculated by the formulæ of § 155.

If all the sides are to be computed, the tangent formulæ are the most convenient, since only four different logarithms are required. If but one side is required, the sine formula will be found to involve the least work.

The triangle is possible for any values of the data, provided that the sum of the angles is between 180° and 540° (§ 131, 4), and that each of the quantities $B + C - A$, $C + A - B$, and $A + B - C$ is between 180° and $-180°$ (§ 134).

For such values of the angles, S is between 90° and 270°, and each of the quantities $S - A$, $S - B$, and $S - C$ between 90° and $-90°$; then, $\cos S$ is $-$, while the cosines of $S - A$, $S - B$, and $S - C$ are $+$ (§ 20).

Hence, the expressions under the radical signs in the formulæ are essentially positive, and no attention need be paid to the algebraic signs.

If all the sides are required, and the tangent formulæ are used, it is convenient to modify them as follows:

By (**109**),
$$\tan \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos^2 (S - A)}{\cos (S - A) \cos (S - B) \cos (S - C)}}$$

$$= \cos (S - A) \sqrt{-\frac{\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}.$$

Denoting $\sqrt{-\dfrac{\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}$ by K,

we have
$$\tan \tfrac{1}{2} a = K \cos (S - A).$$

In like manner, $\tan \frac{1}{2} b = K \cos (S - B)$, and $\tan \frac{1}{2} c = K \cos (S - C)$.

1. Given $A = 150°$, $B = 131°$, $C = 115°$; find a, b, and c.

Here,
$$2 S = A + B + C = 396°.$$

Whence, $S = 198°$, $S - A = 48°$, $S - B = 67°$, $S - C = 83°$.

$$\begin{aligned} \log\cos S &= 9.978206-10 \\ \log\sec(S-A) &= 0.174489 \\ \log\sec(S-B) &= 0.408122 \\ \log\sec(S-C) &= 0.914106 \\ 2)&\overline{1.474923} \\ \log K &= 0.737462 \\ \log\cos(S-A) &= \underline{9.825511-10} \\ \log\tan\tfrac{1}{2}a &= 0.562973 \\ \tfrac{1}{2}a &= 74^\circ\,42'\,4.8''. \\ a &= 149^\circ\,24'\,9.6''. \end{aligned}$$

$$\begin{aligned} \log K &= 0.737462 \\ \log\cos(S-B) &= \underline{9.591878-10} \\ \log\tan\tfrac{1}{2}b &= 0.329340 \\ \tfrac{1}{2}b &= 64^\circ\,53'\,58.0''. \\ b &= 129^\circ\,47'\,56.0''. \\ \log K &= 0.737462 \\ \log\cos(S-C) &= \underline{9.085894-10} \\ \log\tan\tfrac{1}{2}c &= 9.823356-10 \\ \tfrac{1}{2}c &= 33^\circ\,39'\,23.1''. \\ c &= 67^\circ\,18'\,46.2''. \end{aligned}$$

Note 1. By § 35, $\cos 198^\circ = -\sin 108^\circ = -\cos 18^\circ$; whence, without regard to algebraic sign, $\log\cos 198^\circ = \log\cos 18^\circ$.

2. Given $A = 123^\circ$, $B = 45^\circ$, $C = 58^\circ$; find a.

By (**103**), $$\sin\tfrac{1}{2}a = \sqrt{-\frac{\cos S\cos(S-A)}{\sin B\sin C}}.$$

Here, $2S = A + B + C = 226^\circ$; whence, $S = 113^\circ$, and $S - A = -10^\circ$.

$$\begin{aligned} \log\cos S &= 9.591878-10 \\ \log\cos(S-A) &= 9.993351-10 \\ \log\csc B &= 0.150515 \\ \log\csc C &= 0.071580 \\ 2)&\overline{19.807324-20} \\ \log\sin\tfrac{1}{2}a &= 9.903662-10 \end{aligned}$$

$\frac{1}{2}a = 53^\circ\,13'\,51.3''$, and $a = 106^\circ\,27'\,42.6''$.

Note 2. By § 29, $\cos(-10^\circ) = \cos 10^\circ$.

EXAMPLES.

Solve the following spherical triangles:

3. Given $A = 74^\circ$, $B = 82^\circ$, $C = 67^\circ$.

4. Given $A = 120^\circ$, $B = 130^\circ$, $C = 140^\circ$.

5. Given $A = 138^\circ\,16'$, $B = 33^\circ\,11'$, $C = 36^\circ\,53'$.

6. Given $A = 91^\circ\,10'$, $B = 85^\circ\,40'$, $C = 78^\circ\,30'$; find b.

165. Case V. *Given two sides and the angle opposite to one of them.*

1. Given $a = 58^\circ$, $b = 137^\circ$, $B = 131^\circ$; find A, C, and c.

By (**85**), $$\frac{\sin A}{\sin B} = \frac{\sin a}{\sin b}, \text{ or } \sin A = \sin a \csc b \sin B.$$

$$\begin{aligned} \log \sin a &= 9.928420 - 10 \\ \log \csc b &= 0.166217 \\ \log \sin B &= 9.877780 - 10 \\ \hline \log \sin A &= 9.972417 - 10 \end{aligned}$$

$$A = 69^\circ\, 47'\, 41.6'', \text{ or } 110^\circ\, 12'\, 18.4'' \text{ (§ 147).}$$

To find C and c, we have by §§ 156 and 158,

$$\cot \tfrac{1}{2} C = \sin \tfrac{1}{2}(b + a) \csc \tfrac{1}{2}(b - a) \tan \tfrac{1}{2}(B - A),$$

and

$$\tan \tfrac{1}{2} c = \sin \tfrac{1}{2}(B + A) \csc \tfrac{1}{2}(B - A) \tan \tfrac{1}{2}(b - a).$$

Using the first value of A, we have

$$\tfrac{1}{2}(B + A) = 100^\circ\, 23'\, 50.8'', \text{ and } \tfrac{1}{2}(B - A) = 30^\circ\, 36'\, 9.2''.$$

Also, $\tfrac{1}{2}(b + a) = 97^\circ\, 30'$, and $\tfrac{1}{2}(b - a) = 39^\circ\, 30'$.

$$\begin{aligned} \log \sin \tfrac{1}{2}(b + a) &= 9.996269 - 10 \\ \log \csc \tfrac{1}{2}(b - a) &= 0.196489 \\ \log \tan \tfrac{1}{2}(B - A) &= 9.771924 - 10 \\ \hline \log \cot \tfrac{1}{2} C &= 9.964682 - 10 \\ \tfrac{1}{2} C &= 47^\circ\, 19'\, 37.8''. \\ C &= 94^\circ\, 39'\, 15.6''. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(B + A) &= 9.992810 - 10 \\ \log \csc \tfrac{1}{2}(B - A) &= 0.293214 \\ \log \tan \tfrac{1}{2}(b - a) &= 9.916104 - 10 \\ \hline \log \tan \tfrac{1}{2} c &= 0.202128 \\ \tfrac{1}{2} c &= 57^\circ\, 52'\, 35.0''. \\ c &= 115^\circ\, 45'\, 10.0''. \end{aligned}$$

Using the second value of A, we have

$$\tfrac{1}{2}(B + A) = 120^\circ\, 36'\, 9.2'', \text{ and } \tfrac{1}{2}(B - A) = 10^\circ\, 23'\, 50.8''.$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(b + a) &= 9.996269 - 10 \\ \log \csc \tfrac{1}{2}(b - a) &= 0.196489 \\ \log \tan \tfrac{1}{2}(B - A) &= 9.263608 - 10 \\ \hline \log \cot \tfrac{1}{2} C &= 9.456366 - 10 \\ \tfrac{1}{2} C &= 74^\circ\, 2'\, 22.1''. \\ C &= 148^\circ\, 4'\, 44.2''. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(B + A) &= 9.934861 - 10 \\ \log \csc \tfrac{1}{2}(B - A) &= 0.743583 \\ \log \tan \tfrac{1}{2}(b - a) &= 9.916104 - 10 \\ \hline \log \tan \tfrac{1}{2} c &= 0.594548 \\ \tfrac{1}{2} c &= 75^\circ\, 43'\, 43.6''. \\ c &= 151^\circ\, 27'\, 27.2''. \end{aligned}$$

Thus, the two solutions are:

1. $A = 69^\circ\, 47'\, 41.6''$, $C = 94^\circ\, 39'\, 15.6''$, $c = 115^\circ\, 45'\, 10.0''$.
2. $A = 110^\circ\, 12'\, 18.4''$, $C = 148^\circ\, 4'\, 44.2''$, $c = 151^\circ\, 27'\, 27.2''$.

As in the corresponding case in the solution of oblique plane triangles (compare §§ 117 to 120), there may sometimes be two solutions, sometimes only one, and sometimes none, in an example under Case V.

After the two values of A have been obtained, the number of solutions may be readily determined by inspection; for, by § 131, 2, if a is $< b$, A must be $< B$; and if a is $> b$, A must be $> B$.

Hence, *only those values of A can be retained which are greater or less than B according as a is greater or less than b.*

Thus, in Ex. 1, a is given $< b$; and since both values of A are $< B$, we have two solutions.

Again, if the data are such as to make log sin A positive, there will be no solution corresponding.

2. Given $a = 58°$, $c = 116°$, $C = 94° \, 50'$; find A.

In this case, $\dfrac{\sin A}{\sin C} = \dfrac{\sin a}{\sin c}$, or $\sin A = \sin a \csc c \sin C$.

$$\begin{aligned}
\log \sin a &= 9.928420 - 10 \\
\log \csc c &= 0.046340 \\
\log \sin C &= \underline{9.998453 - 10} \\
\log \sin A &= 9.973213 - 10
\end{aligned}$$

$$A = 70° \, 4' \, 57.1'', \text{ or } 109° \, 55' \, 2.9''.$$

Since a is given $< c$, only values of A which are $< C$ can be retained; then the only solution is $A = 70° \, 4' \, 57.1''$.

3. Given $b = 126°$, $c = 70°$, $B = 56°$; find C.

In this case, $\dfrac{\sin C}{\sin B} = \dfrac{\sin c}{\sin b}$, or $\sin C = \sin c \csc b \sin B$.

$$\begin{aligned}
\log \sin c &= 9.972986 - 10 \\
\log \csc b &= 0.092042 \\
\log \sin B &= \underline{9.918574 - 10} \\
\log \sin C &= 9.983602 - 10
\end{aligned}$$

$$C = 74° \, 21' \, 13.8'', \text{ or } 105° \, 38' \, 46.2''.$$

Since both values of C are $> B$, while c is given $< b$, there is no solution.

EXAMPLES.

Solve the following spherical triangles:

4. Given $b = 99° \, 40'$, $c = 64° \, 20'$, $B = 95° \, 40'$.

5. Given $a = 40°$, $b = 118° \, 20'$, $A = 29° \, 40'$.

6. Given $a = 115° \, 20'$, $c = 146° \, 20'$, $C = 141° \, 10'$.

7. Given $a = 109° \, 20'$, $c = 82° \, 1' \, 8''$, $A = 107° \, 40'$.

8. Given $b = 108° \, 30'$, $c = 40° \, 50'$, $C = 39° \, 50'$.

9. Given $a = 162° \, 20'$, $b = 15° \, 40'$, $B = 125°$.

10. Given $a = 55°$, $c = 138° \, 10'$, $A = 42° \, 30'$.

166. Case VI. *Given two angles and the side opposite to one of them.*

1. Given $A = 110°$, $B = 122°$, $b = 129°$; find a, c, and C.

By (85), $\frac{\sin a}{\sin b} = \frac{\sin A}{\sin B}$, or $\sin a = \sin A \csc B \sin b$.

$$\begin{aligned} \log \sin A &= 9.972986 - 10 \\ \log \csc B &= 0.071580 \\ \log \sin b &= \underline{9.890503 - 10} \\ \log \sin a &= 9.935069 - 10 \end{aligned}$$

$$a = 59° 26' 37.6''; \text{ or } 120° 33' 22.4'' \text{ (§ 147).}$$

To find c and C, we have by §§ 156 and 158,

$$\tan \tfrac{1}{2} c = \sin \tfrac{1}{2} (B + A) \csc \tfrac{1}{2} (B - A) \tan \tfrac{1}{2} (b - a),$$

and $$\cot \tfrac{1}{2} C = \sin \tfrac{1}{2} (b + a) \csc \tfrac{1}{2} (b - a) \tan \tfrac{1}{2} (B - A).$$

Using the first value of a, we have

$$\tfrac{1}{2} (b + a) = 94° 13' 18.8'', \text{ and } \tfrac{1}{2} (b - a) = 34° 46' 41.2''.$$

Also, $$\tfrac{1}{2} (B + A) = 116°, \text{ and } \tfrac{1}{2} (B - A) = 6°.$$

$$\begin{aligned} \log \sin \tfrac{1}{2} (B + A) &= 9.953660 - 10 \\ \log \csc \tfrac{1}{2} (B - A) &= 0.980765 \\ \log \tan \tfrac{1}{2} (b - a) &= \underline{9.841642 - 10} \\ \log \tan \tfrac{1}{2} c &= 0.776067 \\ \tfrac{1}{2} c &= 80° 29' 34.8''. \\ c &= 160° 59' \ 9.6''. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2} (b + a) &= 9.998820 - 10 \\ \log \csc \tfrac{1}{2} (b - a) &= 0.243821 \\ \log \tan \tfrac{1}{2} (B - A) &= \underline{9.021620 - 10} \\ \log \cot \tfrac{1}{2} C &= 9.264261 - 10 \\ \tfrac{1}{2} C &= 79° 35' 14.1''. \\ C &= 159° 10' 28.2''. \end{aligned}$$

Using the second value of a, we have

$$\tfrac{1}{2} (b + a) = 124° 46' 41.2'', \text{ and } \tfrac{1}{2} (b - a) = 4° 13' 18.8''.$$

$$\begin{aligned} \log \sin \tfrac{1}{2} (B + A) &= 9.953660 - 10 \\ \log \csc \tfrac{1}{2} (B - A) &= 0.980765 \\ \log \tan \tfrac{1}{2} (b - a) &= \underline{8.868171 - 10} \\ \log \tan \tfrac{1}{2} c &= 9.802596 - 10 \\ \tfrac{1}{2} c &= 32° 24' 17.8''. \\ c &= 64° 48' 35.6''. \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2} (b + a) &= 9.914537 - 10 \\ \log \csc \tfrac{1}{2} (b - a) &= 1.133009 \\ \log \tan \tfrac{1}{2} (B - A) &= \underline{9.021620 - 10} \\ \log \cot \tfrac{1}{2} C &= 0.069166 \\ \tfrac{1}{2} C &= 40° 27' 24.1''. \\ C &= 80° 54' 48.2''. \end{aligned}$$

Thus the two solutions are:

1. $a = 59° 26' 37.6''$, $c = 160° 59' \ 9.6''$, $C = 159° 10' 28.2''$.
2. $a = 120° 33' 22.4''$, $c = 64° 48' 35.6''$, $C = 80° 54' 48.2''$.

In examples in Case VI., as well as in Case V., there may sometimes be two solutions, sometimes only one, and sometimes none.

As in Case V., *only those values of a can be retained which are greater or less than b according as A is greater or less than B.*

Also, if $\log \sin a$ is positive, the triangle is impossible.

EXAMPLES.

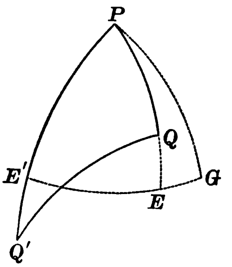
Solve the following spherical triangles:

2.	Given $B = 116°$,	$C = 80°$,	$c = 84°$.
3.	Given $A = 132°$,	$B = 140°$,	$b = 127°$.
4.	Given $A = 62°$,	$C = 101° 58' 24''$,	$a = 64° 30'$.
5.	Given $A = 133° 50'$,	$B = 66° 30'$,	$a = 81° 10'$.
6.	Given $B = 22° 20'$,	$C = 146° 40'$,	$c = 138° 20'$.
7.	Given $A = 61° 40'$,	$C = 140° 20'$,	$c = 150° 20'$.
8.	Given $B = 73°$,	$C = 81° 20'$,	$b = 122° 40'$.

APPLICATIONS.

167. In problems concerning navigation, the earth may be regarded as a sphere.

The *shortest distance* between any two points on the surface is the arc of a great circle which joins them; and the angles between this arc and the meridians of the points determine the *bearings* of the points from each other.



Thus, if Q and Q' are the points, and PQ and PQ' their meridians, the angle PQQ' determines the bearing of Q' from Q, and the angle $PQ'Q$ determines the bearing of Q from Q'.

If the latitudes and longitudes of Q and Q' are known, the arc QQ' and the angles PQQ' and $PQ'Q$ may be determined by the solution of a spherical triangle.

For if EE' is the equator, and PG the meridian of Greenwich, we have

$$\angle QPQ' = \angle Q'PG - \angle QPG = \text{longitude } Q' - \text{longitude } Q.$$

Also, $PQ = PE - QE = 90° -$ latitude Q,

and $PQ' = PE' + Q'E' = 90° +$ latitude Q'.

Thus, in the spherical triangle PQQ', two sides and their included angle are known, and the remaining elements may be computed.

When QQ' has been found in degrees, its length in miles may be calculated by finding the ratio of its arc to 360°, and multiplying the result by the length of the circumference of a great circle; in the following problems, the radius of the earth is taken as 3956 miles.

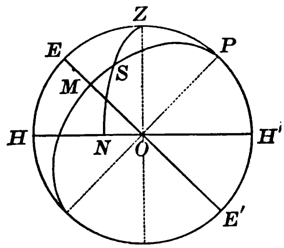
EXAMPLES.

1. Boston lies in lat. 42° 21′ N., lon. 71° 4′ W.; and the latitude of Greenwich is 51° 29′ N. Find the shortest distance in miles between the places, and the bearing of each place from the other.

2. Calcutta lies in lat. 22° 33′ N., lon. 88° 19′ E.; and Valparaiso in lat. 33° 2′ S., lon. 71° 42′ W. Find the shortest distance in miles between the places, and the bearing of each place from the other.

3. Sandy Hook lies in lat. 40° 28′ N., lon. 74° 1′ W.; and Queenstown in lat. 51° 50′ N., lon. 8° 19′ W. In what latitude does a great circle course from Sandy Hook to Queenstown cross the meridian of 50° W.?

168. The Astronomical Triangle.



Let O be the position of an observer on the surface of the earth; P the celestial north-pole; Z the zenith.

The great circle EE', having P for its pole, is called the *celestial equator;* and the great circle HH', having Z for its pole, is called the *horizon.*

Let S be the position of a star; PSM a meridian passing through S; ZSN a quadrant of a great circle passing through Z and S.

The arc SM is called the *declination* of the star; and is called declination *north* or *south,* according as the star is north or south of the celestial equator.

The angle SPZ is called the *hour-angle* of the star; the arc SN is called its *altitude;* the angle PZS, its *bearing* or *azimuth.*

The arc EZ is the latitude of the observer.

The spherical triangle SPZ is called the *Astronomical Triangle;* its sides have the following values:

$$SP = PM - SM = 90° - \text{the declination of the star};$$

$$SZ = ZN - SN = 90° - \text{the altitude of the star};$$

$$PZ = EP - EZ = 90° - \text{the latitude of the observer}.$$

Its angle SPZ is the hour-angle of the star, and its angle SZP the azimuth.

If any three of these five elements are known, the solution of a spherical triangle serves to determine the other two.

169. To Determine the Hour of the Day.

If the altitude and declination of the sun are known, and the latitude of the observer, the three sides of the triangle SPZ are known, and the hour-angle SPZ may be computed.

If 24 hours be multiplied by the ratio of this angle to 360°, we obtain the time required for the sun to move from S to the meridian EP.

If this time be subtracted from 12 o'clock, if the observation is made in the morning, or added, if made in the afternoon, we obtain the *hour of the day* at the time and place of observation.

If the Greenwich time of the observation be noted on a chronometer, the difference between this and the local time as calculated above serves to determine the *longitude* of the place of observation.

In reducing time to longitude, it should be remembered that 24 hours of time correspond to 360° of longitude; that is, one hour of time corresponds to 15° of longitude, one minute to 15′, and one second to 15″.

EXAMPLES.

170. 1. A mariner observes the altitude of the sun to be 14° 18′, its declination being 18° 36′ N. If the latitude of the vessel is 50° 13′ N., and the observation is made in the morning, find the hour of the day. If the observation is taken at 9 A.M., Greenwich time, what is the longitude of the vessel?

2. What will be the altitude of the sun at 4 P.M. in San Francisco, lat. 37° 48′ N., its declination being 12° S.?

3. What will be the bearing of the sun at 9.30 A.M. in Melbourne, lat. 37° 49′ S., if its declination is 6° S.?

4. At what hour will the sun rise in Boston, lat. 42° 21′ N., if its declination is 15° N.?

Note. At sunrise the sun's altitude is 0, so that the arc SZ becomes 90°.

FORMULÆ.

PLANE TRIGONOMETRY.

§ 29. $\left.\begin{array}{lll}\sin(-A)=-\sin A. & \tan(-A)=-\tan A. & \sec(-A)=\sec A.\\ \cos(-A)=\cos A. & \cot(-A)=-\cot A. & \csc(-A)=-\csc A.\end{array}\right\}$ (1)

§ 30. $\left.\begin{array}{ll}\sin(90^\circ+A)=\cos A. & \cot(90^\circ+A)=-\tan A.\\ \cos(90^\circ+A)=-\sin A. & \sec(90^\circ+A)=-\csc A.\\ \tan(90^\circ+A)=-\cot A. & \csc(90^\circ+A)=\sec A.\end{array}\right\}$ (2)

§ 36. $\left.\begin{array}{lll}\sin x=\dfrac{1}{\csc x}. & \tan x=\dfrac{1}{\cot x}. & \sec x=\dfrac{1}{\cos x}.\\ \cos x=\dfrac{1}{\sec x}. & \cot x=\dfrac{1}{\tan x}. & \csc x=\dfrac{1}{\sin x}.\end{array}\right\}$ (3)

§ 37. $$\tan x=\frac{\sin x}{\cos x}. \quad (4)$$

§ 38. $$\cot x=\frac{\cos x}{\sin x}. \quad (5)$$

§ 39. $$\sin^2 x+\cos^2 x=1. \quad (6)$$

§ 40. $\sec^2 x=1+\tan^2 x.$ (7) $\csc^2 x=1+\cot^2 x.$ (8)

§ 41 $$\sin(x+y)=\sin x\cos y+\cos x\sin y. \quad (9)$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y. \quad (10)$$

§ 43. $$\sin(x-y)=\sin x\cos y-\cos x\sin y. \quad (11)$$
$$\cos(x-y)=\cos x\cos y+\sin x\sin y. \quad (12)$$

§ 44. $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}.$ (13) $\cot(x+y)=\dfrac{\cot x\cot y-1}{\cot y+\cot x}.$ (15)

$\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}.$ (14) $\cot(x-y)=\dfrac{\cot x\cot y+1}{\cot y-\cot x}.$ (16)

§ 45. $$\sin x+\sin y=2\sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y). \quad (17)$$
$$\sin x-\sin y=2\cos\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y). \quad (18)$$
$$\cos x+\cos y=2\cos\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y). \quad (19)$$
$$\cos x-\cos y=-2\sin\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y). \quad (20)$$

§ 46. $$\frac{\sin x+\sin y}{\sin x-\sin y}=\frac{\tan\frac{1}{2}(x+y)}{\tan\frac{1}{2}(x-y)}. \quad (21)$$

§ 47. $$\sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y. \quad (22)$$
$$\sin(x+y)\sin(x-y)=\cos^2 y-\cos^2 x. \quad (23)$$
$$\cos(x+y)\cos(x-y)=\cos^2 x-\sin^2 y=\cos^2 y-\sin^2 x. \quad (24)$$

§ 48. $\sin 2x = 2\sin x\cos x.$ (25) $\cos 2x = 2\cos^2 x - 1.$ (28)

$\cos 2x = \cos^2 x - \sin^2 x.$ (26) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}.$ (29)

$\cos 2x = 1 - 2\sin^2 x.$ (27) $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}.$ (30)

§ 49. $2\sin^2 \tfrac{1}{2}x = 1 - \cos x.$ (31) $\tan\tfrac{1}{2}x = \dfrac{1-\cos x}{\sin x}.$ (33)

$2\cos^2\tfrac{1}{2}x = 1 + \cos x.$ (32) $\cot\tfrac{1}{2}x = \dfrac{1+\cos x}{\sin x}.$ (34)

§ 50. $\sin 3x = 3\sin x - 4\sin^3 x.$ (35) $\cos 3x = 4\cos^3 x - 3\cos x.$ (36)

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}. \quad (37)$$

§ 105. $4K = c^2 \sin 2A.$ (38) $2K = a^2\tan B.$ (42)

$4K = c^2\sin 2B.$ (39) $2K = b^2\tan A.$ (43)

$2K = a^2\cot A.$ (40) $2K = a\sqrt{(c+a)(c-a)}.$ (44)

$2K = b^2\cot B.$ (41) $2K = b\sqrt{(c+b)(c-b)}.$ (45)

$$2K = ab. \quad (46)$$

§ 107.

$$a : b = \sin A : \sin B. \quad (47)$$

$$b : c = \sin B : \sin C. \quad (48)$$

$$c : a = \sin C : \sin A. \quad (49)$$

§ 108.

$$\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}. \quad (50)$$

$$\frac{b+c}{b-c} = \frac{\tan\frac{1}{2}(B+C)}{\tan\frac{1}{2}(B-C)}. \quad (51)$$

$$\frac{c+a}{c-a} = \frac{\tan\frac{1}{2}(C+A)}{\tan\frac{1}{2}(C-A)}. \quad (52)$$

§ 109.

$$a^2 = b^2 + c^2 - 2bc\cos A. \quad (53)$$

$$b^2 = c^2 + a^2 - 2ca\cos B. \quad (54)$$

$$c^2 = a^2 + b^2 - 2ab\cos C. \quad (55)$$

§ 110. $\cos A = \dfrac{b^2+c^2-a^2}{2bc}.$ (56) $\cos B = \dfrac{c^2+a^2-b^2}{2ca}.$ (57)

$$\cos C = \frac{a^2+b^2-c^2}{2ab}. \quad (58)$$

§ 111.

$$\sin\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}. \quad (59)$$

$$\sin\tfrac{1}{2}B = \sqrt{\frac{(s-c)(s-a)}{ca}}. \quad (60)$$

$$\sin \tfrac{1}{2} C = \sqrt{\frac{(s-a)(s-b)}{ab}}. \qquad (61)$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}. \qquad (62)$$

$$\cos \tfrac{1}{2} B = \sqrt{\frac{s(s-b)}{ca}}. \qquad (63)$$

$$\cos \tfrac{1}{2} C = \sqrt{\frac{s(s-c)}{ab}}. \qquad (64)$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}. \qquad (65)$$

$$\tan \tfrac{1}{2} B = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}. \qquad (66)$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}. \qquad (67)$$

§ 112. $2K = bc \sin A.$ **(68)** $\quad 2K = \dfrac{a^2 \sin B \sin C}{\sin A}.$ **(71)**

$2K = ca \sin B.$ **(69)** $\quad 2K = \dfrac{b^2 \sin C \sin A}{\sin B}.$ **(72)**

$2K = ab \sin C.$ **(70)** $\quad 2K = \dfrac{c^2 \sin A \sin B}{\sin C}.$ **(73)**

$$K = \sqrt{s(s-a)(s-b)(s-c)}. \qquad (74)$$

SPHERICAL TRIGONOMETRY.

§ 135. $\cos c = \cos a \cos b.$ **(75)**

$\sin A = \dfrac{\sin a}{\sin c}.$ **(76)** $\quad \sin B = \dfrac{\sin b}{\sin c}.$ **(78)**

$\cos A = \dfrac{\tan b}{\tan c}.$ **(77)** $\quad \cos B = \dfrac{\tan a}{\tan c}.$ **(79)**

§ 137. $\tan A = \dfrac{\tan a}{\sin b}.$ **(80)** $\quad \tan B = \dfrac{\tan b}{\sin a}.$ **(81)**

§ 138. $\sin A = \dfrac{\cos B}{\cos b}.$ **(82)** $\quad \sin B = \dfrac{\cos A}{\cos a}.$ **(83)**

§ 139. $\cos c = \cot A \cot B.$ **(84)**

§ 151. $\dfrac{\sin A}{\sin B} = \dfrac{\sin a}{\sin b}.$ **(85)**

$\dfrac{\sin B}{\sin C} = \dfrac{\sin b}{\sin c}.$ **(86)**

$\dfrac{\sin A}{\sin C} = \dfrac{\sin a}{\sin c}.$ **(87)**

§ 152.
$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \tag{88}$$
$$\cos b = \cos c \cos a + \sin c \sin a \cos B. \tag{89}$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \tag{90}$$

§ 153.
$$\cos A = -\cos B \cos C + \sin B \sin C \cos a. \tag{91}$$
$$\cos B = -\cos C \cos A + \sin C \sin A \cos b. \tag{92}$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c. \tag{93}$$

§ 154.
$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}}. \tag{94}$$
$$\sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s-c) \sin (s-a)}{\sin c \sin a}}. \tag{95}$$
$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin (s-a) \sin (s-b)}{\sin a \sin b}}. \tag{96}$$
$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}}. \tag{97}$$
$$\cos \tfrac{1}{2} B = \sqrt{\frac{\sin s \sin (s-b)}{\sin c \sin a}}. \tag{98}$$
$$\cos \tfrac{1}{2} C = \sqrt{\frac{\sin s \sin (s-c)}{\sin a \sin b}}. \tag{99}$$
$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}. \tag{100}$$
$$\tan \tfrac{1}{2} B = \sqrt{\frac{\sin (s-c) \sin (s-a)}{\sin s \sin (s-b)}}. \tag{101}$$
$$\tan \tfrac{1}{2} C = \sqrt{\frac{\sin (s-a) \sin (s-b)}{\sin s \sin (s-c)}}. \tag{102}$$

§ 155.
$$\sin \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos (S-A)}{\sin B \sin C}}. \tag{103}$$
$$\sin \tfrac{1}{2} b = \sqrt{-\frac{\cos S \cos (S-B)}{\sin C \sin A}}. \tag{104}$$
$$\sin \tfrac{1}{2} c = \sqrt{-\frac{\cos S \cos (S-C)}{\sin A \sin B}}. \tag{105}$$
$$\cos \tfrac{1}{2} a = \sqrt{\frac{\cos (S-B) \cos (S-C)}{\sin B \sin C}}. \tag{106}$$

$$\cos \tfrac{1}{2} b = \sqrt{\frac{\cos (S - C) \cos (S - A)}{\sin C \sin A}}. \tag{107}$$

$$\cos \tfrac{1}{2} c = \sqrt{\frac{\cos (S - A) \cos (S - B)}{\sin A \sin B}}. \tag{108}$$

$$\tan \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos (S - A)}{\cos (S - B) \cos (S - C)}}. \tag{109}$$

$$\tan \tfrac{1}{2} b = \sqrt{-\frac{\cos S \cos (S - B)}{\cos (S - C) \cos (S - A)}}. \tag{110}$$

$$\tan \tfrac{1}{2} c = \sqrt{-\frac{\cos S \cos (S - C)}{\cos (S - A) \cos (S - B)}}. \tag{111}$$

§ 156.

$$\frac{\sin \frac{1}{2} (A + B)}{\sin \frac{1}{2} (A - B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2} (a - b)}. \tag{112}$$

§ 157.

$$\frac{\cos \frac{1}{2} (A + B)}{\cos \frac{1}{2} (A - B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2} (a + b)}. \tag{113}$$

§ 158.

$$\frac{\sin \frac{1}{2} (a + b)}{\sin \frac{1}{2} (a - b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2} (A - B)}. \tag{114}$$

$$\frac{\cos \frac{1}{2} (a + b)}{\cos \frac{1}{2} (a - b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2} (A + B)}. \tag{115}$$

ANSWERS.

§ **56**; page **30**.

12. 85° 56′ 37.32″. **14**. 95° 29′ 34.8″. **16**. 130° 55′ 5.952″.

13. 14° 19′ 26.22″. **15**. 20° 27′ 2.52″.

§ **63**; page **39**.

3. $n\pi,\ 2n\pi \pm \frac{\pi}{3}$.

4. $(2n+1)\frac{\pi}{2},\ n\pi + (-1)^n \frac{7\pi}{6}$.

5. $(2n+1)\frac{\pi}{2},\ n\pi \pm \frac{\pi}{4}$.

6. $n\pi, n\pi \pm \frac{\pi}{4}$.

7. $n\pi \pm \frac{\pi}{3}$.

8. $n\pi \pm \frac{\pi}{6}$.

9. $n\pi,\ \pm \tan^{-1}\left(\frac{1}{7}\sqrt{7}\right)$.

10. $\sin^{-1}\frac{\sqrt{5}-1}{2}$.

§ **76**; page **44**.

2. 1.5441.	**6**. 2.1003.	**10**. 2.5104.	**14**. 3.4192.
3. 1.6990.	**7**. 2.2922.	**11**. 2.5774.	**15**. 3.7814.
4. 1.6232.	**8**. 2.3892.	**12**. 2.9421.	**16**. 4.0794.
5. 1.8751.	**9**. 2.3222.	**13**. 2.8363.	**17**. 4.2006.

§ **78**; page **44**.

2. 0.5229.	**5**. 1.1549.	**8**. 0.2831.	**11**. 1.4592.
3. 0.2431.	**6**. 0.2589.	**9**. 0.7939.	**12**. 1.3468.
4. 1.6532.	**7**. 2.3522.	**10**. 2.1303.	**13**. 2.0424.

§ **81**; page **45**.

3. 3.3397.	**8**. 0.5663.	**13**. 0.6171.	**19**. 0.8752.
4. 1.7475.	**9**. 0.0430.	**14**. 0.2918.	**20**. 0.0794.
5. 0.6338.	**10**. 0.1165.	**16**. 0.0495.	**21**. 0.4248.
6. 8.6826.	**11**. 0.0939.	**17**. 0.0365.	**22**. 0.1051.
7. 1.0460.	**12**. 0.5440.	**18**. 0.7007.	**23**. 0.0406.

§ 85; page 47.

2. 0.5562.
3. 1.0491.
4. $9.9242-10$.
5. $8.9912-10$.
6. $7.5353-10$.
7. 3.4592.
8. $8.5932-10$.
9. $6.6074-10$.
10. $9.2885-10$.
11. 2.3064.
12. 0.1151.
13. 0.7782.

§ 86; page 47.

4. 0.011739.
5. 2.527511.
6. $6.780210-10$.
7. 4.812917.
8. 3.960116.
9. $7.013152-10$.
10. $4.942550-10$.
11. 5.863566.
12. $5.640409-10$.
15. 6.61005.
16. 55606.5.
17. .0110890.
18. 186.334.
19. .00223905.
20. .0000100006.
21. 9776.67.
22. 467929.
23. .000342770.
24. .00000130514.

§ 91; pages 50, 51.

1. 1897.85.
2. -193315.
3. .309170.
4. .00110375.
5. 6.36103.
6. .0301742.
7. 31.2004.
8. $-.132693$.
9. .126965.
10. .0235770.
11. -1.16493.
12. $-.00256105$.
13. 3692.77.
14. .277996.
15. -15896.0.
16. .0316228.
17. 244.004.
18. .00279116.
19. .000000237177.
20. 2.23607.
21. 1.14870.
22. -1.22028.
23. 1.77828.
24. .668289.
25. .645831.
26. .137751.
27. $-.370134$.
30. 13.8289.
31. 2.48722.
32. 1.05557.
33. .0000214279.
34. .00710469.
35. .695490.
36. .542699.
37. -36.0189.
38. -11.1122.
39. .943241.
40. 2.62762.
41. 2.53217.
42. -1.79616.
43. 1.03242.
44. .298557.
45. .0448607.
46. .794509.
47. 1.80492.
48. 179.596.
49. 1.88270.
50. .000193152.
51. $-.0995935$.
52. 1.34384.

§ 92; page 52.

3. $x=.2831+$.
4. $x=-2.173+$.
5. $x=1.155+$.
6. $x=-.1765+$.
7. $x=\dfrac{5\log c}{\log a-2\log b}$.
8. $x=\dfrac{3\log a}{4\log n-2\log m}$.
9. $x=\dfrac{1}{2}$.
10. $x=1$ or -5.

§ 93; page 52.

2. 3.7004+. **3.** $-.06546+$. **4.** $-6.059+$. **5.** 3.326+.

6. $-.4601+$. **7.** .3494+. **9.** 4. **10.** $\frac{5}{3}$. **11.** $-\frac{1}{3}$. **12.** $\frac{6}{5}$.

§ 94; page 53.

1. 9.345950 − 10.
2. 0.376890.
3. 9.932630 − 10.
4. 9.865995 − 10.
5. 9.243533 − 10.
6. 9.163433 − 10.
7. 0.302190.
8. 0.153906.
9. 0.002256.
10. 59° 15′ 26.4″.
11. 33° 0′ 16.1″.
12. 81° 7′ 37.9″.
13. 27° 31′ 50.5″.
14. 8° 41′ 32.7″.
15. 75° 45′ 9.8″.
16. 49° 38′ 57.1″.
17. 23° 26′ 30.9″.

§ 95; page 53.

1. .68573.
2. .25232.
3. .06344.
4. .69518.
5. .92163.
6. .86962.
7. 51° 36′ 42.9″.
8. 15° 28′ 22.5″.
9. 66° 14′ 34.3″.
10. 29° 9′ 13.8″.

§ 96; page 53.

1. 8.338076 − 10.
2. 8.810945 − 10.
3. 1.369926.
4. 0° 58′ 51.06″.
5. 0° 24′ 53.79″.
6. 1° 37′ 41.93″.

§ 102; pages 56 to 58.

1. $a = 1.8117$, $b = 6.7615$.
2. $b = 11.7793$, $c = 12.7965$.
3. $a = 16.7820$, $c = 26.1081$.
4. $A = 34° 22' 7.1''$, $b = .511764$.
5. $A = 33° 8' 56.3''$, $c = 499.252$.
6. $b = 10.3547$, $c = 13.1404$.
7. $a = .0036235$, $b = .013523$.
8. $A = 39° 49' 24.6''$, $a = 48.8645$.
9. $a = 148.407$, $c = 948.680$.
10. $A = 49° 53' 54.9''$, $c = 4.46330$.
11. $b = .000336374$, $c = .00336715$.
12. $a = 3821.55$, $b = 3641.34$.
13. $A = 35° 53' 55.2''$, $b = 731.237$.
14. $a = 176.533$, $c = 191.993$.
15. $a = 20455.6$, $c = 21405.6$.
16. $a = 2.40989$, $b = .812578$.
17. $A = 19° 31' 57.2''$, $c = .000505172$.
18. $b = 77.6330$, $c = 91.2952$.
19. $A = 32° 10' 16.5''$, $a = 388.471$.
20. $b = 644.109$, $c = 650.272$.
21. $a = 34308.0$, $b = 23381.6$.
22. $b = 4.48174$, $c = 8.5085$.
23. $A = 39° 21' 54.1''$, $b = 121.240$.
24. $a = .00247181$, $c = .00360016$.
25. $a = 16001.6$, $c = 85725.1$.
26. $a = 3624500$, $b = 8821960$.

27. $A = 76° 33' 49.0''$, $a = 24234.4$.
28. $a = 207302$, $b = 421170$.
29. $a = .507624$, $c = .525355$.
30. $A = 60° 14' 12.9''$, $c = 774.563$.
31. $c = 252.103$.
32. $a = 1.73561$.
33. $c = 122748$.
34. $A = 47° 42' 47.8''$.
35. $a = .344647$.
36. $a = 4925.31$.
37. 20.573.
38. 83.271 ft.
39. 31° 47′ 24.5″.
40. 36° 37′ 58.0″.
41. 99.4565 mi.
42. 10.2352.
43. 19° 49′ 46.7″.
44. 365.64 ft.
45. 56° 18′ 35.7″.
46. 25.2230 mi., 30.0750 mi.
47. 21.6514.
48. 14.4853, 15.6787.
49. 517.51 ft.
50. 17.2624.
51. 420.867 ft.
52. 437.605.
53. 10.392.
54. 482.1 ft.
55. Rate, 6.79668 miles an hour; bearing, N. 63° 8′ 28.5″ W.

§ 104; page 60.

2. $B = 89° 59' 42.8''$.
3. $B = 89° 23' 22.6''$.
4. $A = 89° 59' 37.2''$.
5. $B = 89° 59' 59.0''$.
6. $A = 89° 43' 13.6''$.

§ 106; page 61.

2. 6.9066.
3. .151079.
4. 5699.7.
5. .089433.
6. 8130.9.
7. .0067825.
8. 2.18876.
9. 107.762.
10. .0487840.

§ 114; page 67.

2. $b = 283.331$, $c = 267.677$.
3. $a = .340132$, $c = .986084$.
4. $a = 29.0595$, $b = 18.3742$.
5. $a = .0313440$, $c = .0498733$.
6. $b = 5.76721$, $c = 2.16917$.
7. $a = 5058.5$, $b = 3683.53$.
8. $a = .299674$, $b = .731538$.
9. $a = 4.01036$, $c = 3.55195$.
10. $b = 56719.9$, $c = 23073.5$.

§ 115; pages 68, 69.

2. $A = 118° 17' 57.4''$, $b = 44.7274$.
3. $A = 60° 44' 39.5''$, $c = 965.282$.
4. $C = 63° 49' 9.3''$, $a = 4.48237$.
5. $B = 28° 43' 49.0''$, $c = 1.44246$.
6. $B = 145° 35' 24.7''$, $a = 1045.74$.
7. $C = 63° 48' 28.1''$, $b = 13.7387$.
8. $A = 67° 55' 16.9''$, $c = 85.3596$.
9. $C = 46° 13' 20.9''$, $a = .0759588$.
10. $C = 134° 36' 27.4''$, $b = 27335.0$.

§ 116; page 70.

3. $A = 28° 57' 18.0''$, $B = 46° 34' 2.8''$, $C = 104° 28' 39.0''$.
4. $A = 44° 24' 54.8''$, $B = 78° 27' 47.0''$, $C = 57° 7' 17.6''$.
5. $A = 71° 47' 24.4''$, $B = 58° 45' 5.4''$, $C = 49° 27' 30.0''$.
6. $A = 74° 40' 16.4''$, $B = 47° 46' 39.0''$, $C = 57° 33' 4.8''$.
7. $A = 59° 19' 11.8''$, $B = 68° 34' 7.6''$, $C = 52° 6' 40.6''$.
8. $A = 45° 11' 46.6''$, $B = 101° 22' 17.8''$, $C = 33° 25' 56.4''$.
9. $A = 71° 33' 49.2''$. 10. $B = 30° 47' 22.8''$. 11. $C = 25° 56' 54.2''$.

§ 121; pages 73, 74.

1. $B = 32° 36' 9.4''$, $c = 6.62085$.
2. $B_1 = 31° 57' 47.8''$, $a_1 = 120.313$;
   $B_2 = 148° 2' 12.2''$, $a_2 = 11.3800$.
3. $C = 23° 33' 18.2''$, $a = .183882$.
4. $A = 34° 29' 48.2''$, $b = 7.12905$.
5. Impossible.
6. Impossible.
7. $B = 48° 34' 38.4''$, $a = 76.0172$.
8. $C = 90°$, $b = 5.51109$.
9. $C_1 = 46° 18' 35.5''$, $a_1 = 6.94575$;
   $C_2 = 133° 41' 24.5''$, $a_2 = .699906$.
10. $A = 25° 32' 50.9''$, $c = 278.193$.
11. Impossible.
12. $C = 14° 4' 7.7''$, $b = 1.43516$.
13. $B = 90°$, $c = 137.872$.
14. $A_1 = 70° 12' 46.7''$, $b_1 = .287904$;
    $A_2 = 109° 47' 13.3''$, $b_2 = .104539$.
15. $B = 45° 38' 30.2''$, $a = 16214.3$.

§ 122; pages 74, 75.

2. 197.656. 5. 165917. 8. .078614. 11. 4000.81.
3. 14.9812. 6. 2878.31. 9. 860.006. 12. .000329015.
4. 16.6843. 7. 1.30108. 10. .0448746. 13. 25.6249.

§ 123; pages 75, 76.

1. Height, 153.629 ft.; distances, 117.246 ft., 217.246 ft.
2. $AD = 44.9525$. 4. $47° 52' 2.1''$. 6. 56.6547, 49.3482.
3. 29799.9 sq. rd. 5. 247.741 ft. 7. 32.5255 mi.

8. Two angles, $74^\circ 12' 20.0''$, $58^\circ 23' 48.0''$; third side, .430133.

9. N. $47^\circ 32' 33.1''$ W.

10. 9.8995 mi., 19.1244 mi.

11. One angle, $101^\circ 13' 45.8''$; diagonal, 136.187.

12. 297.954 ft.

13. Sides, 26.5604, 90.5154; one angle, $119^\circ 5' 14.6''$.

14. 91.6364 ft., 33.8973 ft.

15. 17.64934, 8.77461.

16. 1113.34 ft.

17. Diagonal, 52.9024; side, 41.9506.

18. 247.998 ft.

19. $AD = 88.1534$, $A = 56^\circ 1' 10.7''$.

20. 1569.948 sq. rd.

§ 126; page 79.

2. 2.11491, -1.86081, $-.254102$.

3. 2.14510, .523978, -2.66907.

4. .47761, -6.1364, $-.34120$.

5. 3.49086, $-.83425$, .343379.

§ 148; page 93.

5.	$A = 36^\circ 58' 50.0''$,	$B = 63^\circ 42' 34.0''$,	$b = 42^\circ 34' 54.4''$.
6.	$a = 27^\circ 49' 17.9''$,	$b = 42^\circ 29' 21.8''$,	$c = 49^\circ 17' 42.4''$.
7.	$B = 68^\circ 37' 18.1''$,	$b = 44^\circ 56' 46.7''$,	$c = 49^\circ 20' 41.8''$;
or,	$B = 111^\circ 22' 41.9''$,	$b = 135^\circ 3' 13.3''$,	$c = 130^\circ 39' 18.2''$.
8.	$A = 68^\circ 10' 4.4''$,	$b = 163^\circ 42' 32.1''$,	$c = 141^\circ 50' 15.2''$.
9.	$A = 15^\circ 34' 32.3''$,	$B = 94^\circ 14' 40.0''$,	$c = 105^\circ 26' 27.5''$.
10.	$a = 170^\circ 13' 25.6''$,	$B = 78^\circ 34' 3.4''$,	$b = 40^\circ 1' 8.6''$.
11.	$A = 21^\circ 11' 12.7''$,	$a = 19^\circ 50' 30.4''$,	$c = 69^\circ 54' 41.6''$;
or,	$A = 158^\circ 48' 47.3''$,	$a = 160^\circ 9' 29.6''$,	$c = 110^\circ 5' 18.4''$.
12.	$A = 82^\circ 8' 19.3''$,	$a = 73^\circ 38' 54.4''$,	$b = 28^\circ 4' 23.5''$.
13.	$A = 122^\circ 34' 33.5''$,	$a = 132^\circ 24' 39.6''$,	$B = 52^\circ 58' 9.5''$.
14.	$A = 153^\circ 10' 2.8''$,	$B = 115^\circ 25' 2.8''$,	$c = 20^\circ 2' 40.3''$.
15.	$A = 165^\circ 50' 26.0''$,	$b = 139^\circ 10' 11.5''$,	$c = 41^\circ 42' 23.4''$.
16.	$a = 112^\circ 16' 49.7''$,	$b = 145^\circ 51' 35.5''$,	$c = 71^\circ 42' 41.1''$.
17.	$A = 55^\circ 58' 5.5''$,	$B = 34^\circ 41' 20.4''$,	$c = 12^\circ 39' 44.7''$.
18.	$a = 54^\circ 0' 24.8''$,	$B = 84^\circ 43' 10.5''$,	$c = 86^\circ 10' 32.3''$.
19.	$a = 41^\circ 29' 25.7''$,	$b = 133^\circ 39' 29\ 8''$,	$c = 121^\circ 8' 21.5''$.
20.	$a = 152^\circ 35' 19.0''$,	$B = 108^\circ 7' 8.6''$,	$b = 125^\circ 24' 13.7''$.
21.	$A = 20^\circ 3' 21.5''$,	$a = 14^\circ 58' 21.1''$,	$c = 131^\circ 7' 4.9''$;
or,	$A = 159^\circ 56' 38.5''$,	$a = 165^\circ 1' 38.9''$,	$c = 48^\circ 52' 55.1''$.
22.	$a = 110^\circ 57' 15.6''$,	$B = 165^\circ 10' 31.9''$,	$c = 69^\circ 41' 7.1''$.
23.	$A = 111^\circ 53' 21.2''$,	$B = 115^\circ 40' 6.8''$,	$b = 117^\circ 49' 41.2''$.

24.	$A=165°\ 3'\ 57.9''$,	$a=168°\ 8'\ 48.3''$,	$b=51°\ 53'\ 53.3''$.
25.	$B=22°\ 13'\ 3.9''$,	$b=20°\ 34'\ 38.3''$,	$c=111°\ 38'\ 31.1''$;
or,	$B=157°\ 46'\ 56.1''$,	$b=159°\ 25'\ 21.7''$,	$c=68°\ 21'\ 28.9''$.
26.	$A=64°\ 30'\ 52.0''$,	$a=38°\ 32'\ 30.5''$,	$B=146°\ 37'\ 27.3''$.

§ 149 ; page 94.

2.	$a=103°\ 25'\ 57.4''$,	$B=157°\ 31'\ 44.4''$,	$C=119°\ 19'\ 11.3''$.
3.	$a=57°\ 43'\ 57.2''$,	$b=129°\ 56'\ 31.7''$,	$C=58°\ 4'\ 55.6''$.
4.	$A=19°\ 56'\ 45.0''$,	$B=141°\ 38'\ 20.3''$,	$b=113°\ 18'\ 58.3''$.
5.	$A=44°\ 41'\ 15.9''$,	$a=51°\ 37'\ 1.9''$,	$b=60°\ 51'\ 3.4''$.
6.	$B=80°\ 27'\ 25.7''$,	$b=80°\ 46'\ 54.3''$,	$C=87°\ 31'\ 12.5''$;
or,	$B=99°\ 32'\ 34.3''$,	$b=99°\ 13'\ 5.7''$,	$C=92°\ 28'\ 47.5''$.
7.	$A=67°\ 11'\ 45.0''$,	$B=80°\ 58'\ 16.5''$,	$C=93°\ 29'\ 13.4''$.

§ 150 ; page 95.

2.	$a=69°\ 55'\ 43.2''$,	$C=159°\ 59'\ 40.6''$.
3.	$A=120°\ 41'\ 19.6''$,	$c=30°\ 14'\ 37.4''$.
4.	$A=140°\ 35'\ 4.5''$,	$C=145°\ 11'\ 50.4''$.
5.	$C=148°\ 19'\ 24.8''$,	$c=80°\ 47'\ 39.8''$.

§ 161 ; page 104.

2.	$a=95°\ 37'\ 51.0''$,	$b=41°\ 52'\ 22.2''$,	$C=110°\ 48'\ 24.0''$.
3.	$b=98°\ 30'\ 32.4''$,	$c=56°\ 42'\ 47.0''$,	$A=59°\ 38'\ 53.2''$.
4.	$c=64°\ 19'\ 27.8''$,	$a=34°\ 3'\ 11.8''$,	$B=37°\ 39'\ 27.2''$.
5.	$b=146°\ 25'\ 1.4''$,	$a=69°\ 4'\ 38.2''$,	$C=125°\ 11'\ 41.8''$.

§ 162 ; page 105.

2.	$A=121°\ 32'\ 41.3''$,	$B=40°\ 56'\ 48.5''$,	$c=37°\ 25'\ 48.8''$.
3.	$A=86°\ 59'\ 48.8''$,	$C=60°\ 50'\ 54.8''$,	$b=111°\ 16'\ 42.4''$.
4.	$C=134°\ 57'\ 31.3''$,	$B=50°\ 40'\ 48.3''$,	$a=69°\ 7'\ 34.6''$.
5.	$B=163°\ 8'\ 48.4''$,	$A=147°\ 29'\ 24.2''$,	$c=76°\ 8'\ 49.0''$.

§ 163 ; page 106.

2.	$A=51°\ 58'\ 28.0''$,	$B=58°\ 53'\ 13.2''$,	$C=83°\ 54'\ 31.6''$.
3.	$A=142°\ 32'\ 37.8''$,	$B=27°\ 52'\ 36.0''$,	$C=32°\ 26'\ 52.8''$.
4.	$A=142°\ 23'\ 44.0''$,	$B=159°\ 15'\ 41.6''$,	$C=133°\ 14'\ 4.2''$.
5.	$A=47°\ 21'\ 11.8''$.		

§ 164; page 107.

3.	$a = 68° 46' 28.4''$,	$b = 73° 47' 57.8''$,	$c = 63° 12' 24.6''$.
4.	$a = 90° 53' 2.6''$,	$b = 117° 48' 59.6''$,	$c = 132° 5' 10.0''$.
5.	$a = 103° 31' 33.8''$,	$b = 53° 4' 26.2''$,	$c = 61° 14' 18.2''$.
6.	$b = 85° 48' 53.8''$.		

§ 165; page 109.

4.	$C = 65° 29' 1.0''$,	$A = 97° 18' 33.8''$,	$a = 100° 42' 23.4''$.
5.	$B = 42° 40' 9.2''$,	$C = 159° 54' 3.6''$,	$c = 153° 29' 39.8''$;
or,	$B = 137° 19' 50.8''$,	$C = 50° 21' 16.4''$,	$c = 90° 8' 51.4''$.
6.	Impossible.		
7.	$C = 90°$,	$B = 113° 33' 15.5''$,	$b = 114° 47' 47.5''$.
8.	$B = 68° 17' 2.4''$,	$A = 132° 35' 12.4''$,	$a = 131° 16' 32.2''$;
or,	$B = 111° 42' 57.6''$,	$A = 77° 3' 48.0''$,	$a = 95° 48' 41.8''$.
9.	Impossible.		
10.	$C = 146° 37' 40.2''$,	$B = 55° 1' 11.8''$,	$b = 96° 33' 16.2''$.

§ 166; page 111.

2.	$b = 114° 48' 57.9''$,	$a = 82° 54' 0.0''$,	$A = 79° 18' 29.0''$.
3.	$a = 67° 25' 2.3''$,	$c = 160° 6' 10.0''$,	$C = 164° 6' 8.4''$;
or,	$a = 112° 34' 57.7''$,	$c = 103° 6' 20.4''$,	$C = 128° 22' 54.8''$.
4.	$c = 90°$,	$B = 63° 46' 30.2''$,	$b = 66° 29' 37.6''$.
5.	Impossible.		
6.	$b = 27° 22' 7.6''$,	$a = 117° 9' 5.2''$,	$A = 47° 20' 57.2''$.
7.	$a = 43° 2' 23.6''$,	$b = 129° 9' 46.0''$,	$B = 89° 23' 51.8''$;
or,	$a = 136° 57' 36.4''$,	$b = 20° 34' 54.2''$,	$B = 26° 57' 36.4''$.
8.	Impossible.		

§ 167; page 112.

1. Distance, 3275.20 mi.; bearing of Boston from Greenwich, N. 71° 38′ 53.7″ W.; of Greenwich from Boston, N. 53° 6′ 31.9″ E.

2. Distance, 11012.9 mi.; bearing of Calcutta from Valparaiso, S. 64° 20′ 17.4″ E.; of Valparaiso from Calcutta, S. 54° 54′ 25.2″ W.

3. Latitude, 49° 58′ 23.1″ N.

§ 170; page 113.

1. Time, 6 h. 0 m. 43 s. A.M.; longitude, 44° 49′ 18″ W.

2. 15° 0′ 41.4″. **3.** N. 56° 28′ 8.5″ E. **4.** 5 h. 3 m. 27 s. A.M.

WELLS' MATHEMATICAL SERIES.

SIX PLACE

LOGARITHMIC TABLES,

TOGETHER WITH A

TABLE OF NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

PREPARED BY

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INTRODUCTION.

I. USE OF THE TABLE OF LOGARITHMS OF NUMBERS.

This table (pages 2 to 16) gives the mantissæ of the logarithms of all numbers of four figures from 1000 to 10000, calculated to six places of decimals.

To find the logarithm of any number of four figures.

Find in the column N. the first three figures of the given number. Then the required mantissa will be found in the corresponding horizontal line, in the vertical column headed by the fourth figure of the number.

If only the last four figures of the mantissa are found, the first two may be obtained from the nearest mantissa above, in the same column, which contains six figures.

Finally, prefix the proper characteristic.

For example, $\log 140.8 = 2.148603$;

$\log .05837 = 8.766190 - 10.$

For numbers of one, two, or three figures, the column headed 0 may be used; for log 167 has the same mantissa as log 1670, log 8.3 the same mantissa as log 8300, and log .9 the same mantissa as log 9000; thus,

$$\log 167 = 2.222716, \quad \log 8.3 = 0.919078, \text{ and } \log .9 = 9.954243 - 10.$$

To find the logarithm of a number of more than four figures.

Required the logarithm of 3296.78.

We find from the table, $\log 3296 = 3.517987$;

$\log 3297 = 3.518119.$

That is, an increase of one unit in the number produces an increase of .000132 in the logarithm.

Then an increase of .78 of a unit in the number will produce an increase of $.78 \times .000132$ in the logarithm, or .000103 to the nearest sixth decimal place.

Whence, $\log 3296.78 = 3.517987 + .000103 = 3.518090.$

Note I. The foregoing method is based on the assumption that the differences of logarithms are proportional to the differences of their corresponding numbers, which, though not strictly accurate, is sufficiently exact for practical purposes.

Note II. The difference between any mantissa in the table and the mantissa of the next higher number of four figures, is called the *tabular difference.*

The following rule is derived from the above:

Find from the table the mantissa of the first four significant figures, and the tabular difference. (See Note III.)

Multiply the latter by the remaining figures of the number with a decimal point before them. (See Note IV.)

Add the result to the mantissa of the first four figures, and prefix the proper characteristic.

Example. Find the logarithm of .002243076.

Mantissa of 2243 = 350829	Tabular difference = 194
15	.076
350844	1 164
	13 58
	Correction = 14.744
Result, 7.350844 − 10.	= 15, nearly.

Note III. The tabular difference may be conveniently found as follows:

Subtract the last figure of the mantissa from the last figure of the next greater, and then take the nearest integer, ending in that figure, to the number in the column D. in the same line.

Thus, in the above example, the last figure of the mantissa of 2243 is 9, and of the next greater mantissa, 3; 9 from 13 leaves 4, and the nearest integer, ending in 4, to 193, the number in the column D., is 194, the proper tabular difference.

Note IV. In finding the correction to the nearest unit's figure, the decimal portion may be omitted provided that, if it is equal to or greater than .5, the unit's figure is increased by 1.

Thus, 13.26 would be taken as 13; 30.5 as 31; and 22.803 as 23.

To find the number corresponding to a logarithm.

1. Required the number whose logarithm is 1.693551.

Find in the table the mantissa 693551.

In the corresponding line, in the column N., we find 493, the first three figures of the required number, and at the head of the column we find 8, the fourth figure.

Since the characteristic is 1, there must be two figures to the left of the decimal point.

Whence, number corresponding to 1.693551 = 49.38.

2. Required the number whose logarithm is 3.950185.

We find in the table the mantissa 950170, whose corresponding number is 8916, and the mantissa 950219, whose corresponding number is 8917.

That is, an increase of 49 in the mantissa produces an increase of one unit in the number corresponding.

Then an increase of 15 in the mantissa will produce an increase of $\frac{15}{49}$ of a unit in the number corresponding, or .31 nearly.

Whence, number corresponding $= 8916 + .31 = 8916.31$.

The following rule is derived from the above:

Find from the table the next less mantissa, the four figures corresponding, and the tabular difference. (See Note III.)

Subtract the next less mantissa from the given mantissa, and divide the remainder by the tabular difference. (See Note VI.)

Annex the quotient to the first four figures of the number, and point off the result. (See Note V.)

Note V. The rules for pointing off are the reverse of the rules for characteristic; they may be stated as follows:

I. *If* -10 *is not written after the mantissa, add* 1 *to the characteristic, giving the number of places to the left of the decimal point.*

II. *If* -10 *is written after the mantissa, subtract the positive part of the characteristic from* 9, *giving the number of ciphers to be placed between the decimal point and first significant figure.*

Example. Find the number whose logarithm is $7.427662 - 10$.

$$
\begin{array}{ll}
 & 427662 \\
\text{Next less mantissa} = & \underline{427648}\text{; four figures corresponding} = 2677. \\
\text{Tabular difference} = & 163)14.000(.085 = .09\text{, nearly.} \\
 & \quad\ \underline{13\ 04} \\
 & \qquad\ 960
\end{array}
$$

Result, .00267709.

Note VI. The correction can usually be depended upon to two decimal places; the division should be carried out to three decimal places in order to determine the last figure accurately. (See Note IV.)

II. USE OF THE TABLE OF LOGARITHMIC SINES, COSINES, ETC.

This table (pages 18 to 62) gives the logarithms of the sines, cosines, tangents, and cotangents of all angles at intervals of one minute from 0° to 90°.

For angles between 0° and 45°, the degrees will be found at the *top* of the page, the minutes in the *left-hand* column, and the functions in the columns designated by the names at the *top;* that is, sines in the first column, cosines in the second, tangents in the third, and cotangents in the fourth.

For angles between 45° and 90°, the degrees will be found at the *foot* of the page, the minutes in the *right-hand* column, and the functions in the columns designated by the names at the *foot;* that is, cosines in the first column, sines in the second, cotangents in the third, and tangents in the fourth.

The sines and cosines of all acute angles, the tangents of angles between 0° and 45°, and the cotangents of angles between 45° and 90°, being less than unity, the characteristics of their logarithms have been increased by 10, and -10 must be written after their mantissæ; in all other cases, the true value of the characteristic is given in the table.

Thus,
$$\log \sin 38°\ 37' = 9.795259 - 10;$$
$$\log \tan 66°\ 20' = 0.358253;$$
$$\log \cot 79°\ \ 3' = 9.286624 - 10;$$
$$\log \cos 85°\ 51' = 8.859546 - 10.$$

To find the logarithmic sine, cosine, tangent, or cotangent of any acute angle expressed in degrees, minutes, and seconds.

Find from the table the logarithmic sine, cosine, tangent, or cotangent of the degrees and minutes, and the difference for 1″ *corresponding.* (See Note VII. below.)

Multiply this difference by the number of seconds. (See Note IV.)

If sine or tangent, add } *this correction.*
If cosine or cotangent, subtract }

Note VII. The columns immediately to the right of those headed "Sin.," "Cos.," and "Tan.," contain the respective differences for 1″; the right-hand column of differences is also to be used with the column headed "Cot."

It will be observed that the differences do not stand in the same horizontal line with the logarithms, but opposite the intervals between consecutive logarithms. With the degrees at the *top* of the page, the difference next *below* should be taken; with the degrees at the *foot* of the page, the difference next *above*.

Note VIII. The rule given above assumes that the differences of the logarithmic functions are proportional to the differences of their corresponding angles, which, unless the angle is very near to 0° or 90°, is in general sufficiently exact for practical purposes. (See page x.)

1. Find $\log\tan 17^\circ\ 13'\ 51''$.

$$\begin{array}{lr}
\log\tan 17^\circ 13' = & 9.491180 - 10 \\
 & 380 \\
\hline
\text{Result,} & 9.491560 - 10
\end{array}
\qquad
\begin{array}{r}
\text{D. } 1'' = 7.45 \\
51 \\
\hline
7\ 45 \\
372\ 5 \\
\hline
379.95 = 380, \text{ nearly.}
\end{array}$$

2. Find $\log\cos 66^\circ\ 38'\ 23''$.

$$\begin{array}{lr}
\log\cos 66^\circ 38' = & 9.598368 - 10 \\
 & 112 \\
\hline
\text{Result,} & 9.598256 - 10
\end{array}
\qquad
\begin{array}{r}
\text{D. } 1'' = 4.88 \\
23 \\
\hline
14\ 64 \\
97\ 6 \\
\hline
112.24 = 112, \text{ nearly.}
\end{array}$$

To find the acute angle corresponding to a given logarithmic sine, cosine, tangent, or cotangent.

Take from the table, if sine or tangent, the next less, if cosine or cotangent, the next greater, logarithmic function, the degrees and minutes corresponding, and the difference for 1″. (See Note IX. below.)

Find the difference between the given logarithm and that taken from the table, and divide it by the difference for 1″, *giving the correction in seconds.*

Add the result to the degrees and minutes.

Note IX. In searching for the next less (or greater) logarithm, attention must be paid to the fact that the functions are found in different columns according as the angle is below or above 45°.

If, for example, the next less logarithmic sine is found in the column with "Sin." at the *top*, the degrees must be taken from the *top* of the page, and the minutes from the *left-hand* column; but if it is found in the column with "Sin." at the *foot*, the degrees must be taken from the *foot* of the page, and the minutes from the *right-hand* column. Similar considerations hold with respect to the other three functions.

1. Find the angle whose $\log\sin = 9.959345 - 10$.

$$\begin{array}{rl}
 & 9.959345 - 10 \\
\text{Next less } \log\sin = & \underline{9.959310 - 10}; \text{ angle corresponding} = 65^\circ\ 35'. \\
\text{D. } 1'' = & .97)35\ \ (36.08 = 36.1, \text{ nearly.} \\
 & \quad 291 \\
 & \quad \overline{\ 590} \\
 & \quad\ 582 \\
 & \quad \overline{\ \ 800}
\end{array}$$

Result, $65^\circ\ 35'\ 36.1''$.

2. Find the angle whose $\log\cot = 0.169602$.

Next greater $\log\cot = 0.169651$; angle corresponding $= 34°\ 5'$.

$$\begin{array}{l} 0.169651 \\ \underline{0.169602} \\ \text{D. } 1'' = 4.53)49(10.81 = 10.8, \text{ nearly.} \\ \quad 453 \\ \quad \overline{\ 3700} \\ \quad \underline{\ 3624} \\ \quad\ \ 760 \end{array}$$

Result, $34°\ 5'\ 10.8''$.

Note X. In finding the logarithmic sine of an angle between 85° and 90°, or the logarithmic cosine of an angle between 0° and 5°, it is better to obtain the correction by multiplying the difference between the next less and next greater logarithms by the number of seconds, and dividing the result by 60.

In finding the angle corresponding in the same cases, the correction in seconds may be obtained by multiplying the difference between the given logarithm and that taken from the table by 60, and dividing the result by the difference between the next less and next greater logarithms.

To find the logarithmic secant or cosecant of any acuto angle.

Since $\sec x = \dfrac{1}{\cos x}$ and $\csc x = \dfrac{1}{\sin x}$, we have

$$\log\sec x = \operatorname{colog}\cos x, \text{ and } \log\csc x = \operatorname{colog}\sin x.$$

Hence, *to find the logarithmic secant, subtract the logarithmic cosine from* $10 - 10$; *and to find the logarithmic cosecant, subtract the logarithmic sine from* $10 - 10$.

Example. Find $\log\sec 22°\ 38'$.

From the table, we find $\log\cos 22°\ 38' = 9.965195 - 10$.

Subtracting from $10 - 10$, $\log\sec 22°\ 38' = 0.034805$.

Note XI. The logarithmic cotangent of an angle may be obtained by subtracting the logarithmic tangent from $10 - 10$.

To find the logarithmic functions of an angle not lying between the limits 0° and 90°.

Any function of any angle may be expressed as a function of a certain acute angle; and hence the table of functions of acute angles serves to determine the functions of angles of any magnitude whatever, positive or negative.

Let it be required, for example, to find $\log\sin 152°\ 16'$.

We have, $\sin 152°\ 16' = \sin(90° + 62°\ 16') = \cos 62°\ 16'$.

Whence, $\log \sin 152° 16' = \log \cos 62° 16' = 9.667786 - 10.$

Or we may proceed as follows:

$$\sin 152° 16' = \sin(180° - 27° 44') = \sin 27° 44'.$$

Note XII. If the natural function is *negative*, as for example in the case of the cosine of an angle between 90° and 180°, there is no logarithmic function, strictly speaking.

In the solution of examples involving such functions, we may proceed as if the functions were positive, and determine the algebraic sign of the result irrespective of the logarithmic work.

III. USE OF THE TABLE OF NATURAL SINES, COSINES, ETC.

This table (pages 64 to 78) gives the natural values of the sines, cosines, tangents, and cotangents of all angles at intervals of 1′ from 0° to 90°, calculated for sines, cosines, and tangents to five places of decimals, and for cotangents to five significant figures.

Its use is similar to that of the table of logarithmic functions, except that the tabular differences for 1″ are not given, but are to be calculated from the table when required.

1. Required $\tan 41° 27' 14''$.

$$\tan 41° 27' = .88317.$$

The difference between this and $\tan 41° 28'$ is 52.

Correction for $14'' = \frac{14}{60} \times 52 = 12$, nearly.

$$\begin{array}{r} .88317 \\ 12 \\ \hline \text{Result, } .88329 \end{array}$$

2. Required the angle whose $\cos = .45854$.

Next greater $\cos = .45865$; angle corresponding $= 62° 42'$.

$$\begin{array}{r} .45854 \\ \hline 11 \end{array}$$

The difference between $\cos 62° 42'$ and $\cos 62° 43'$ is 26.

Correction in seconds $= \frac{11}{26} \times 60 = 25.4$, nearly.

Result, $62° 42' 25.4''$.

Note XIII. To find a natural function to a greater degree of accuracy than is possible from the table of natural functions, we may find the logarithmic function of the angle, and take the number corresponding to the result.

IV. USE OF THE AUXILIARY TABLE FOR SMALL ANGLES.

This table (page 79) gives the values of the expressions

$$10 + \log \sin x - \log x \text{ and } 10 + \log \tan x - \log x,$$

x being expressed in seconds, for all angles at intervals of 1′ from 0° to 4° 59′.

It may be used to find the logarithmic sines or tangents of angles between 0° and 5°, or the angles corresponding in the same cases, to a greater degree of accuracy than is possible from the table of logarithmic functions. (See Note VIII.)

To find the logarithmic sine or tangent of an angle between 0° and 5°.

Find from the auxiliary table the logarithm corresponding to the given function, add to the result the logarithm of the number of seconds in the angle, and write -10 *after the mantissa.*

Example. Find log tan 0° 43′ 37″.

The logarithms corresponding to tan 0° 43′ and tan 0° 44′ are 4.685597 and 4.685599; the difference between which is 2.

Correction for $37'' = \frac{37}{60} \times 2 = 1$, nearly.

Adding to 4.685597, the result is 4.685598.
The given angle, reduced to seconds, is 2617″.

$$\begin{array}{rl} & 4.685598 - 10 \\ \log 2617 = & 3.417804 \\ \hline \text{Result,} & 8.103402 - 10 \end{array}$$

This is correct to the sixth place of decimals; the result by the table of logarithmic tangents is 8.103375 − 10.

To find the angle corresponding to a given logarithmic sine or tangent, when between 0° and 5°.

Find from the table of logarithmic functions the angle corresponding to the given logarithm, to the nearest second.

Take from the auxiliary table the logarithm corresponding to this angle.

Subtract the result from the given logarithm, and find the number corresponding to the difference, giving the required angle in seconds.

Example. Find the angle whose log sin = 7.632366 − 10.

The angle corresponding is 0° 14′ 45″, to the nearest second.

The logarithm corresponding to sin 0° 14 45″ is 4.685573 − 10.

$$\begin{array}{r} 7.632366 - 10 \\ \underline{4.685573 - 10} \\ 2.946793 \end{array}$$

The number corresponding to this logarithm is 884.69.

Then the required angle is 884.69″, or 0° 14′ 44.69″.

This is correct to the second decimal place of seconds; the result by the table of logarithmic sines is 0° 14′ 45.08″.

Note XIV. The above methods serve to determine with accuracy the logarithmic cosine or cotangent of an angle between 85° and 90°, or the angle corresponding in the same cases.

To find accurately the logarithmic tangent of an angle between 85° and 90°, find the logarithmic cotangent of the angle as above, and subtract the result from 10 − 10. (Note XI.)

To find the angle corresponding to a logarithmic tangent in the same case, find the logarithmic cotangent of the angle (Note XI.), and find the angle corresponding to the result.

These methods also serve to determine the logarithmic cotangent of an angle between 0° and 5°, or the angle corresponding in the same case.

A TABLE

CONTAINING THE

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

N.	0	1	2	3	4	5	6	7	8	9	D.
100	00 0000	00 0434	00 0868	00 1301	00 1734	00 2166	00 2598	00 3029	00 3461	00 3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	01 0300	01 0724	01 1147	01 1570	01 1993	01 2415	424
103	01 2837	01 3259	01 3680	01 4100	4521	4940	5360	5779	6197	6616	420
104	7033	7451	7868	8284	8700	9116	9532	9947	02 0361	02 0775	416
105	02 1189	02 1603	02 2016	02 2428	02 2841	02 3252	02 3664	02 4075	02 4486	02 4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	03 0195	03 0600	03 1004	03 1408	03 1812	03 2216	03 2619	03 3021	404
108	03 3424	03 3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	04 0207	04 0602	04 0998	397
110	04 1393	04 1787	04 2182	04 2576	04 2969	04 3362	04 3755	04 4148	04 4540	04 4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
112	9218	9606	9993	05 0380	05 0766	05 1153	05 1538	05 1924	05 2309	05 2694	386
113	05 3078	05 3463	05 3846	4230	4613	4996	5378	5760	6142	6524	383
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	06 0320	379
115	06 0698	06 1075	06 1452	06 1829	06 2206	06 2582	06 2958	06 3333	06 3709	06 4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	8186	8557	8928	9298	9668	07 0038	07 0407	07 0776	07 1145	07 1514	370
118	07 1882	07 2250	07 2617	07 2985	07 3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	07 9181	07 9543	07 9904	08 0266	08 0626	08 0987	08 1347	08 1707	08 2067	08 2426	360
121	08 2785	08 3144	08 3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	09 0258	09 0611	09 0963	09 1315	09 1667	09 2018	09 2370	09 2721	09 3071	352
124	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	09 6910	09 7257	09 7604	09 7951	09 8298	09 8644	09 8990	09 9335	09 9681	10 0026	346
126	10 0371	10 0715	10 1059	10 1403	10 1747	10 2091	10 2434	10 2777	10 3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	11 0253	338
129	11 0590	11 0926	11 1263	11 1599	11 1934	11 2270	11 2605	11 2940	11 3275	3609	335
130	11 3943	11 4277	11 4611	11 4944	11 5278	11 5611	11 5943	11 6276	11 6608	11 6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	12 0245	330
132	12 0574	12 0903	12 1231	12 1560	12 1888	12 2216	12 2544	12 2871	12 3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	13 0012	323
135	13 0334	13 0655	13 0977	13 1298	13 1619	13 1939	13 2260	13 2580	13 2900	13 3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	9879	14 0194	14 0508	14 0822	14 1136	14 1450	14 1763	14 2076	14 2389	14 2702	314
139	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	14 6128	14 6438	14 6748	14 7058	14 7367	14 7676	14 7985	14 8294	14 8603	14 8911	309
141	9219	9527	9835	15 0142	15 0449	15 0756	15 1063	15 1370	15 1676	15 1982	307
142	15 2288	15 2594	15 2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	16 0168	16 0469	16 0769	16 1068	301
145	16 1368	16 1667	16 1967	16 2266	16 2564	16 2863	16 3161	16 3460	16 3758	16 4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	17 0262	17 0555	17 0848	17 1141	17 1434	17 1726	17 2019	17 2311	17 2603	17 2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	17 6091	17 6381	17 6670	17 6959	17 7248	17 7536	17 7825	17 8113	17 8401	17 8689	289
151	8977	9264	9552	9839	18 0126	18 0413	18 0699	18 0986	18 1272	18 1558	287
152	18 1844	18 2129	18 2415	18 2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	19 0051	281
155	19 0332	19 0612	19 0892	19 1171	19 1451	19 1730	19 2010	19 2289	19 2567	19 2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	20 0029	20 0303	20 0577	20 0850	20 1124	274
159	20 1397	20 1670	20 1943	20 2216	20 2488	2761	3033	3305	3577	3848	272
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
160	20 4120	20 4391	20 4663	20 4934	20 5204	20 5475	20 5746	20 6016	20 6286	20 6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	21 0051	21 0319	21 0586	21 0853	21 1121	21 1388	21 1654	21 1921	267
163	21 2188	21 2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	21 7484	21 7747	21 8010	21 8273	21 8536	21 8798	21 9060	21 9323	21 9585	21 9846	262
166	22 0108	22 0370	22 0631	22 0892	22 1153	22 1414	22 1675	22 1936	22 2196	22 2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	23 0193	256
170	23 0449	23 0704	23 0960	23 1215	23 1470	23 1724	23 1979	23 2234	23 2488	23 2742	255
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	24 0050	24 0300	250
174	24 0549	24 0799	24 1048	24 1297	24 1546	24 1795	24 2044	24 2293	2541	2790	249
175	24 3038	24 3286	24 3534	24 3782	24 4030	24 4277	24 4525	24 4772	24 5019	24 5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	25 0176	245
178	25 0420	25 0664	25 0908	25 1151	25 1395	25 1638	25 1881	25 2125	25 2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	25 5273	25 5514	25 5755	25 5996	25 6237	25 6477	25 6718	25 6958	25 7198	25 7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26 0071	26 0310	26 0548	26 0787	26 1025	26 1263	26 1501	26 1739	26 1976	26 2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	26 7172	26 7406	26 7641	26 7875	26 8110	26 8344	26 8578	26 8812	26 9046	26 9279	234
186	9513	9746	9980	27 0213	27 0446	27 0679	27 0912	27 1144	27 1377	27 1609	233
187	27 1842	27 2074	27 2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	27 8754	27 8982	27 9211	27 9439	27 9667	27 9895	28 0123	28 0351	28 0578	28 0806	228
191	28 1033	28 1261	28 1488	28 1715	28 1942	28 2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	29 0257	29 0480	29 0702	29 0925	29 1147	29 1369	29 1591	29 1813	29 2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	30 0161	30 0378	30 0595	30 0813	218
200	30 1030	30 1247	30 1464	30 1681	30 1898	30 2114	30 2331	30 2547	30 2764	30 2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	31 0056	31 0268	31 0481	31 0693	31 0906	31 1118	31 1330	31 1542	212
205	31 1754	31 1966	31 2177	31 2389	31 2600	31 2812	31 3023	31 3234	31 3445	31 3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32 0146	32 0354	32 0562	32 0769	32 0977	32 1184	32 1391	32 1598	32 1805	32 2012	207
210	32 2219	32 2426	32 2633	32 2839	32 3046	32 3252	32 3458	32 3665	32 3871	32 4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	33 0008	33 0211	203
214	33 0414	33 0617	33 0819	33 1022	33 1225	33 1427	33 1630	33 1832	2034	2236	202
215	33 2438	33 2640	33 2842	33 3044	33 3246	33 3447	33 3649	33 3850	33 4051	33 4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	34 0047	34 0246	199
219	34 0444	34 0642	34 0841	34 1039	34 1237	34 1435	34 1632	34 1830	2028	2225	198
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
220	34 2423	34 2620	34 2817	34 3014	34 3212	34 3409	34 3606	34 3802	34 3999	34 4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	35 0054	194
224	35 0248	35 0442	35 0636	35 0829	35 1023	35 1216	35 1410	35 1603	35 1796	1989	193
225	35 2183	35 2375	35 2568	35 2761	35 2954	35 3147	35 3339	35 3532	35 3724	35 3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835	36 0025	36 0215	36 0404	36 0593	36 0783	36 0972	36 1161	36 1350	36 1539	189
230	36 1728	36 1917	36 2105	36 2294	36 2482	36 2671	36 2859	36 3048	36 3236	36 3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	37 0143	37 0328	37 0513	37 0698	37 0883	185
235	37 1068	37 1253	37 1437	37 1622	37 1806	37 1991	37 2175	37 2360	37 2544	37 2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	38 0030	181
240	38 0211	38 0392	38 0573	38 0754	38 0934	38 1115	38 1296	38 1476	38 1656	38 1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	38 9166	38 9343	38 9520	38 9698	38 9875	39 0051	39 0228	39 0405	39 0582	39 0759	177
246	39 0935	39 1112	39 1288	39 1464	39 1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
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251	9674	9847	40 0020	40 0192	40 0365	40 0538	40 0711	40 0883	40 1056	40 1228	173
252	40 1401	40 1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
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265	42 3246	42 3410	42 3574	42 3737	42 3901	42 4065	42 4228	42 4392	42 4555	42 4718	164
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310	49 1362	49 1502	49 1642	49 1782	49 1922	49 2062	49 2201	49 2341	49 2481	49 2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
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353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
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363	9907	56 0026	56 0146	56 0265	56 0385	56 0504	56 0624	56 0743	56 0863	56 0982	119
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367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
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369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
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371	9374	9491	9608	9725	9842	9959	57 0076	57 0193	57 0309	57 0426	117
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423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
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427	63 0428	63 0530	63 0631	63 0733	63 0835	63 0936	1038	1139	1241	1342	102
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458	66 0865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
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462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
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466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
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469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	67 2098	67 2190	67 2283	67 2375	67 2467	67 2560	67 2652	67 2744	67 2836	67 2929	92
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475	67 6694	67 6785	67 6876	67 6968	67 7059	67 7151	67 7242	67 7333	67 7424	67 7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
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478	9428	9519	9610	9700	9791	9882	9973	68 0063	68 0154	68 0245	91
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487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
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515	71 1807	71 1892	71 1976	71 2060	71 2144	71 2229	71 2313	71 2397	71 2481	71 2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
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522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	72 0077	83
525	72 0159	72 0242	72 0325	72 0407	72 0490	72 0573	72 0655	72 0738	72 0821	72 0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	72 4276	72 4358	72 4440	72 4522	72 4604	72 4685	72 4767	72 4849	72 4931	72 5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
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535	72 8354	72 8435	72 8516	72 8597	72 8678	72 8759	72 8841	72 8922	72 9003	72 9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	73 0055	73 0136	73 0217	73 0298	73 0378	73 0459	73 0540	73 0621	73 0702	81
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540	73 2394	73 2474	73 2555	73 2635	73 2715	73 2796	73 2876	73 2956	73 3037	73 3117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
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545	73 6397	73 6476	73 6556	73 6635	73 6715	73 6795	73 6874	73 6954	73 7034	73 7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
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550	74 0363	74 0442	74 0521	74 0600	74 0678	74 0757	74 0836	74 0915	74 0994	74 1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	74 4293	74 4371	74 4449	74 4528	74 4606	74 4684	74 4762	74 4840	74 4919	74 4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	74 8188	74 8266	74 8343	74 8421	74 8498	74 8576	74 8653	74 8731	74 8808	74 8885	77
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562	9736	9814	9891	9968	75 0045	75 0123	75 0200	75 0277	75 0354	75 0431	77
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564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	75 2048	75 2125	75 2202	75 2279	75 2356	75 2433	75 2509	75 2586	75 2663	75 2740	77
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567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
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569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
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571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
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576	76 0422	76 0498	76 0573	76 0649	76 0724	0799	0875	0950	1025	1101	75
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580	76 3428	76 3503	76 3578	76 3653	76 3727	76 3802	76 3877	76 3952	76 4027	76 4101	75
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582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
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585	76 7156	76 7230	76 7304	76 7379	76 7453	76 7527	76 7601	76 7675	76 7749	76 7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	77 0042	74
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590	77 0852	77 0926	77 0999	77 1073	77 1146	77 1220	77 1293	77 1367	77 1440	77 1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	77 4517	77 4590	77 4663	77 4736	77 4809	77 4882	77 4955	77 5028	77 5100	77 5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
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599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
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601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
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603	78 0317	78 0389	78 0461	78 0533	78 0605	78 0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	78 1755	78 1827	78 1899	78 1971	78 2042	78 2114	78 2186	78 2258	78 2329	78 2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	78 5330	78 5401	78 5472	78 5543	78 5615	78 5686	78 5757	78 5828	78 5899	78 5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	78 8875	78 8946	78 9016	78 9087	78 9157	78 9228	78 9299	78 9369	78 9440	78 9510	71
616	9581	9651	9722	9792	9863	9933	79 0004	79 0074	79 0144	79 0215	70
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619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
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621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
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624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	79 5880	79 5949	79 6019	79 6088	79 6158	79 6227	79 6297	79 6366	79 6436	79 6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
635	80 2774	80 2842	80 2910	80 2979	80 3047	80 3116	80 3184	80 3252	80 3321	80 3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
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641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	80 9560	80 9627	80 9694	80 9762	80 9829	80 9896	80 9964	81 0031	81 0098	81 0165	67
646	81 0233	81 0300	81 0367	81 0434	81 0501	81 0569	81 0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	81 2913	81 2980	81 3047	81 3114	81 3181	81 3247	81 3314	81 3381	81 3448	81 3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	81 6241	81 6308	81 6374	81 6440	81 6506	81 6573	81 6639	81 6705	81 6771	81 6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	81 9544	81 9610	81 9676	81 9741	81 9807	81 9873	81 9939	82 0004	82 0070	82 0136	66
661	82 0201	82 0267	82 0333	82 0399	82 0464	82 0530	82 0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	82 2822	82 2887	82 2952	82 3018	82 3083	82 3148	82 3213	82 3279	82 3344	82 3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	82 6075	82 6140	82 6204	82 6269	82 6334	82 6399	82 6464	82 6528	82 6593	82 6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	82 9304	82 9368	82 9432	82 9497	82 9561	82 9625	82 9690	82 9754	82 9818	82 9882	64
676	9947	83 0011	83 0075	83 0139	83 0204	83 0268	83 0332	83 0396	83 0460	83 0525	64
677	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	83 2509	83 2573	83 2637	83 2700	83 2764	83 2828	83 2892	83 2956	83 3020	83 3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	83 5691	83 5754	83 5817	83 5881	83 5944	83 6007	83 6071	83 6134	83 6197	83 6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	83 8849	83 8912	83 8975	83 9038	83 9101	83 9164	83 9227	83 9289	83 9352	83 9415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	84 0043	63
692	84 0106	84 0169	84 0232	84 0294	84 0357	84 0420	84 0482	84 0545	84 0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	84 1985	84 2047	84 2110	84 2172	84 2235	84 2297	84 2360	84 2422	84 2484	84 2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
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701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	84 8189	84 8251	84 8312	84 8374	84 8435	84 8497	84 8559	84 8620	84 8682	84 8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85 0033	85 0095	85 0156	85 0217	85 0279	85 0340	85 0401	85 0462	85 0524	85 0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	85 1258	85 1320	85 1381	85 1442	85 1503	85 1564	85 1625	85 1686	85 1747	85 1809	61
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714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	85 4306	85 4367	85 4428	85 4488	85 4549	85 4610	85 4670	85 4731	85 4792	85 4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
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729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	86 3323	86 3382	86 3442	86 3501	86 3561	86 3620	86 3680	86 3739	86 3799	86 3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	86 6287	86 6346	86 6405	86 6465	86 6524	86 6583	86 6642	86 6701	86 6760	86 6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
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741	9818	9877	9935	9994	87 0053	87 0111	87 0170	87 0228	87 0287	87 0345	59
742	87 0404	87 0462	87 0521	87 0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	87 2156	87 2215	87 2273	87 2331	87 2389	87 2448	87 2506	87 2564	87 2622	87 2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	87 5061	87 5119	87 5177	87 5235	87 5293	87 5351	87 5409	87 5466	87 5524	87 5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	87 7947	87 8004	87 8062	87 8119	87 8177	87 8234	87 8292	87 8349	87 8407	87 8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	88 0013	88 0070	88 0127	88 0185	57
759	88 0242	88 0299	88 0356	88 0413	88 0471	88 0528	0585	0642	0699	0756	57
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761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	88 3661	88 3718	88 3775	88 3832	88 3888	88 3945	88 4002	88 4059	88 4115	88 4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	88 6491	88 6547	88 6604	88 6660	88 6716	88 6773	88 6829	88 6885	88 6942	88 6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	88 9302	88 9358	88 9414	88 9470	88 9526	88 9582	88 9638	88 9694	88 9750	88 9806	56
776	9862	9918	9974	89 0030	89 0086	89 0141	89 0197	89 0253	89 0309	89 0365	56
777	89 0421	89 0477	89 0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	89 2095	89 2150	89 2206	89 2262	89 2317	89 2373	89 2429	89 2484	89 2540	89 2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	89 4870	89 4925	89 4980	89 5036	89 5091	89 5146	89 5201	89 5257	89 5312	89 5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	89 7627	89 7682	89 7737	89 7792	89 7847	89 7902	89 7957	89 8012	89 8067	89 8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	90 0039	90 0094	90 0149	90 0203	90 0258	90 0312	55
795	90 0367	90 0422	90 0476	90 0531	90 0586	90 0640	90 0695	90 0749	90 0804	90 0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	90 3090	90 3144	90 3199	90 3253	90 3307	90 3361	90 3416	90 3470	90 3524	90 3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	90 5796	90 5850	90 5904	90 5958	90 6012	90 6066	90 6119	90 6173	90 6227	90 6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
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813	91 0091	91 0144	91 0197	91 0251	91 0304	91 0358	91 0411	91 0464	91 0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	91 1158	91 1211	91 1264	91 1317	91 1371	91 1424	91 1477	91 1530	91 1584	91 1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
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821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	91 6454	91 6507	91 6559	91 6612	91 6664	91 6717	91 6770	91 6822	91 6875	91 6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	91 9078	91 9130	91 9183	91 9235	91 9287	91 9340	91 9392	91 9444	91 9496	91 9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	92 0019	92 0071	52
832	92 0123	92 0176	92 0228	92 0280	92 0332	92 0384	92 0436	92 0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	92 1686	92 1738	92 1790	92 1842	92 1894	92 1946	92 1998	92 2050	92 2102	92 2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	92 4279	92 4331	92 4383	92 4434	92 4486	92 4538	92 4589	92 4641	92 4693	92 4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	92 6857	92 6908	92 6959	92 7011	92 7062	92 7114	92 7165	92 7216	92 7268	92 7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	92 9419	92 9470	92 9521	92 9572	92 9623	92 9674	92 9725	92 9776	92 9827	92 9879	51
851	9930	9981	93 0032	93 0083	93 0134	93 0185	93 0236	93 0287	93 0338	93 0389	51
852	93 0440	93 0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	93 1966	93 2017	93 2068	93 2118	93 2169	93 2220	93 2271	93 2322	93 2372	93 2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	93 4498	93 4549	93 4599	93 4650	93 4700	93 4751	93 4801	93 4852	93 4902	93 4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
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864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
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866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
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874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	94 2008	94 2058	94 2107	94 2157	94 2207	94 2256	94 2306	94 2355	94 2405	94 2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
880	94 4483	94 4532	94 4581	94 4631	94 4680	94 4729	94 4779	94 4828	94 4877	94 4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	94 6943	94 6992	94 7041	94 7090	94 7140	94 7189	94 7238	94 7287	94 7336	94 7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	94 9390	94 9439	94 9488	94 9536	94 9585	94 9634	94 9683	94 9731	94 9780	94 9829	49
891	9878	9926	9975	95 0024	95 0073	95 0121	95 0170	95 0219	95 0267	95 0316	49
892	95 0365	95 0414	95 0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	95 1823	95 1872	95 1920	95 1969	95 2017	95 2066	95 2114	95 2163	95 2211	95 2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	95 4243	95 4291	95 4339	95 4387	95 4435	95 4484	95 4532	95 4580	95 4628	95 4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	95 6649	95 6697	95 6745	95 6793	95 6840	95 6888	95 6936	95 6984	95 7032	95 7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	95 9041	95 9089	95 9137	95 9185	95 9232	95 9280	95 9328	95 9375	95 9423	95 9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	96 0042	96 0090	96 0138	96 0185	96 0233	96 0280	96 0328	96 0376	96 0423	48
913	96 0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
915	96 1421	96 1469	96 1516	96 1563	96 1611	96 1658	96 1706	96 1753	96 1801	96 1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	96 3788	96 3835	96 3882	96 3929	96 3977	96 4024	96 4071	96 4118	96 4165	96 4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	96 6142	96 6189	96 6236	96 6283	96 6329	96 6376	96 6423	96 6470	96 6517	96 6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	96 8483	96 8530	96 8576	96 8623	96 8670	96 8716	96 8763	96 8810	96 8856	96 8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	97 0021	97 0068	97 0114	97 0161	97 0207	97 0254	97 0300	47
934	97 0347	97 0393	97 0440	0486	0533	0579	0626	0672	0719	0765	46
935	97 0812	97 0858	97 0904	97 0951	97 0997	97 1044	97 1090	97 1137	97 1183	97 1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
940	97 3128	97 3174	97 3220	97 3266	97 3313	97 3359	97 3405	97 3451	97 3497	97 3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	97 5432	97 5478	97 5524	97 5570	97 5616	97 5662	97 5707	97 5753	97 5799	97 5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	97 7724	97 7769	97 7815	97 7861	97 7906	97 7952	97 7998	97 8043	97 8089	97 8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	98 0003	98 0049	98 0094	98 0140	98 0185	98 0231	98 0276	98 0322	98 0367	98 0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	98 2271	98 2316	98 2362	98 2407	98 2452	98 2497	98 2543	98 2588	98 2633	98 2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	98 4527	98 4572	98 4617	98 4662	98 4707	98 4752	98 4797	98 4842	98 4887	98 4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	98 6772	98 6817	98 6861	98 6906	98 6951	98 6996	98 7040	98 7085	98 7130	98 7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	98 9005	98 9049	98 9094	98 9138	98 9183	98 9227	98 9272	98 9316	98 9361	98 9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	99 0028	99 0072	99 0117	99 0161	99 0206	99 0250	99 0294	44
978	99 0339	99 0383	99 0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	99 1226	99 1270	99 1315	99 1359	99 1403	99 1448	99 1492	99 1536	99 1580	99 1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	99 3436	99 3480	99 3524	99 3568	99 3613	99 3657	99 3701	99 3745	99 3789	99 3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	99 5635	99 5679	99 5723	99 5767	99 5811	99 5854	99 5898	99 5942	99 5986	99 6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	99 7823	99 7867	99 7910	99 7954	99 7998	99 8041	99 8085	99 8129	99 8172	99 8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

A TABLE

OF THE

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS,

FOR EVERY

DEGREE AND MINUTE FROM 0° TO 90°.

0°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	— ∞		10.000 000		— ∞		— ∞	60
1	6.463 726	5017.17	.000 000	.00	6.463 726	5017.17	3.536 274	59
2	.764 756	2934.85	.000 000	.00	.764 756	2934.85	.235 244	58
3	.940 847	2082.32	.000 000	.00	.940 847	2082.32	.059 153	57
4	7.065 786	1615.17	.000 000	.00	7.065 786	1615.17	2.934 214	56
5	7.162 696	1319.68	10.000 000	.02	7.162 696	1319.70	2.837 304	55
6	.241 877	1115.78	9.999 999	.00	.241 878	1115.78	.758 122	54
7	.308 824	966.53	.999 999	.00	.308 825	966.53	.691 175	53
8	.366 816	852.53	.999 999	.00	.366 817	852.55	.633 183	52
9	.417 968	762.63	.999 999	.02	.417 970	762.62	.582 030	51
10	7.463 726	689.87	9.999 998	.00	7.463 727	689.88	2.536 273	50
11	.505 118	629.80	.999 998	.02	.505 120	629.82	.494 880	49
12	.542 906	579.37	.999 997	.00	.542 909	579.38	.457 091	48
13	.577 668	536.42	.999 997	.02	.577 672	536.42	.422 328	47
14	.609 853	499.38	.999 996	.00	.609 857	499.38	.390 143	46
15	7.639 816	467.15	9.999 996	.02	7.639 820	467.15	2.360 180	45
16	.667 845	438.80	.999 995	.00	.667 849	438.83	.332 151	44
17	.694 173	413.73	.999 995	.02	.694 179	413.73	.305 821	43
18	.718 997	391.35	.999 994	.02	.719 003	391.35	.280 997	42
19	.742 478	371.27	.999 993	.00	.742 484	371.28	.257 516	41
20	7.764 754	353.15	9.999 993	.02	7.764 761	353.17	2.235 239	40
21	.785 943	336.72	.999 992	.02	.785 951	336.73	.214 049	39
22	.806 146	321.75	.999 991	.02	.806 155	321.75	.193 845	38
23	.825 451	308.05	.999 990	.02	.825 460	308.07	.174 540	37
24	.843 934	295.47	.999 989	.00	.843 944	295.50	.156 056	36
25	7.861 662	283.88	9.999 989	.02	7.861 674	283.90	2.138 326	35
26	.878 695	273.17	.999 988	.02	.878 708	273.18	.121 292	34
27	.895 085	263.23	.999 987	.02	.895 099	263.25	.104 901	33
28	.910 879	254.00	.999 986	.02	.910 894	254.00	.089 106	32
29	.926 119	245.38	.999 985	.03	.926 134	245.40	.073 866	31
30	7.940 842	237.33	9.999 983	.02	7.940 858	237.37	2.059 142	30
31	.955 082	229.80	.999 982	.02	.955 100	229.82	.044 900	29
32	.968 870	222.72	.999 981	.02	.968 889	222.73	.031 111	28
33	.982 233	216.08	.999 980	.02	.982 253	216.10	.017 747	27
34	.995 198	209.82	.999 979	.03	.995 219	209.83	.004 781	26
35	8.007 787	203.90	9.999 977	.02	8.007 809	203.92	1.992 191	25
36	.020 021	198.30	.999 976	.02	.020 044	198.35	.979 956	24
37	.031 919	193.03	.999 975	.03	.031 945	193.03	.968 055	23
38	.043 501	188.00	.999 973	.02	.043 527	188.03	.956 473	22
39	.054 781	183.25	.999 972	.02	.054 809	183.28	.945 191	21
40	8.065 776	178.73	9.999 971	.03	8.065 806	178.75	1.934 194	20
41	.076 500	174.42	.999 969	.02	.076 531	174.43	.923 469	19
42	.086 965	170.30	.999 968	.03	.086 997	170.33	.913 003	18
43	.097 183	166.40	.999 966	.03	.097 217	166.43	.902 783	17
44	.107 167	162.65	.999 964	.02	.107 203	162.67	.892 797	16
45	8.116 926	159.08	9.999 963	.03	8.116 963	159.12	1.883 037	15
46	.126 471	155.65	.999 961	.03	.126 510	155.68	.873 490	14
47	.135 810	152.38	.999 959	.02	.135 851	152.42	.864 149	13
48	.144 953	149.23	.999 958	.03	.144 996	149.27	.855 004	12
49	.153 907	146.23	.999 956	.03	.153 952	146.25	.846 048	11
50	8.162 681	143.32	9.999 954	.03	8.162 727	143.35	1.837 273	10
51	.171 280	140.55	.999 952	.03	.171 328	140.58	.828 672	9
52	.179 713	137.87	.999 950	.03	.179 763	137.88	.820 237	8
53	.187 985	135.28	.999 948	.03	.188 036	135.33	.811 964	7
54	.196 102	132.80	.999 946	.03	.196 156	132.83	.803 844	6
55	8.204 070	130.42	9.999 944	.03	8.204 126	130.45	1.795 874	5
56	.211 895	128.10	.999 942	.03	.211 953	128.13	.788 047	4
57	.219 581	125.88	.999 940	.03	.219 641	125.90	.780 359	3
58	.227 134	123.72	.999 938	.03	.227 195	123.77	.772 805	2
59	.234 557	121.63	.999 936	.03	.234 621	121.67	.765 379	1
60	8.241 855		9.999 934		8.241 921		1.758 079	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

89°

1°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.241 855	119.63	9.999 934	.03	8.241 921	119.68	1.758 079	60
1	.249 033	117.68	.999 932	.05	.249 102	117.72	.750 898	59
2	.256 094	115.80	.999 929	.03	.256 165	115.83	.743 835	58
3	.263 042	113.98	.999 927	.03	.263 115	114.02	.736 885	57
4	.269 881	112.22	.999 925	.05	.269 956	112.25	.730 044	56
5	8.276 614	110.48	9.999 922	.03	8.276 691	110.53	1.723 309	55
6	.283 243	108.83	.999 920	.03	.283 323	108.88	.716 677	54
7	.289 773	107.23	.999 918	.05	.289 856	107.27	.710 144	53
8	.296 207	105.65	.999 915	.03	.296 292	105.70	.703 708	52
9	.302 546	104.13	.999 913	.05	.302 634	104.17	.697 366	51
10	8.308 794	102.67	9.999 910	.05	8.308 884	102.70	1.691 116	50
11	.314 954	101.22	.999 907	.03	.315 046	101.27	.684 954	49
12	.321 027	99.82	.999 905	.05	.321 122	99.87	.678 878	48
13	.327 016	98.47	.999 902	.05	.327 114	98.52	.672 886	47
14	.332 924	97.15	.999 899	.03	.333 025	97.18	.666 975	46
15	8.338 753	95.85	9.999 897	.05	8.338 856	95.90	1.661 144	45
16	.344 504	94.62	.999 894	.05	.344 610	94.65	.655 390	44
17	.350 181	93.37	.999 891	.05	.350 289	93.43	.649 711	43
18	.355 783	92.20	.999 888	.05	.355 895	92.25	.644 105	42
19	.361 315	91.03	.999 885	.05	.361 430	91.08	.638 570	41
20	8.366 777	89.90	9.999 882	.05	8.366 895	89.95	1.633 105	40
21	.372 171	88.80	.999 879	.05	.372 292	88.83	.627 708	39
22	.377 499	87.72	.999 876	.05	.377 622	87.78	.622 378	38
23	.382 762	86.67	.999 873	.05	.382 889	86.72	.617 111	37
24	.387 962	85.65	.999 870	.05	.388 092	85.70	.611 908	36
25	8.393 101	84.63	9.999 867	.05	8.393 234	84.68	1.606 766	35
26	.398 179	83.67	.999 864	.05	.398 315	83.72	.601 685	34
27	.403 199	82.70	.999 861	.05	.403 338	82.77	.596 662	33
28	.408 161	81.78	.999 858	.07	.408 304	81.82	.591 696	32
29	.413 068	80.85	.999 854	.05	.413 213	80.92	.586 787	31
30	8.417 919	79.97	9.999 851	.05	8.418 068	80.02	1.581 932	30
31	.422 717	79.08	.999 848	.07	.422 869	79.15	.577 131	29
32	.427 462	78.23	.999 844	.05	.427 618	78.28	.572 382	28
33	.432 156	77.40	.999 841	.05	.432 315	77.45	.567 685	27
34	.436 800	76.57	.999 838	.07	.436 962	76.63	.563 038	26
35	8.441 394	75.78	9.999 834	.05	8.441 560	75.83	1.558 440	25
36	.445 941	74.98	.999 831	.07	.446 110	75.05	.553 890	24
37	.450 440	74.22	.999 827	.05	.450 613	74.28	.549 387	23
38	.454 893	73.47	.999 824	.07	.455 070	73.52	.544 930	22
39	.459 301	72.73	.999 820	.07	.459 481	72.80	.540 519	21
40	8.463 665	72.00	9.999 816	.05	8.463 849	72.05	1.536 151	20
41	.467 985	71.30	.999 813	.07	.468 172	71.37	.531 828	19
42	.472 263	70.58	.999 809	.07	.472 454	70.65	.527 546	18
43	.476 498	69.92	.999 805	.07	.476 693	69.98	.523 307	17
44	.480 693	69.25	.999 801	.07	.480 892	69.30	.519 108	16
45	8.484 848	68.58	9.999 797	.05	8.485 050	68.67	1.514 950	15
46	.488 963	67.95	.999 794	.07	.489 170	68.00	.510 830	14
47	.493 040	67.30	.999 790	.07	.493 250	67.38	.506 750	13
48	.497 078	66.70	.999 786	.07	.497 293	66.75	.502 707	12
49	.501 080	66.08	.999 782	.07	.501 298	66.15	.498 702	11
50	8.505 045	65.48	9.999 778	.07	8.505 267	65.55	1.494 733	10
51	.508 974	64.88	.999 774	.08	.509 200	64.97	.490 800	9
52	.512 867	64.32	.999 769	.07	.513 098	64.38	.486 902	8
53	.516 726	63.75	.999 765	.07	.516 961	63.82	.483 039	7
54	.520 551	63.20	.999 761	.07	.520 790	63.27	.479 210	6
55	8.524 343	62.65	9.999 757	.07	8.524 586	62.72	1.475 414	5
56	.528 102	62.10	.999 753	.08	.528 349	62.18	.471 651	4
57	.531 828	61.58	.999 748	.07	.532 080	61.65	.467 920	3
58	.535 523	61.05	.999 744	.07	.535 779	61.13	.464 221	2
59	.539 186	60.55	.999 740	.08	.539 447	60.62	.460 553	1
60	8.542 819		9.999 735		8.543 084		1.456 916	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

88°

2°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.542 819	60.05	9.999 735	.07	8.543 084	60.12	1.456 916	60
1	.546 422	59.55	.999 731	.08	.546 691	59.62	.453 309	59
2	.549 995	59.07	.999 726	.07	.550 268	59.15	.449 732	58
3	.553 539	58.58	.999 722	.08	.553 817	58.65	.446 183	57
4	.557 054	58.10	.999 717	.07	.557 336	58.20	.442 664	56
5	8.560 540	57.65	9.999 713	.08	8.560 828	57.72	1.439 172	55
6	.563 999	57.20	.999 708	.07	.564 291	57.27	.435 709	54
7	.567 431	56.75	.999 704	.08	.567 727	56.83	.432 273	53
8	.570 836	56.30	.999 699	.08	.571 137	56.38	.428 863	52
9	.574 214	55.87	.999 694	.08	.574 520	55.95	.425 480	51
10	8.577 566	55.43	9.999 689	.07	8.577 877	55.52	1.422 123	50
11	.580 892	55.02	.999 685	.08	.581 208	55.10	.418 792	49
12	.584 193	54.60	.999 680	.08	.584 514	54.68	.415 486	48
13	.587 469	54.20	.999 675	.08	.587 795	54.27	.412 205	47
14	.590 721	53.78	.999 670	.08	.591 051	53.87	.408 949	46
15	8.593 948	53.40	9.999 665	.08	8.594 283	53.48	1.405 717	45
16	.597 152	53.00	.999 660	.08	.597 492	53.08	.402 508	44
17	.600 332	52.62	.999 655	.08	.600 677	52.70	.399 323	43
18	.603 489	52.23	.999 650	.08	.603 839	52.32	.396 161	42
19	.606 623	51.85	.999 645	.08	.606 978	51.93	.393 022	41
20	8.609 734	51.48	9.999 640	.08	8.610 094	51.58	1.389 906	40
21	.612 823	51.13	.999 635	.10	.613 189	51.22	.386 811	39
22	.615 891	50.77	.999 629	.08	.616 262	50.85	.383 738	38
23	.618 937	50.42	.999 624	.08	.619 313	50.50	.380 687	37
24	.621 962	50.05	.999 619	.08	.622 343	50.15	.377 657	36
25	8.624 965	49.72	9.999 614	.10	8.625 352	49.80	1.374 648	35
26	.627 948	49.38	.999 608	.08	.628 340	49.47	.371 660	34
27	.630 911	49.05	.999 603	.10	.631 308	49.13	.368 692	33
28	.633 854	48.70	.999 597	.08	.634 256	48.80	.365 744	32
29	.636 776	48.40	.999 592	.10	.637 184	48.48	.362 816	31
30	8.639 680	48.05	9.999 586	.08	8.640 093	48.15	1.359 907	30
31	.642 563	47.75	.999 581	.10	.642 982	47.85	.357 018	29
32	.645 428	47.43	.999 575	.08	.645 853	47.52	.354 147	28
33	.648 274	47.13	.999 570	.10	.648 704	47.22	.351 296	27
34	.651 102	46.82	.999 564	.10	.651 537	46.92	.348 463	26
35	8.653 911	46.52	9.999 558	.08	8.654 352	46.62	1.345 648	25
36	.656 702	46.22	.999 553	.10	.657 149	46.32	.342 851	24
37	.659 475	45.92	.999 547	.10	.659 928	46.02	.340 072	23
38	.662 230	45.63	.999 541	.10	.662 689	45.73	.337 311	22
39	.664 968	45.35	.999 535	.10	.665 433	45.45	.334 567	21
40	8.667 689	45.07	9.999 529	.08	8.668 160	45.17	1.331 840	20
41	.670 393	44.78	.999 524	.10	.670 870	44.88	.329 130	19
42	.673 080	44.52	.999 518	.10	.673 563	44.60	.326 437	18
43	.675 751	44.23	.999 512	.10	.676 239	44.35	.323 761	17
44	.678 405	43.97	.999 506	.10	.678 900	44.07	.321 100	16
45	8.681 043	43.70	9.999 500	.12	8.681 544	43.80	1.318 456	15
46	.683 665	43.45	.999 493	.10	.684 172	43.53	.315 828	14
47	.686 272	43.18	.999 487	.10	.686 784	43.28	.313 216	13
48	.688 863	42.92	.999 481	.10	.689 381	43.03	.310 619	12
49	.691 438	42.67	.999 475	.10	.691 963	42.77	.308 037	11
50	8.693 998	42.42	9.999 469	.10	8.694 529	42.53	1.305 471	10
51	.696 543	42.17	.999 463	.12	.697 081	42.27	.302 919	9
52	.699 073	41.93	.999 456	.10	.699 617	42.03	.300 383	8
53	.701 589	41.68	.999 450	.12	.702 139	41.78	.297 861	7
54	.704 090	41.45	.999 443	.10	.704 646	41.57	.295 354	6
55	8.706 577	41.20	9.999 437	.10	8.707 140	41.30	1.292 860	5
56	.709 049	40.97	.999 431	.12	.709 618	41.08	.290 382	4
57	.711 507	40.75	.999 424	.10	.712 083	40.85	.287 917	3
58	.713 952	40.52	.999 418	.12	.714 534	40.63	.285 466	2
59	.716 383	40.28	.999 411	.12	.716 972	40.40	.283 028	1
60	8.718 800		9.999 404		8.719 396		1.280 604	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

87°

3°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.718 800	40.07	9.999 404	.10	8.719 396	40.17	1.280 604	60
1	.721 204	39.85	.999 398	.12	.721 806	39.97	.278 194	59
2	.723 595	39.62	.999 391	.12	.724 204	39.73	.275 796	58
3	.725 972	39.42	.999 384	.10	.726 588	39.52	.273 412	57
4	.728 337	39.18	.999 378	.12	.728 959	39.30	.271 041	56
5	8.730 688	38.98	9.999 371	.12	8.731 317	39.10	1.268 683	55
6	.733 027	38.78	.999 364	.12	.733 663	38.88	.266 337	54
7	.735 354	38.55	.999 357	.12	.735 996	38.68	.264 004	53
8	.737 667	38.37	.999 350	.12	.738 317	38.48	.261 683	52
9	.739 969	38.17	.999 343	.12	.740 626	38.27	.259 374	51
10	8.742 259	37.95	9.999 336	.12	8.742 922	38.08	1.257 078	50
11	.744 536	37.77	.999 329	.12	.745 207	37.87	.254 793	49
12	.746 802	37.55	.999 322	.12	.747 479	37.68	.252 521	48
13	.749 055	37.37	.999 315	.12	.749 740	37.48	.250 260	47
14	.751 297	37.18	.999 308	.12	.751 989	37.30	.248 011	46
15	8.753 528	36.98	9.999 301	.12	8.754 227	37.10	1.245 773	45
16	.755 747	36.80	.999 294	.12	.756 453	36.92	.243 547	44
17	.757 955	36.60	.999 287	.13	.758 668	36.73	.241 332	43
18	.760 151	36.43	.999 279	.12	.760 872	36.55	.239 128	42
19	.762 337	36.23	.999 272	.12	.763 065	36.35	.236 935	41
20	8.764 511	36.07	9.999 265	.13	8.765 246	36.18	1.234 754	40
21	.766 675	35.88	.999 257	.12	.767 417	36.02	.232 583	39
22	.768 828	35.70	.999 250	.13	.769 578	35.82	.230 422	38
23	.770 970	35.52	.999 242	.12	.771 727	35.65	.228 273	37
24	.773 101	35.37	.999 235	.13	.773 866	35.48	.226 134	36
25	8.775 223	35.17	9.999 227	.12	8.775 995	35.32	1.224 005	35
26	.777 333	35.02	.999 220	.13	.778 114	35.13	.221 886	34
27	.779 434	34.83	.999 212	.12	.780 222	34.97	.219 778	33
28	.781 524	34.68	.999 205	.13	.782 320	34.80	.217 680	32
29	.783 605	34.50	.999 197	.13	.784 408	34.63	.215 592	31
30	8.785 675	34.35	9.999 189	.13	8.786 486	34.47	1.213 514	30
31	.787 736	34.18	.999 181	.12	.788 554	34.32	.211 446	29
32	.789 787	34.02	.999 174	.13	.790 613	34.15	.209 387	28
33	.791 828	33.85	.999 166	.13	.792 662	33.98	.207 338	27
34	.793 859	33.70	.999 158	.13	.794 701	33.83	.205 299	26
35	8.795 881	33.55	9.999 150	.13	8.796 731	33.68	1.203 269	25
36	.797 894	33.38	.999 142	.13	.798 752	33.52	.201 248	24
37	.799 897	33.25	.999 134	.13	.800 763	33.37	.199 237	23
38	.801 892	33.07	.999 126	.13	.802 765	33.22	.197 235	22
39	.803 876	32.93	.999 118	.13	.804 758	33.07	.195 242	21
40	8.805 852	32.78	9.999 110	.13	8.806 742	32.92	1.193 258	20
41	.807 819	32.63	.999 102	.13	.808 717	32.77	.191 283	19
42	.809 777	32.48	.999 094	.13	.810 683	32.63	.189 317	18
43	.811 726	32.35	.999 086	.15	.812 641	32.47	.187 359	17
44	.813 667	32.20	.999 077	.13	.814 589	32.33	.185 411	16
45	8.815 599	32.05	9.999 069	.13	8.816 529	32.20	1.183 471	15
46	.817 522	31.90	.999 061	.13	.818 461	32.05	.181 539	14
47	.819 436	31.78	.999 053	.15	.820 384	31.90	.179 616	13
48	.821 343	31.62	.999 044	.13	.822 298	31.78	.177 702	12
49	.823 240	31.50	.999 036	.15	.824 205	31.63	.175 795	11
50	8.825 130	31.35	9.999 027	.13	8.826 103	31.48	1.173 897	10
51	.827 011	31.22	.999 019	.15	.827 992	31.37	.172 008	9
52	.828 884	31.08	.999 010	.13	.829 874	31.23	.170 126	8
53	.830 749	30.97	.999 002	.15	.831 748	31.08	.168 252	7
54	.832 607	30.82	.998 993	.15	.833 613	30.97	.166 387	6
55	8.834 456	30.68	9.998 984	.13	8.835 471	30.83	1.164 529	5
56	.836 297	30.55	.998 976	.15	.837 321	30.70	.162 679	4
57	.838 130	30.43	.998 967	.15	.839 163	30.58	.160 837	3
58	.839 956	30.30	.998 958	.13	.840 998	30.45	.159 002	2
59	.841 774	30.18	.998 950	.15	.842 825	30.32	.157 175	1
60	8.843 585		9.998 941		8.844 644		1.155 356	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

86°

4°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.843 585	30.03	9.998 941	.15	8.844 644	30.18	1.155 356	60
1	.845 387	29.93	.998 932	.15	.846 455	30.08	.153 545	59
2	.847 183	29.80	.998 923	.15	.848 260	29.95	.151 740	58
3	.848 971	29.67	.998 914	.15	.850 057	29.82	.149 943	57
4	.850 751	29.57	.998 905	.15	.851 846	29.70	.148 154	56
5	8.852 525	29.43	9.998 896	.15	8.853 628	29.58	1.146 372	55
6	.854 291	29.30	.998 887	.15	.855 403	29.47	.144 597	54
7	.856 049	29.20	.998 878	.15	.857 171	29.35	.142 829	53
8	.857 801	29.08	.998 869	.15	.858 932	29.23	.141 068	52
9	.859 546	28.95	.998 860	.15	.860 686	29.12	.139 314	51
10	8.861 283	28.85	9.998 851	.17	8.862 433	29.00	1.137 567	50
11	.863 014	28.73	.998 841	.15	.864 173	28.88	.135 827	49
12	.864 738	28.62	.998 832	.15	.865 906	28.77	.134 094	48
13	.866 455	28.50	.998 823	.17	.867 632	28.65	.132 368	47
14	.868 165	28.38	.998 813	.15	.869 351	28.55	.130 649	46
15	8.869 868	28.28	9.998 804	.15	8.871 064	28.43	1.128 936	45
16	.871 565	28.17	.998 795	.17	.872 770	28.32	.127 230	44
17	.873 255	28.05	.998 785	.15	.874 469	28.22	.125 531	43
18	.874 938	27.95	.998 776	.17	.876 162	28.12	.123 838	42
19	.876 615	27.83	.998 766	.15	.877 849	28.00	.122 151	41
20	8.878 285	27.73	9.998 757	.17	8.879 529	27.88	1.120 471	40
21	.879 949	27.63	.998 747	.15	.881 202	27.78	.118 798	39
22	.881 607	27.52	.998 738	.17	.882 869	27.68	.117 131	38
23	.883 258	27.42	.998 728	.17	.884 530	27.58	.115 470	37
24	.884 903	27.32	.998 718	.17	.886 185	27.47	.113 815	36
25	8.886 542	27.20	9.998 708	.15	8.887 833	27.38	1.112 167	35
26	.888 174	27.12	.998 699	.17	.889 476	27.27	.110 524	34
27	.889 801	27.00	.998 689	.17	.891 112	27.17	.108 888	33
28	.891 421	26.90	.998 679	.17	.892 742	27.07	.107 258	32
29	.893 035	26.80	.998 669	.17	.894 366	26.97	.105 634	31
30	8.894 643	26.72	9.998 659	.17	8.895 984	26.87	1.104 016	30
31	.896 246	26.60	.998 649	.17	.897 596	26.78	.102 404	29
32	.897 842	26.50	.998 639	.17	.899 203	26.67	.100 797	28
33	.899 432	26.42	.998 629	.17	.900 803	26.58	.099 197	27
34	.901 017	26.32	.998 619	.17	.902 398	26.48	.097 602	26
35	8.902 596	26.22	9.998 609	.17	8.903 987	26.38	1.096 013	25
36	.904 169	26.12	.998 599	.17	.905 570	26.28	.094 430	24
37	.905 736	26.02	.998 589	.18	.907 147	26.20	.092 853	23
38	.907 297	25.93	.998 578	.17	.908 719	26.10	.091 281	22
39	.908 853	25.85	.998 568	.17	.910 285	26.02	.089 715	21
40	8.910 404	25.75	9.998 558	.17	8.911 846	25.92	1.088 154	20
41	.911 949	25.65	.998 548	.18	.913 401	25.83	.086 599	19
42	.913 488	25.57	.998 537	.17	.914 951	25.73	.085 049	18
43	.915 022	25.47	.998 527	.18	.916 495	25.65	.083 505	17
44	.916 550	25.38	.998 516	.17	.918 034	25.57	.081 966	16
45	8.918 073	25.30	9.998 506	.18	8.919 568	25.47	1.080 432	15
46	.919 591	25.20	.998 495	.17	.921 096	25.38	.078 904	14
47	.921 103	25.12	.998 485	.18	.922 619	25.28	.077 381	13
48	.922 610	25.03	.998 474	.17	.924 136	25.22	.075 864	12
49	.924 112	24.95	.998 464	.18	.925 649	25.12	.074 351	11
50	8.925 609	24.85	9.998 453	.18	8.927 156	25.03	1.072 844	10
51	.927 100	24.78	.998 442	.18	.928 658	24.95	.071 342	9
52	.928 587	24.68	.998 431	.17	.930 155	24.87	.069 845	8
53	.930 068	24.60	.998 421	.18	.931 647	24.78	.068 353	7
54	.931 544	24.52	.998 410	.18	.933 134	24.70	.066 866	6
55	8.933 015	24.43	9.998 399	.18	8.934 616	24.62	1.065 384	5
56	.934 481	24.35	.998 388	.18	.936 093	24.53	.063 907	4
57	.935 942	24.27	.998 377	.18	.937 565	24.45	.062 435	3
58	.937 398	24.20	.998 366	.18	.939 032	24.37	.060 968	2
59	.938 850	24.10	.998 355	.18	.940 494	24.30	.059 506	1
60	8.940 296		9.998 344		8.941 952		1.058 048	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

85°

5°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	8.940 296	24.03	9.998 344	.18	8.941 052	24.20	1.058 948	60
1	.941 738	23.93	.998 333	.18	.943 404	24.13	.056 596	59
2	.943 174	23.87	.998 322	.18	.944 852	24.05	.055 148	58
3	.944 606	23.80	.998 311	.18	.946 295	23.98	.053 705	57
4	.946 034	23.70	.998 300	.18	.947 734	23.90	.052 266	56
5	8.947 456	23.63	9.998 289	.20	8.949 168	23.82	1.050 832	55
6	.948 874	23.55	.998 277	.18	.950 597	23.73	.049 403	54
7	.950 287	23.48	.998 266	.18	.952 021	23.67	.047 979	53
8	.951 696	23.40	.998 255	.20	.953 441	23.58	.046 559	52
9	.953 100	23.32	.998 243	.18	.954 856	23.52	.045 144	51
10	8.954 499	23.25	9.998 232	.20	8.956 267	23.45	1.043 733	50
11	.955 894	23.17	.998 220	.18	.957 674	23.35	.042 326	49
12	.957 284	23.10	.998 209	.20	.959 075	23.30	.040 925	48
13	.958 670	23.03	.998 197	.18	.960 473	23.22	.039 527	47
14	.960 052	22.95	.998 186	.20	.961 866	23.15	.038 134	46
15	8.961 429	22.87	9.998 174	.18	8.963 255	23.07	1.036 745	45
16	.962 801	22.82	.998 163	.20	.964 639	23.00	.035 361	44
17	.964 170	22.73	.998 151	.20	.966 019	22.92	.033 981	43
18	.965 534	22.65	.998 139	.18	.967 394	22.87	.032 606	42
19	.966 893	22.60	.998 128	.20	.968 766	22.78	.031 234	41
20	8.968 249	22.52	9.998 116	.20	8.970 133	22.72	1.029 867	40
21	.969 600	22.45	.998 104	.20	.971 496	22.65	.028 504	39
22	.970 947	22.37	.998 092	.20	.972 855	22.57	.027 145	38
23	.972 289	22.32	.998 080	.20	.974 209	22.52	.025 791	37
24	.973 628	22.23	.998 068	.20	.975 560	22.43	.024 440	36
25	8.974 962	22.18	9.998 056	.20	8.976 906	22.37	1.023 094	35
26	.976 293	22.10	.998 044	.20	.978 248	22.30	.021 752	34
27	.977 619	22.03	.998 032	.20	.979 586	22.25	.020 414	33
28	.978 941	21.97	.998 020	.20	.980 921	22.17	.019 079	32
29	.980 259	21.90	.998 008	.20	.982 251	22.10	.017 749	31
30	8.981 573	21.83	9.997 996	.20	8.983 577	22.03	1.016 423	30
31	.982 883	21.77	.997 984	.20	.984 899	21.97	.015 101	29
32	.984 189	21.70	.997 972	.22	.986 217	21.92	.013 783	28
33	.985 491	21.63	.997 959	.20	.987 532	21.83	.012 468	27
34	.986 789	21.57	.997 947	.20	.988 842	21.78	.011 158	26
35	8.988 083	21.52	9.997 935	.22	8.990 149	21.70	1.009 851	25
36	.989 374	21.43	.997 922	.20	.991 451	21.65	.008 549	24
37	.990 660	21.38	.997 910	.22	.992 750	21.58	.007 250	23
38	.991 943	21.32	.997 897	.20	.994 045	21.53	.005 955	22
39	.993 222	21.25	.997 885	.22	.995 337	21.45	.004 663	21
40	8.994 497	21.18	9.997 872	.20	8.996 624	21.40	1.003 376	20
41	.995 768	21.13	.997 860	.22	.997 908	21.33	.002 092	19
42	.997 036	21.05	.997 847	.20	.999 188	21.28	.000 812	18
43	.998 299	21.02	.997 835	.22	9.000 465	21.22	0.999 535	17
44	.999 560	20.93	.997 822	.22	.001 738	21.15	.998 262	16
45	9.000 816	20.88	9.997 809	.20	9.003 007	21.08	0.996 993	15
46	.002 069	20.82	.997 797	.22	.004 272	21.03	.995 728	14
47	.003 318	20.75	.997 784	.22	.005 534	20.97	.994 466	13
48	.004 563	20.70	.997 771	.22	.006 792	20.92	.993 208	12
49	.005 805	20.65	.997 758	.22	.008 047	20.85	.991 953	11
50	9.007 044	20.57	9.997 745	.22	9.009 298	20.80	0.990 702	10
51	.008 278	20.53	.997 732	.22	.010 546	20.73	.989 454	9
52	.009 510	20.45	.997 719	.22	.011 790	20.68	.988 210	8
53	.010 737	20.42	.997 706	.22	.013 031	20.62	.986 969	7
54	.011 962	20.33	.997 693	.22	.014 268	20.57	.985 732	6
55	9.013 182	20.30	9.997 680	.22	9.015 502	20.50	0.984 498	5
56	.014 400	20.22	.997 667	.22	.016 732	20.45	.983 268	4
57	.015 613	20.18	.997 654	.22	.017 959	20.40	.982 041	3
58	.016 824	20.12	.997 641	.22	.019 183	20.33	.980 817	2
59	.018 031	20.07	.997 628	.23	.020 403	20.28	.979 597	1
60	9.019 235		9.997 614		9.021 620		0.978 380	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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6°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.019 235	20.00	9.997 614	.22	9.021 620	20.23	0.978 380	60
1	.020 435	19.95	.997 601	.22	.022 834	20.17	.977 166	59
2	.021 632	19.88	.997 588	.23	.024 044	20.12	.975 956	58
3	.022 825	19.85	.997 574	.22	.025 251	20.07	.974 749	57
4	.024 016	19.78	.997 561	.23	.026 455	20.00	.973 545	56
5	9.025 203	19.72	9.997 547	.22	9.027 655	19.95	0.972 345	55
6	.026 386	19.68	.997 534	.23	.028 852	19.90	.971 148	54
7	.027 567	19.62	.997 520	.22	.030 046	19.85	.969 954	53
8	.028 744	19.57	.997 507	.23	.031 237	19.80	.968 763	52
9	.029 918	19.52	.997 493	.22	.032 425	19.73	.967 575	51
10	9.031 089	19.47	9.997 480	.23	9.033 609	19.70	0.966 391	50
11	.032 257	19.40	.997 466	.23	.034 791	19.63	.965 209	49
12	.033 421	19.35	.997 452	.22	.035 969	19.58	.964 031	48
13	.034 582	19.32	.997 439	.23	.037 144	19.53	.962 856	47
14	.035 741	19.25	.997 425	.23	.038 316	19.48	.961 684	46
15	9.036 896	19.20	9.997 411	.23	9.039 485	19.43	0.960 515	45
16	.038 048	19.15	.997 397	.23	.040 651	19.37	.959 349	44
17	.039 197	19.08	.997 383	.23	.041 813	19.33	.958 187	43
18	.040 342	19.05	.997 369	.23	.042 973	19.28	.957 027	42
19	.041 485	19.00	.997 355	.23	.044 130	19.23	.955 870	41
20	9.042 625	18.95	9.997 341	.23	9.045 284	19.17	0.954 716	40
21	.043 762	18.88	.997 327	.23	.046 434	19.13	.953 566	39
22	.044 895	18.85	.997 313	.23	.047 582	19.08	.952 418	38
23	.046 026	18.80	.997 299	.23	.048 727	19.03	.951 273	37
24	.047 154	18.75	.997 285	.23	.049 869	18.98	.950 131	36
25	9.048 279	18.68	9.997 271	.23	9.051 008	18.93	0.948 992	35
26	.049 400	18.65	.997 257	.25	.052 144	18.88	.947 856	34
27	.050 519	18.60	.997 242	.23	.053 277	18.83	.946 723	33
28	.051 635	18.57	.997 228	.23	.054 407	18.80	.945 593	32
29	.052 749	18.50	.997 214	.25	.055 535	18.73	.944 465	31
30	9.053 859	18.45	9.997 199	.23	9.056 659	18.70	0.943 341	30
31	.054 966	18.42	.997 185	.25	.057 781	18.65	.942 219	29
32	.056 071	18.35	.997 170	.23	.058 900	18.60	.941 100	28
33	.057 172	18.32	.997 156	.25	.060 016	18.57	.939 984	27
34	.058 271	18.27	.997 141	.23	.061 130	18.50	.938 870	26
35	9.059 367	18.22	9.997 127	.25	9.062 240	18.47	0.937 760	25
36	.060 460	18.18	.997 112	.23	.063 348	18.42	.936 652	24
37	.061 551	18.13	.997 098	.25	.064 453	18.38	.935 547	23
38	.062 639	18.08	.997 083	.25	.065 556	18.32	.934 444	22
39	.063 724	18.03	.997 068	.25	.066 655	18.28	.933 345	21
40	9.064 806	17.98	9.997 053	.23	9.067 752	18.23	0.932 248	20
41	.065 885	17.95	.997 039	.25	.068 846	18.20	.931 154	19
42	.066 962	17.90	.997 024	.25	.069 938	18.15	.930 062	18
43	.068 036	17.85	.997 009	.25	.071 027	18.10	.928 973	17
44	.069 107	17.82	.996 994	.25	.072 113	18.07	.927 887	16
45	9.070 176	17.77	9.996 979	.25	9.073 197	18.02	0.926 803	15
46	.071 242	17.73	.996 964	.25	.074 278	17.97	.925 722	14
47	.072 306	17.67	.996 949	.25	.075 356	17.93	.924 644	13
48	.073 366	17.63	.996 934	.25	.076 432	17.88	.923 568	12
49	.074 424	17.60	.996 919	.25	.077 505	17.85	.922 495	11
50	9.075 480	17.55	9.996 904	.25	9.078 576	17.80	0.921 424	10
51	.076 533	17.50	.996 889	.25	.079 644	17.77	.920 356	9
52	.077 583	17.47	.996 874	.27	.080 710	17.72	.919 290	8
53	.078 631	17.42	.996 858	.25	.081 773	17.67	.918 227	7
54	.079 676	17.38	.996 843	.25	.082 833	17.63	.917 167	6
55	9.080 719	17.33	9.996 828	.27	9.083 891	17.60	0.916 109	5
56	.081 759	17.30	.996 812	.25	.084 947	17.55	.915 053	4
57	.082 797	17.25	.996 797	.25	.086 000	17.50	.914 000	3
58	.083 832	17.20	.996 782	.27	.087 050	17.47	.912 950	2
59	.084 864	17.17	.996 766	.25	.088 098	17.43	.911 902	1
60	9.085 894		9.996 751		9.089 144		0.910 856	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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7°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.085 894	17.13	9.996 751	.27	9.089 144	17.38	0.910 856	60
1	.086 922	17.08	.996 735	.25	.090 187	17.35	.909 813	59
2	.087 947	17.05	.996 720	.27	.091 228	17.30	.908 772	58
3	.088 970	17.00	.996 704	.27	.092 266	17.27	.907 734	57
4	.089 990	16.97	.996 688	.25	.093 302	17.23	.906 698	56
5	9.091 008	16.93	9.996 673	.27	9.094 336	17.18	0.905 664	55
6	.092 024	16.88	.996 657	.27	.095 367	17.13	.904 633	54
7	.093 037	16.83	.996 641	.27	.096 395	17.12	.903 605	53
8	.094 047	16.82	.996 625	.25	.097 422	17.07	.902 578	52
9	.095 056	16.77	.996 610	.27	.098 446	17.03	.901 554	51
10	9.096 062	16.72	9.996 594	.27	9.099 468	16.98	0.900 532	50
11	.097 065	16.68	.996 578	.27	.100 487	16.95	.899 513	49
12	.098 066	16.65	.996 562	.27	.101 504	16.92	.898 496	48
13	.099 065	16.62	.996 546	.27	.102 519	16.88	.897 481	47
14	.100 062	16.57	.996 530	.27	.103 532	16.83	.896 468	46
15	9.101 056	16.53	9.996 514	.27	9.104 542	16.80	0.895 458	45
16	.102 048	16.48	.996 498	.27	.105 550	16.77	.894 450	44
17	.103 037	16.47	.996 482	.28	.106 556	16.72	.893 444	43
18	.104 025	16.42	.996 465	.27	.107 559	16.68	.892 441	42
19	.105 010	16.37	.996 449	.27	.108 560	16.65	.891 440	41
20	9.105 992	16.35	9.996 433	.27	9.109 559	16.62	0.890 441	40
21	.106 973	16.30	.996 417	.28	.110 556	16.58	.889 444	39
22	.107 951	16.27	.996 400	.27	.111 551	16.53	.888 449	38
23	.108 927	16.23	.996 384	.27	.112 543	16.50	.887 457	37
24	.109 901	16.20	.996 368	.28	.113 533	16.47	.886 467	36
25	9.110 873	16.15	9.996 351	.27	9.114 521	16.43	0.885 479	35
26	.111 842	16.12	.996 335	.28	.115 507	16.40	.884 493	34
27	.112 809	16.08	.996 318	.27	.116 491	16.35	.883 509	33
28	.113 774	16.05	.996 302	.28	.117 472	16.33	.882 528	32
29	.114 737	16.02	.996 285	.27	.118 452	16.28	.881 548	31
30	9.115 698	15.97	9.996 269	.28	9.119 429	16.25	0.880 571	30
31	.116 656	15.95	.996 252	.28	.120 404	16.22	.879 596	29
32	.117 613	15.90	.996 235	.27	.121 377	16.18	.878 623	28
33	.118 567	15.87	.996 219	.28	.122 348	16.15	.877 652	27
34	.119 519	15.83	.996 202	.28	.123 317	16.12	.876 683	26
35	9.120 469	15.80	9.996 185	.28	9.124 284	16.08	0.875 716	25
36	.121 417	15.75	.996 168	.28	.125 249	16.03	.874 751	24
37	.122 362	15.73	.996 151	.28	.126 211	16.02	.873 789	23
38	.123 306	15.70	.996 134	.28	.127 172	15.97	.872 828	22
39	.124 248	15.65	.996 117	.28	.128 130	15.95	.871 870	21
40	9.125 187	15.63	9.996 100	.28	9.129 087	15.90	0.870 913	20
41	.126 125	15.58	.996 083	.28	.130 041	15.88	.869 959	19
42	.127 060	15.55	.996 066	.28	.130 994	15.83	.869 006	18
43	.127 993	15.53	.996 049	.28	.131 944	15.82	.868 056	17
44	.128 925	15.48	.996 032	.28	.132 893	15.77	.867 107	16
45	9.129 854	15.45	9.996 015	.28	9.133 839	15.75	0.866 161	15
46	.130 781	15.42	.995 998	.30	.134 784	15.70	.865 216	14
47	.131 706	15.40	.995 980	.28	.135 726	15.68	.864 274	13
48	.132 630	15.35	.995 963	.28	.136 667	15.63	.863 333	12
49	.133 551	15.32	.995 946	.30	.137 605	15.62	.862 395	11
50	9.134 470	15.28	9.995 928	.28	9.138 542	15.57	0.861 458	10
51	.135 387	15.27	.995 911	.28	.139 476	15.55	.860 524	9
52	.136 303	15.22	.995 894	.30	.140 409	15.52	.859 591	8
53	.137 216	15.20	.995 876	.28	.141 340	15.48	.858 660	7
54	.138 128	15.15	.995 859	.30	.142 269	15.45	.857 731	6
55	9.139 037	15.12	9.995 841	.30	9.143 196	15.42	0.856 804	5
56	.139 944	15.10	.995 823	.28	.144 121	15.38	.855 879	4
57	.140 850	15.07	.995 806	.30	.145 044	15.37	.854 956	3
58	.141 754	15.02	.995 788	.28	.145 966	15.32	.854 034	2
59	.142 655	15.00	.995 771	.30	.146 885	15.30	.853 115	1
60	9.143 555		9.995 753		9.147 803		0.852 197	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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8°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.143 555	14.97	9.995 753	.30	9.147 803	15.25	0.852 197	60
1	.144 453	14.93	.995 735	.30	.148 718	15.23	.851 282	59
2	.145 349	14.90	.995 717	.30	.149 632	15.20	.850 368	58
3	.146 243	14.88	.995 699	.30	.150 544	15.17	.849 456	57
4	.147 136	14.83	.995 681	.28	.151 454	15.15	.848 546	56
5	9.148 026	14.82	9.995 664	.30	9.152 363	15.10	0.847 637	55
6	.148 915	14.78	.995 646	.30	.153 269	15.08	.846 731	54
7	.149 802	14.73	.995 628	.30	.154 174	15.05	.845 826	53
8	.150 686	14.72	.995 610	.32	.155 077	15.02	.844 923	52
9	.151 569	14.70	.995 591	.30	.155 978	14.98	.844 022	51
10	9.152 451	14.65	9.995 573	.30	9.156 877	14.97	0.843 123	50
11	.153 330	14.63	.995 555	.30	.157 775	14.93	.842 225	49
12	.154 208	14.58	.995 537	.30	.158 671	14.90	.841 329	48
13	.155 083	14.57	.995 519	.30	.159 565	14.87	.840 435	47
14	.155 957	14.55	.995 501	.32	.160 457	14.83	.839 543	46
15	9.156 830	14.50	9.995 482	.30	9.161 347	14.82	0.838 653	45
16	.157 700	14.48	.995 464	.30	.162 236	14.78	.837 764	44
17	.158 569	14.43	.995 446	.32	.163 123	14.75	.836 877	43
18	.159 435	14.43	.995 427	.30	.164 008	14.73	.835 992	42
19	.160 301	14.38	.995 409	.32	.164 892	14.70	.835 108	41
20	9.161 164	14.35	9.995 390	.30	9.165 774	14.67	0.834 226	40
21	.162 025	14.33	.995 372	.32	.166 654	14.63	.833 346	39
22	.162 885	14.30	.995 353	.32	.167 532	14.62	.832 468	38
23	.163 743	14.28	.995 334	.30	.168 409	14.58	.831 591	37
24	.164 600	14.23	.995 316	.32	.169 284	14.55	.830 716	36
25	9.165 454	14.22	9.995 297	.32	9.170 157	14.53	0.829 843	35
26	.166 307	14.20	.995 278	.30	.171 029	14.50	.828 971	34
27	.167 159	14.15	.995 260	.32	.171 899	14.47	.828 101	33
28	.168 008	14.13	.995 241	.32	.172 767	14.45	.827 233	32
29	.168 856	14.10	.995 222	.32	.173 634	14.42	.826 366	31
30	9.169 702	14.08	9.995 203	.32	9.174 499	14.38	0.825 501	30
31	.170 547	14.03	.995 184	.32	.175 362	14.37	.824 638	29
32	.171 389	14.02	.995 165	.32	.176 224	14.33	.823 776	28
33	.172 230	14.00	.995 146	.32	.177 084	14.30	.822 916	27
34	.173 070	13.97	.995 127	.32	.177 942	14.28	.822 058	26
35	9.173 908	13.93	9.995 108	.32	9.178 799	14.27	0.821 201	25
36	.174 744	13.90	.995 089	.32	.179 655	14.22	.820 345	24
37	.175 578	13.88	.995 070	.32	.180 508	14.20	.819 492	23
38	.176 411	13.85	.995 051	.32	.181 360	14.18	.818 640	22
39	.177 242	13.83	.995 032	.32	.182 211	14.13	.817 789	21
40	9.178 072	13.80	9.995 013	.33	9.183 059	14.13	0.816 941	20
41	.178 900	13.77	.994 993	.32	.183 907	14.08	.816 093	19
42	.179 726	13.75	.994 974	.32	.184 752	14.08	.815 248	18
43	.180 551	13.72	.994 955	.33	.185 597	14.03	.814 403	17
44	.181 374	13.70	.994 935	.32	.186 439	14.02	.813 561	16
45	9.182 196	13.67	9.994 916	.33	9.187 280	14.00	0.812 720	15
46	.183 016	13.63	.994 896	.32	.188 120	13.97	.811 880	14
47	.183 834	13.62	.994 877	.33	.188 958	13.93	.811 042	13
48	.184 651	13.58	.994 857	.32	.189 794	13.92	.810 206	12
49	.185 466	13.57	.994 838	.33	.190 629	13.88	.809 371	11
50	9.186 280	13.53	9.994 818	.33	9.191 462	13.87	0.808 538	10
51	.187 092	13.52	.994 798	.32	.192 294	13.83	.807 706	9
52	.187 903	13.48	.994 779	.33	.193 124	13.82	.806 876	8
53	.188 712	13.45	.994 759	.33	.193 953	13.78	.806 047	7
54	.189 519	13.43	.994 739	.32	.194 780	13.77	.805 220	6
55	9.190 325	13.42	9.994 720	.33	9.195 606	13.73	0.804 394	5
56	.191 130	13.38	.994 700	.33	.196 430	13.72	.803 570	4
57	.191 933	13.35	.994 680	.33	.197 253	13.68	.802 747	3
58	.192 734	13.33	.994 660	.33	.198 074	13.67	.801 926	2
59	.193 534	13.30	.994 640	.33	.198 894	13.65	.801 106	1
60	9.194 332		9.994 620		9.199 713		0.800 287	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

81°

9°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.194 332	13.28	9.994 620	.33	9.199 713	13.60	0.800 287	60
1	.195 129	13.27	.994 600	.33	.200 529	13.60	.799 471	59
2	.195 925	13.23	.994 580	.33	.201 345	13.57	.798 655	58
3	.196 719	13.20	.994 560	.33	.202 159	13.53	.797 841	57
4	.197 511	13.18	.994 540	.35	.202 971	13.52	.797 029	56
5	9.198 302	13.15	9.994 519	.33	9.203 782	13.50	0.796 218	55
6	.199 091	13.13	.994 499	.33	.204 592	13.47	.795 408	54
7	.199 879	13.12	.994 479	.33	.205 400	13.45	.794 600	53
8	.200 666	13.08	.994 459	.35	.206 207	13.43	.793 793	52
9	.201 451	13.05	.994 438	.33	.207 013	13.40	.792 987	51
10	9.202 234	13.05	9.994 418	.33	9.207 817	13.37	0.792 183	50
11	.203 017	13.00	.994 398	.35	.208 619	13.35	.791 381	49
12	.203 797	13.00	.994 377	.33	.209 420	13.33	.790 580	48
13	.204 577	12.95	.994 357	.35	.210 220	13.30	.789 780	47
14	.205 354	12.95	.994 336	.33	.211 018	13.28	.788 982	46
15	9.206 131	12.92	9.994 316	.35	9.211 815	13.27	0.788 185	45
16	.206 906	12.88	.994 295	.35	.212 611	13.23	.787 389	44
17	.207 679	12.88	.994 274	.33	.213 405	13.22	.786 595	43
18	.208 452	12.83	.994 254	.35	.214 198	13.18	.785 802	42
19	.209 222	12.83	.994 233	.35	.214 989	13.18	.785 011	41
20	9.209 992	12.80	9.994 212	.35	9.215 780	13.13	0.784 220	40
21	.210 760	12.77	.994 191	.33	.216 568	13.13	.783 432	39
22	.211 526	12.75	.994 171	.35	.217 356	13.10	.782 644	38
23	.212 291	12.73	.994 150	.35	.218 142	13.07	.781 858	37
24	.213 055	12.72	.994 129	.35	.218 926	13.07	.781 074	36
25	9.213 818	12.68	9.994 108	.35	9.219 710	13.03	0.780 290	35
26	.214 579	12.65	.994 087	.35	.220 492	13.00	.779 508	34
27	.215 338	12.65	.994 066	.35	.221 272	13.00	.778 728	33
28	.216 097	12.62	.994 045	.35	.222 052	12.97	.777 948	32
29	.216 854	12.58	.994 024	.35	.222 830	12.95	.777 170	31
30	9.217 609	12.57	9.994 003	.35	9.223 607	12.92	0.776 393	30
31	.218 363	12.55	.993 982	.37	.224 382	12.90	.775 618	29
32	.219 116	12.53	.993 960	.35	.225 156	12.88	.774 844	28
33	.219 868	12.50	.993 939	.35	.225 929	12.85	.774 071	27
34	.220 618	12.48	.993 918	.35	.226 700	12.85	.773 300	26
35	9.221 367	12.47	9.993 897	.37	9.227 471	12.80	0.772 529	25
36	.222 115	12.43	.993 875	.35	.228 239	12.80	.771 761	24
37	.222 861	12.42	.993 854	.37	.229 007	12.77	.770 993	23
38	.223 606	12.38	.993 832	.35	.229 773	12.77	.770 227	22
39	.224 349	12.38	.993 811	.37	.230 539	12.72	.769 461	21
40	9.225 092	12.35	9.993 789	.35	9.231 302	12.72	0.768 698	20
41	.225 833	12.33	.993 768	.37	.232 065	12.68	.767 935	19
42	.226 573	12.30	.993 746	.35	.232 826	12.67	.767 174	18
43	.227 311	12.28	.993 725	.37	.233 586	12.65	.766 414	17
44	.228 048	12.27	.993 703	.37	.234 345	12.63	.765 655	16
45	9.228 784	12.23	9.993 681	.35	9.235 103	12.60	0.764 897	15
46	.229 518	12.23	.993 660	.37	.235 859	12.58	.764 141	14
47	.230 252	12.20	.993 638	.37	.236 614	12.57	.763 386	13
48	.230 984	12.18	.993 616	.37	.237 368	12.53	.762 632	12
49	.231 715	12.15	.993 594	.37	.238 120	12.53	.761 880	11
50	9.232 444	12.13	9.993 572	.37	9.238 872	12.50	0.761 128	10
51	.233 172	12.12	.993 550	.37	.239 622	12.48	.760 378	9
52	.233 899	12.10	.993 528	.37	.240 371	12.45	.759 629	8
53	.234 625	12.07	.993 506	.37	.241 118	12.45	.758 882	7
54	.235 349	12.07	.993 484	.37	.241 865	12.42	.758 135	6
55	9.236 073	12.03	9.993 462	.37	9.242 610	12.40	0.757 390	5
56	.236 795	12.00	.993 440	.37	.243 354	12.38	.756 646	4
57	.237 515	12.00	.993 418	.37	.244 097	12.37	.755 903	3
58	.238 235	11.97	.993 396	.37	.244 839	12.33	.755 161	2
59	.238 953	11.95	.993 374	.38	.245 579	12.33	.754 421	1
60	9.239 670		9.993 351		9.246 319		0.753 681	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

80°

10°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.239 670	11.93	9.993 351	.37	9.246 319	12.30	0.753 681	60
1	.240 386	11.92	.993 329	.37	.247 057	12.28	.752 943	59
2	.241 101	11.88	.993 307	.38	.247 794	12.27	.752 206	58
3	.241 814	11.87	.993 284	.37	.248 530	12.23	.751 470	57
4	.242 526	11.85	.993 262	.37	.249 264	12.23	.750 736	56
5	9.243 237	11.83	9.993 240	.38	9.249 998	12.20	0.750 002	55
6	.243 947	11.82	.993 217	.37	.250 730	12.18	.749 270	54
7	.244 656	11.78	.993 195	.38	.251 461	12.17	.748 539	53
8	.245 363	11.77	.993 172	.38	.252 191	12.15	.747 809	52
9	.246 069	11.77	.993 149	.37	.252 920	12.13	.747 080	51
10	9.246 775	11.72	9.993 127	.38	9.253 648	12.10	0.746 352	50
11	.247 478	11.72	.993 104	.38	.254 374	12.10	.745 626	49
12	.248 181	11.70	.993 081	.37	.255 100	12.07	.744 900	48
13	.248 883	11.67	.993 059	.38	.255 824	12.05	.744 176	47
14	.249 583	11.65	.993 036	.38	.256 547	12.03	.743 453	46
15	9.250 282	11.63	9.993 013	.38	9.257 269	12.02	0.742 731	45
16	.250 980	11.62	.992 990	.38	.257 990	12.00	.742 010	44
17	.251 677	11.60	.992 967	.38	.258 710	11.98	.741 290	43
18	.252 373	11.57	.992 944	.38	.259 429	11.95	.740 571	42
19	.253 067	11.57	.992 921	.38	.260 146	11.95	.739 854	41
20	9.253 761	11.53	9.992 898	.38	9.260 863	11.92	0.739 137	40
21	.254 453	11.52	.992 875	.38	.261 578	11.90	.738 422	39
22	.255 144	11.50	.992 852	.38	.262 292	11.88	.737 708	38
23	.255 834	11.48	.992 829	.38	.263 005	11.87	.736 995	37
24	.256 523	11.47	.992 806	.38	.263 717	11.85	.736 283	36
25	9.257 211	11.45	9.992 783	.40	9.264 428	11.83	0.735 572	35
26	.257 898	11.42	.992 759	.38	.265 138	11.82	.734 862	34
27	.258 583	11.42	.992 736	.38	.265 847	11.80	.734 153	33
28	.259 268	11.38	.992 713	.38	.266 555	11.77	.733 445	32
29	.259 951	11.37	.992 690	.40	.267 261	11.77	.732 739	31
30	9.260 633	11.35	9.992 666	.38	9.267 967	11.73	0.732 033	30
31	.261 314	11.33	.992 643	.40	.268 671	11.73	.731 329	29
32	.261 994	11.32	.992 619	.38	.269 375	11.70	.730 625	28
33	.262 673	11.30	.992 596	.40	.270 077	11.70	.729 923	27
34	.263 351	11.27	.992 572	.38	.270 779	11.67	.729 221	26
35	9.264 027	11.27	9.992 549	.40	9.271 479	11.65	0.728 521	25
36	.264 703	11.23	.992 525	.40	.272 178	11.63	.727 822	24
37	.265 377	11.23	.992 501	.38	.272 876	11.62	.727 124	23
38	.266 051	11.20	.992 478	.40	.273 573	11.60	.726 427	22
39	.266 723	11.20	.992 454	.40	.274 269	11.58	.725 731	21
40	9.267 395	11.17	9.992 430	.40	9.274 964	11.57	0.725 036	20
41	.268 065	11.15	.992 406	.40	.275 658	11.55	.724 342	19
42	.268 734	11.13	.992 382	.38	.276 351	11.53	.723 649	18
43	.269 402	11.12	.992 359	.40	.277 043	11.52	.722 957	17
44	.270 069	11.10	.992 335	.40	.277 734	11.50	.722 266	16
45	9.270 735	11.08	9.992 311	.40	9.278 424	11.48	0.721 576	15
46	.271 400	11.07	.992 287	.40	.279 113	11.47	.720 887	14
47	.272 064	11.03	.992 263	.40	.279 801	11.45	.720 199	13
48	.272 726	11.03	.992 239	.42	.280 488	11.43	.719 512	12
49	.273 388	11.02	.992 214	.40	.281 174	11.40	.718 826	11
50	9.274 049	10.98	9.992 190	.40	9.281 858	11.40	0.718 142	10
51	.274 708	10.98	.992 166	.40	.282 542	11.38	.717 458	9
52	.275 367	10.97	.992 142	.40	.283 225	11.37	.716 775	8
53	.276 025	10.93	.992 118	.42	.283 907	11.35	.716 093	7
54	.276 681	10.93	.992 093	.40	.284 588	11.33	.715 412	6
55	9.277 337	10.90	9.992 069	.42	9.285 268	11.32	0.714 732	5
56	.277 991	10.90	.992 044	.40	.285 947	11.28	.714 053	4
57	.278 645	10.87	.992 020	.40	.286 624	11.28	.713 376	3
58	.279 297	10.85	.991 996	.42	.287 301	11.27	.712 699	2
59	.279 948	10.85	.991 971	.40	.287 977	11.25	.712 023	1
60	9.280 599		9.991 947		9.288 652		0.711 348	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

79°

11°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.280 599	10.82	9.991 947	.42	9.288 652	11.23	0.711 348	60
1	.281 248	10.82	.991 922	.42	.289 326	11.22	.710 674	59
2	.281 897	10.78	.991 897	.40	.289 999	11.20	.710 001	58
3	.282 544	10.77	.991 873	.42	.290 671	11.18	.709 329	57
4	.283 190	10.77	.991 848	.42	.291 342	11.18	.708 658	56
5	9.283 836	10.73	9.991 823	.40	9.292 013	11.15	0.707 987	55
6	.284 480	10.73	.991 799	.42	.292 682	11.13	.707 318	54
7	.285 124	10.70	.991 774	.42	.293 350	11.12	.706 650	53
8	.285 766	10.70	.991 749	.42	.294 017	11.12	.705 983	52
9	.286 408	10.67	.991 724	.42	.294 684	11.08	.705 316	51
10	9.287 048	10.67	9.991 699	.42	9.295 349	11.07	0.704 651	50
11	.287 688	10.63	.991 674	.42	.296 013	11.07	.703 987	49
12	.288 326	10.63	.991 649	.42	.296 677	11.03	.703 323	48
13	.288 964	10.60	.991 624	.42	.297 339	11.03	.702 661	47
14	.289 600	10.60	.991 599	.42	.298 001	11.02	.701 999	46
15	9.290 236	10.57	9.991 574	.42	9.298 662	11.00	0.701 338	45
16	.290 870	10.57	.991 549	.42	.299 322	10.97	.700 678	44
17	.291 504	10.55	.991 524	.43	.299 980	10.97	.700 020	43
18	.292 137	10.52	.991 498	.42	.300 638	10.95	.699 362	42
19	.292 768	10.52	.991 473	.42	.301 295	10.93	.698 705	41
20	9.293 399	10.50	9.991 448	.43	9.301 951	10.93	0.698 049	40
21	.294 029	10.48	.991 422	.42	.302 607	10.90	.697 393	39
22	.294 658	10.47	.991 397	.42	.303 261	10.88	.696 739	38
23	.295 286	10.45	.991 372	.43	.303 914	10.88	.696 086	37
24	.295 913	10.43	.991 346	.42	.304 567	10.85	.695 433	36
25	9.296 539	10.42	9.991 321	.43	9.305 218	10.85	0.694 782	35
26	.297 164	10.40	.991 295	.42	.305 869	10.83	.694 131	34
27	.297 788	10.40	.991 270	.43	.306 519	10.82	.693 481	33
28	.298 412	10.37	.991 244	.43	.307 168	10.80	.692 832	32
29	.299 034	10.35	.991 218	.42	.307 816	10.78	.692 184	31
30	9.299 655	10.35	9.991 193	.43	9.308 463	10.77	0.691 537	30
31	.300 276	10.32	.991 167	.43	.309 109	10.75	.690 891	29
32	.300 895	10.32	.991 141	.43	.309 754	10.75	.690 246	28
33	.301 514	10.30	.991 115	.42	.310 399	10.72	.689 601	27
34	.302 132	10.27	.991 090	.43	.311 042	10.72	.688 958	26
35	9.302 748	10.27	9.991 064	.43	9.311 685	10.70	0.688 315	25
36	.303 364	10.25	.991 038	.43	.312 327	10.68	.687 673	24
37	.303 979	10.23	.991 012	.43	.312 968	10.67	.687 032	23
38	.304 593	10.23	.990 986	.43	.313 608	10.65	.686 392	22
39	.305 207	10.20	.990 960	.43	.314 247	10.63	.685 753	21
40	9.305 819	10.18	9.990 934	.43	9.314 885	10.63	0.685 115	20
41	.306 430	10.18	.990 908	.43	.315 523	10.60	.684 477	19
42	.307 041	10.15	.990 882	.45	.316 159	10.60	.683 841	18
43	.307 650	10.15	.990 855	.43	.316 795	10.58	.683 205	17
44	.308 259	10.13	.990 829	.43	.317 430	10.57	.682 570	16
45	9.308 867	10.12	9.990 803	.43	9.318 064	10.55	0.681 936	15
46	.309 474	10.10	.990 777	.45	.318 697	10.55	.681 303	14
47	.310 080	10.08	.990 750	.43	.319 330	10.52	.680 670	13
48	.310 685	10.07	.990 724	.45	.319 961	10.52	.680 039	12
49	.311 289	10.07	.990 697	.43	.320 592	10.50	.679 408	11
50	9.311 893	10.03	9.990 671	.43	9.321 222	10.48	0.678 778	10
51	.312 495	10.03	.990 645	.45	.321 851	10.47	.678 149	9
52	.313 097	10.02	.990 618	.45	.322 479	10.45	.677 521	8
53	.313 698	9.98	.990 591	.43	.323 106	10.45	.676 894	7
54	.314 297	10.00	.990 565	.45	.323 733	10.42	.676 267	6
55	9.314 897	9.97	9.990 538	.45	9.324 358	10.42	0.675 642	5
56	.315 495	9.95	.990 511	.43	.324 983	10.40	.675 017	4
57	.316 092	9.95	.990 485	.45	.325 607	10.40	.674 393	3
58	.316 689	9.92	.990 458	.45	.326 231	10.37	.673 769	2
59	.317 284	9.92	.990 431	.45	.326 853	10.37	.673 147	1
60	9.317 879		9.990 404		9.327 475		0.672 525	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

78°

12°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.317 879	9.90	9.990 404	.43	9.327 475	10.33	0.672 525	60
1	.318 473	9.88	.990 378	.45	.328 095	10.33	.671 905	59
2	.319 066	9.87	.990 351	.45	.328 715	10.32	.671 285	58
3	.319 658	9.85	.990 324	.45	.329 334	10.32	.670 666	57
4	.320 249	9.85	.990 297	.45	.329 953	10.28	.670 047	56
5	9.320 840	9.83	9.990 270	.45	9.330 570	10.28	0.669 430	55
6	.321 430	9.82	.990 243	.47	.331 187	10.27	.668 813	54
7	.322 019	9.80	.990 215	.45	.331 803	10.25	.668 197	53
8	.322 607	9.78	.990 188	.45	.332 418	10.25	.667 582	52
9	.323 194	9.77	.990 161	.45	.333 033	10.22	.666 967	51
10	9.323 780	9.77	9.990 134	.45	9.333 646	10.22	0.666 354	50
11	.324 366	9.73	.990 107	.47	.334 259	10.20	.665 741	49
12	.324 950	9.73	.990 079	.45	.334 871	10.18	.665 129	48
13	.325 534	9.72	.990 052	.45	.335 482	10.18	.664 518	47
14	.326 117	9.72	.990 025	.47	.336 093	10.15	.663 907	46
15	9.326 700	9.68	9.989 997	.45	9.336 702	10.15	0.663 298	45
16	.327 281	9.68	.989 970	.47	.337 311	10.13	.662 689	44
17	.327 862	9.67	.989 942	.45	.337 919	10.13	.662 081	43
18	.328 442	9.65	.989 915	.47	.338 527	10.10	.661 473	42
19	.329 021	9.63	.989 887	.45	.339 133	10.10	.660 867	41
20	9.329 599	9.62	9.989 860	.47	9.339 739	10.08	0.660 261	40
21	.330 176	9.62	.989 832	.47	.340 344	10.07	.659 656	39
22	.330 753	9.60	.989 804	.45	.340 948	10.07	.659 052	38
23	.331 329	9.57	.989 777	.47	.341 552	10.05	.658 448	37
24	.331 903	9.58	.989 749	.47	.342 155	10.03	.657 845	36
25	9.332 478	9.55	9.989 721	.47	9.342 757	10.02	0.657 243	35
26	.333 051	9.55	.989 693	.47	.343 358	10.00	.656 642	34
27	.333 624	9.52	.989 665	.47	.343 958	10.00	.656 042	33
28	.334 195	9.53	.989 637	.45	.344 558	9.98	.655 442	32
29	.334 767	9.50	.989 610	.47	.345 157	9.97	.654 843	31
30	9.335 337	9.48	9.989 582	.48	9.345 755	9.97	0.654 245	30
31	.335 906	9.48	.989 553	.47	.346 353	9.93	.653 647	29
32	.336 475	9.47	.989 525	.47	.346 949	9.93	.653 051	28
33	.337 043	9.45	.989 497	.47	.347 545	9.93	.652 455	27
34	.337 610	9.43	.989 469	.47	.348 141	9.90	.651 859	26
35	9.338 176	9.43	9.989 441	.47	9.348 735	9.90	0.651 265	25
36	.338 742	9.42	.989 413	.47	.349 329	9.88	.650 671	24
37	.339 307	9.40	.989 385	.48	.349 922	9.87	.650 078	23
38	.339 871	9.38	.989 356	.47	.350 514	9.87	.649 486	22
39	.340 434	9.37	.989 328	.47	.351 106	9.85	.648 894	21
40	9.340 996	9.37	9.989 300	.48	9.351 697	9.83	0.648 303	20
41	.341 558	9.35	.989 271	.47	.352 287	9.82	.647 713	19
42	.342 119	9.33	.989 243	.48	.352 876	9.82	.647 124	18
43	.342 679	9.33	.989 214	.47	.353 465	9.80	.646 535	17
44	.343 239	9.30	.989 186	.48	.354 053	9.78	.645 947	16
45	9.343 797	9.30	9.989 157	.48	9.354 640	9.78	0.645 360	15
46	.344 355	9.28	.989 128	.47	.355 227	9.77	.644 773	14
47	.344 912	9.28	.989 100	.48	.355 813	9.75	.644 187	13
48	.345 469	9.25	.989 071	.48	.356 398	9.73	.643 602	12
49	.346 024	9.25	.989 042	.47	.356 982	9.73	.643 018	11
50	9.346 579	9.25	9.989 014	.48	9.357 566	9.72	0.642 434	10
51	.347 134	9.22	.988 985	.48	.358 149	9.70	.641 851	9
52	.347 687	9.22	.988 956	.48	.358 731	9.70	.641 269	8
53	.348 240	9.20	.988 927	.48	.359 313	9.67	.640 687	7
54	.348 792	9.18	.988 898	.48	.359 893	9.68	.640 107	6
55	9.349 343	9.17	9.988 869	.48	9.360 474	9.65	0.639 526	5
56	.349 893	9.17	.988 840	.48	.361 053	9.65	.638 947	4
57	.350 443	9.15	.988 811	.48	.361 632	9.63	.638 368	3
58	.350 992	9.13	.988 782	.48	.362 210	9.62	.637 790	2
59	.351 540	9.13	.988 753	.48	.362 787	9.62	.637 213	1
60	9.352 088		9.988 724		9.363 364		0.636 636	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

77°

13°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.352 088	9.12	9.988 724	.48	9.363 364	9.60	0.636 636	60
1	.352 635	9.10	.988 695	.48	.363 940	9.58	.636 060	59
2	.353 181	9.08	.988 666	.50	.364 515	9.58	.635 485	58
3	.353 726	9.08	.988 636	.48	.365 090	9.57	.634 910	57
4	.354 271	9.07	.988 607	.48	.365 664	9.55	.634 336	56
5	9.354 815	9.05	9.988 578	.50	9.366 237	9.55	0.633 763	55
6	.355 358	9.05	.988 548	.48	.366 810	9.53	.633 190	54
7	.355 901	9.03	.988 519	.50	.367 382	9.52	.632 618	53
8	.356 443	9.02	.988 489	.48	.367 953	9.52	.632 047	52
9	.356 984	9.00	.988 460	.50	.368 524	9.50	.631 476	51
10	9.357 524	9.00	9.988 430	.48	9.369 094	9.48	0.630 906	50
11	.358 064	8.98	.988 401	.50	.369 663	9.48	.630 337	49
12	.358 603	8.97	.988 371	.48	.370 232	9.45	.629 768	48
13	.359 141	8.95	.988 342	.50	.370 799	9.47	.629 201	47
14	.359 678	8.95	.988 312	.50	.371 367	9.43	.628 633	46
15	9.360 215	8.95	9.988 282	.50	9.371 933	9.43	0.628 067	45
16	.360 752	8.92	.988 252	.48	.372 499	9.42	.627 501	44
17	.361 287	8.92	.988 223	.50	.373 064	9.42	.626 936	43
18	.361 822	8.90	.988 193	.50	.373 629	9.40	.626 371.	42
19	.362 356	8.88	.988 163	.50	.374 193	9.38	.625 807	41
20	9.362 889	8.88	9.988 133	.50	9.374 756	9.38	0.625 244	40
21	.363 422	8.87	.988 103	.50	.375 319	9.37	.624 681	39
22	.363 954	8.85	.988 073	.50	.375 881	9.35	.624 119	38
23	.364 485	8.85	.988 043	.50	.376 442	9.35	.623 558	37
24	.365 016	8.83	.988 013	.50	.377 003	9.33	.622 997	36
25	9.365 546	8.82	9.987 983	.50	9.377 563	9.32	0.622 437	35
26	.366 075	8.82	.987 953	.52	.378 122	9.32	.621 878	34
27	.366 604	8.78	.987 922	.50	.378 681	9.30	.621 319	33
28	.367 131	8.80	.987 892	.50	.379 239	9.30	.620 761	32
29	.367 659	8.77	.987 862	.50	.379 797	9.28	.620 203	31
30	9.368 185	8.77	9.987 832	.52	9.380 354	9.27	0.619 646	30
31	.368 711	8.75	.987 801	.50	.380 910	9.27	.619 090	29
32	.369 236	8.75	.987 771	.52	.381 466	9.23	.618 534	28
33	.369 761	8.73	.987 740	.50	.382 020	9.25	.617 980	27
34	.370 285	8.72	.987 710	.52	.382 575	9.23	.617 425	26
35	9.370 808	8.70	9.987 679	.50	9.383 129	9.22	0.616 871	25
36	.371 330	8.70	.987 649	.52	.383 682	9.20	.616 318	24
37	.371 852	8.68	.987 618	.50	.384 234	9.20	.615 766	23
38	.372 373	8.68	.987 588	.52	.384 786	9.18	.615 214	22
39	.372 894	8.67	.987 557	.52	.385 337	9.18	.614 663	21
40	9.373 414	8.65	9.987 526	.50	9.385 888	9.17	0.614 112	20
41	.373 933	8.65	.987 496	.52	.386 438	9.15	.613 562	19
42	.374 452	8.63	.987 465	.52	.386 987	9.15	.613 013	18
43	.374 970	8.62	.987 434	.52	.387 536	9.13	.612 464	17
44	.375 487	8.60	.987 403	.52	.388 084	9.12	.611 916	16
45	9.376 003	8.60	9.987 372	.52	9.388 631	9.12	0.611 369	15
46	.376 519	8.60	.987 341	.52	.389 178	9.10	.610 822	14
47	.377 035	8.57	.987 310	.52	.389 724	9.10	.610 276	13
48	.377 549	8.57	.987 279	.52	.390 270	9.08	.609 730	12
49	.378 063	8.57	.987 248	.52	.390 815	9.08	.609 185	11
50	9.378 577	8.53	9.987 217	.52	9.391 360	9.05	0.608 640	10
51	.379 089	8.53	.987 186	.52	.391 903	9.07	.608 097	9
52	.379 601	8.53	.987 155	.52	.392 447	9.03	.607 553	8
53	.380 113	8.52	.987 124	.53	.392 989	9.03	.607 011	7
54	.380 624	8.50	.987 092	.52	.393 531	9.03	.606 469	6
55	9.381 134	8.48	9.987 061	.52	9.394 073	9.02	0.605 927	5
56	.381 643	8.48	.987 030	.53	.394 614	9.00	.605 386	4
57	.382 152	8.48	.986 998	.52	.395 154	9.00	.604 846	3
58	.382 661	8.45	.986 967	.52	.395 694	8.98	.604 306	2
59	.383 168	8.45	.986 936	.53	.396 233	8.97	.603 767	1
60	9.383 675		9.986 904		9.396 771		0.603 229	0
	Cos.	D. 1″.	Sin.	D 1″.	Cot.	D. 1″.	Tan.	M.

76°

14°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.383 675	8.45	9.986 904	.52	9.396 771	8.97	0.603 229	60
1	.384 182	8.42	.986 873	.53	.397 309	8.95	.602 691	59
2	.384 687	8.42	.986 841	.53	.397 846	8.95	.602 154	58
3	.385 192	8.42	.986 809	.52	.398 383	8.93	.601 617	57
4	.385 697	8.40	.986 778	.53	.398 919	8.93	.601 081	56
5	9.386 201	8.38	9.986 746	.53	9.399 455	8.92	0.600 545	55
6	.386 704	8.38	.986 714	.52	.399 990	8.90	.600 010	54
7	.387 207	8.37	.986 683	.53	.400 524	8.90	.599 476	53
8	.387 709	8.35	.986 651	.53	.401 058	8.88	.598 942	52
9	.388 210	8.35	.986 619	.53	.401 591	8.88	.598 409	51
10	9.388 711	8.33	9.986 587	.53	9.402 124	8.87	0.597 876	50
11	.389 211	8.33	.986 555	.53	.402 656	8.85	.597 344	49
12	.389 711	8.32	.986 523	.53	.403 187	8.85	.596 813	48
13	.390 210	8.30	.986 491	.53	.403 718	8.85	.596 282	47
14	.390 708	8.30	.986 459	.53	.404 249	8.82	.595 751	46
15	9.391 206	8.28	9.986 427	.53	9.404 778	8.83	0.595 222	45
16	.391 703	8.27	.986 395	.53	.405 308	8.80	.594 692	44
17	.392 199	8.27	.986 363	.53	.405 836	8.80	.594 164	43
18	.392 695	8.27	.986 331	.53	.406 364	8.80	.593 636	42
19	.393 191	8.23	.986 299	.55	.406 892	8.78	.593 108	41
20	9.393 685	8.23	9.986 266	.53	9.407 419	8.77	0.592 581	40
21	.394 179	8.23	.986 234	.53	.407 945	8.77	.592 055	39
22	.394 673	8.22	.986 202	.55	.408 471	8.75	.591 529	38
23	.395 166	8.20	.986 169	.53	.408 996	8.75	.591 004	37
24	.395 658	8.20	.986 137	.55	.409 521	8.73	.590 479	36
25	9.396 150	8.18	9.986 104	.53	9.410 045	8.73	0.589 955	35
26	.396 641	8.18	.986 072	.55	.410 569	8.72	.589 431	34
27	.397 132	8.15	.986 039	.53	.411 092	8.72	.588 908	33
28	.397 621	8.17	.986 007	.55	.411 615	8.70	.588 385	32
29	.398 111	8.15	.985 974	.53	.412 137	8.68	.587 863	31
30	9.398 600	8.13	9.985 942	.55	9.412 658	8.68	0.587 342	30
31	.399 088	8.12	.985 909	.55	.413 179	8.67	.586 821	29
32	.399 575	8.12	.985 876	.55	.413 699	8.67	.586 301	28
33	.400 062	8.12	.985 843	.53	.414 219	8.65	.585 781	27
34	.400 549	8.10	.985 811	.55	.414 738	8.65	.585 262	26
35	9.401 035	8.08	9.985 778	.55	9.415 257	8.63	0.584 743	25
36	.401 520	8.08	.985 745	.55	.415 775	8.63	.584 225	24
37	.402 005	8.07	.985 712	.55	.416 293	8.62	.583 707	23
38	.402 489	8.05	.985 679	.55	.416 810	8.60	.583 190	22
39	.402 972	8.05	.985 646	.55	.417 326	8.60	.582 674	21
40	9.403 455	8.05	9.985 613	.55	9.417 842	8.60	0.582 158	20
41	.403 938	8.03	.985 580	.55	.418 358	8.58	.581 642	19
42	.404 420	8.02	.985 547	.55	.418 873	8.57	.581 127	18
43	.404 901	8.02	.985 514	.57	.419 387	8.57	.580 613	17
44	.405 382	8.00	.985 480	.55	.419 901	8.57	.580 099	16
45	9.405 862	7.98	9.985 447	.55	9.420 415	8.53	0.579 585	15
46	.406 341	7.98	.985 414	.55	.420 927	8.55	.579 073	14
47	.406 820	7.98	.985 381	.57	.421 440	8.53	.578 560	13
48	.407 299	7.97	.985 347	.55	.421 952	8.52	.578 048	12
49	.407 777	7.95	.985 314	.57	.422 463	8.52	.577 537	11
50	9.408 254	7.95	9.985 280	.55	9.422 974	8.50	0.577 026	10
51	.408 731	7.93	.985 247	.57	.423 484	8.48	.576 516	9
52	.409 207	7.92	.985 213	.55	.423 993	8.50	.576 007	8
53	.409 682	7.92	.985 180	.57	.424 503	8.47	.575 497	7
54	.410 157	7.92	.985 146	.55	.425 011	8.47	.574 989	6
55	9.410 632	7.90	9.985 113	.57	9.425 519	8.47	0.574 481	5
56	.411 106	7.88	.985 079	.57	.426 027	8.45	.573 973	4
57	.411 579	7.88	.985 045	.57	.426 534	8.45	.573 466	3
58	.412 052	7.87	.985 011	.55	.427 041	8.43	.572 959	2
59	.412 524	7.87	.984 978	.57	.427 547	8.42	.572 453	1
60	9.412 996		9.984 944		9.428 052		0.571 948	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

75°

15°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.412 996	7.85	9.984 944	.57	9.428 052	8.43	0.571 948	60
1	.413 467	7.85	.984 910	.57	.428 558	8.40	.571 442	59
2	.413 938	7.83	.984 876	.57	.429 062	8.40	.570 938	58
3	.414 408	7.83	.984 842	.57	.429 566	8.40	.570 434	57
4	.414 878	7.82	.984 808	.57	.430 070	8.38	.569 930	56
5	9.415 347	7.80	9.984 774	.57	9.430 573	8.37	0.569 427	55
6	.415 815	7.80	.984 740	.57	.431 075	8.37	.568 925	54
7	.416 283	7.80	.984 706	.57	.431 577	8.37	.568 423	53
8	.416 751	7.77	.984 672	.57	.432 079	8.35	.567 921	52
9	.417 217	7.78	.984 638	.58	.432 580	8.33	.567 420	51
10	9.417 684	7.77	9.984 603	.57	9.433 080	8.33	0.566 920	50
11	.418 150	7.75	.984 569	.57	.433 580	8.33	.566 420	49
12	.418 615	7.73	.984 535	.58	.434 080	8.32	.565 920	48
13	.419 079	7.75	.984 500	.57	.434 579	8.32	.565 421	47
14	.419 544	7.72	.984 466	.57	.435 078	8.30	.564 922	46
15	9.420 007	7.72	9.984 432	.58	9.435 576	8.28	0.564 424	45
16	.420 470	7.72	.984 397	.57	.436 073	8.28	.563 927	44
17	.420 933	7.70	.984 363	.58	.436 570	8.28	.563 430	43
18	.421 395	7.70	.984 328	.57	.437 067	8.27	.562 933	42
19	.421 857	7.68	.984 294	.58	.437 563	8.27	.562 437	41
20	9.422 318	7.67	9.984 259	.58	9.438 059	8.25	0.561 941	40
21	.422 778	7.67	.984 224	.57	.438 554	8.23	.561 446	39
22	.423 238	7.65	.984 190	.58	.439 048	8.25	.560 952	38
23	.423 697	7.65	.984 155	.58	.439 543	8.22	.560 457	37
24	.424 156	7.65	.984 120	.58	.440 036	8.22	.559 964	36
25	9.424 615	7.63	9.984 085	.58	9.440 529	8.22	0.559 471	35
26	.425 073	7.62	.984 050	.58	.441 022	8.20	.558 978	34
27	.425 530	7.62	.984 015	.57	.441 514	8.20	.558 486	33
28	.425 987	7.60	.983 981	.58	.442 006	8.18	.557 994	32
29	.426 443	7.60	.983 946	.58	.442 497	8.18	.557 503	31
30	9.426 899	7.58	9.983 911	.60	9.442 988	8.18	0.557 012	30
31	.427 354	7.58	.983 875	.58	.443 479	8.15	.556 521	29
32	.427 809	7.57	.983 840	.58	.443 968	8.17	.556 032	28
33	.428 263	7.57	.983 805	.58	.444 458	8.15	.555 542	27
34	.428 717	7.55	.983 770	.58	.444 947	8.13	.555 053	26
35	9.429 170	7.55	9.983 735	.58	9.445 435	8.13	0.554 565	25
36	.429 623	7.53	.983 700	.60	.445 923	8.13	.554 077	24
37	.430 075	7.53	.983 664	.58	.446 411	8.12	.553 589	23
38	.430 527	7.52	.983 629	.58	.446 898	8.10	.553 102	22
39	.430 978	7.52	.983 594	.60	.447 384	8.10	.552 616	21
40	9.431 429	7.50	9.983 558	.58	9.447 870	8.10	0.552 130	20
41	.431 879	7.50	.983 523	.60	.448 356	8.08	.551 644	19
42	.432 329	7.48	.983 487	.58	.448 841	8.08	.551 159	18
43	.432 778	7.47	.983 452	.60	.449 326	8.07	.550 674	17
44	.433 226	7.48	.983 416	.58	.449 810	8.07	.550 190	16
45	9.433 675	7.45	9.983 381	.60	9.450 294	8.05	0.549 706	15
46	.434 122	7.45	.983 345	.60	.450 777	8.05	.549 223	14
47	.434 569	7.45	.983 309	.60	.451 260	8.05	.548 740	13
48	.435 016	7.43	.983 273	.58	.451 743	8.03	.548 257	12
49	.435 462	7.43	.983 238	.60	.452 225	8.02	.547 775	11
50	9.435 908	7.42	9.983 202	.60	9.452 706	8.02	0.547 294	10
51	.436 353	7.42	.983 166	.60	.453 187	8.02	.546 813	9
52	.436 798	7.40	.983 130	.60	.453 668	8.00	.546 332	8
53	.437 242	7.40	.983 094	.60	.454 148	8.00	.545 852	7
54	.437 686	7.38	.983 058	.60	.454 628	7.98	.545 372	6
55	9.438 129	7.38	9.983 022	.60	9.455 107	7.98	0.544 893	5
56	.438 572	7.37	.982 986	.60	.455 586	7.97	.544 414	4
57	.439 014	7.37	.982 950	.60	.456 064	7.97	.543 936	3
58	.439 456	7.35	.982 914	.60	.456 542	7.95	.543 458	2
59	.439 897	7.35	.982 878	.60	.457 019	7.95	.542 981	1
60	9.440 338		9.982 842		9.457 496		0.542 504	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

74°

16°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.440 338	7.33	9.982 842	.62	9.457 496	7.95	0.542 504	60
1	.440 778	7.33	.982 805	.60	.457 973	7.93	.542 027	59
2	.441 218	7.33	.982 769	.60	.458 449	7.93	.541 551	58
3	.441 658	7.30	.982 733	.62	.458 925	7.92	.541 075	57
4	.442 096	7.32	.982 696	.60	.459 400	7.92	.540 600	56
5	9.442 535	7.30	9.982 660	.60	9.459 875	7.90	0.540 125	55
6	.442 973	7.28	.982 624	.62	.460 349	7.90	.539 651	54
7	.443 410	7.28	.982 587	.60	.460 823	7.90	.539 177	53
8	.443 847	7.28	.982 551	.62	.461 297	7.88	.538 703	52
9	.444 284	7.27	.982 514	.62	.461 770	7.87	.538 230	51
10	9.444 720	7.25	9.982 477	.60	9.462 242	7.88	0.537 758	50
11	.445 155	7.25	.982 441	.62	.462 715	7.85	.537 285	49
12	.445 590	7.25	.982 404	.62	.463 186	7.87	.536 814	48
13	.446 025	7.23	.982 367	.60	.463 658	7.83	.536 342	47
14	.446 459	7.23	.982 331	.62	.464 128	7.85	.535 872	46
15	9.446 893	7.22	9.982 294	.62	9.464 599	7.83	0.535 401	45
16	.447 326	7.22	.982 257	.62	.465 069	7.83	.534 931	44
17	.447 759	7.20	.982 220	.62	.465 539	7.82	.534 461	43
18	.448 191	7.20	.982 183	.62	.466 008	7.82	.533 992	42
19	.448 623	7.18	.982 146	.62	.466 477	7.80	.533 523	41
20	9.449 054	7.18	9.982 109	.62	9.466 945	7.80	0.533 055	40
21	.449 485	7.17	.982 072	.62	.467 413	7.78	.532 587	39
22	.449 915	7.17	.982 035	.62	.467 880	7.78	.532 120	38
23	.450 345	7.17	.981 998	.62	.468 347	7.78	.531 653	37
24	.450 775	7.15	.981 961	.62	.468 814	7.77	.531 186	36
25	9.451 204	7.13	9.981 924	.63	9.469 280	7.77	0.530 720	35
26	.451 632	7.13	.981 886	.62	.469 746	7.75	.530 254	34
27	.452 060	7.13	.981 849	.62	.470 211	7.75	.529 789	33
28	.452 488	7.12	.981 812	.63	.470 676	7.75	.529 324	32
29	.452 915	7.12	.981 774	.62	.471 141	7.73	.528 859	31
30	9.453 342	7.10	9.981 737	.62	9.471 605	7.73	0.528 395	30
31	.453 768	7.10	.981 700	.63	.472 069	7.72	.527 931	29
32	.454 194	7.08	.981 662	.62	.472 532	7.72	.527 468	28
33	.454 619	7.08	.981 625	.63	.472 995	7.70	.527 005	27
34	.455 044	7.08	.981 587	.63	.473 457	7.70	.526 543	26
35	9.455 469	7.07	9.981 549	.62	9.473 919	7.70	0.526 081	25
36	.455 893	7.05	.981 512	.63	.474 381	7.68	.525 619	24
37	.456 316	7.05	.981 474	.63	.474 842	7.68	.525 158	23
38	.456 739	7.05	.981 436	.62	.475 303	7.67	.524 697	22
39	.457 162	7.03	.981 399	.63	.475 763	7.67	.524 237	21
40	9.457 584	7.03	9.981 361	.63	9.476 223	7.67	0.523 777	20
41	.458 006	7.02	.981 323	.63	.476 683	7.65	.523 317	19
42	.458 427	7.02	.981 285	.63	.477 142	7.65	.522 858	18
43	.458 848	7.00	.981 247	.63	.477 601	7.63	.522 399	17
44	.459 268	7.00	.981 209	.63	.478 059	7.63	.521 941	16
45	9.459 688	7.00	9.981 171	.63	9.478 517	7.63	0.521 483	15
46	.460 108	6.98	.981 133	.63	.478 975	7.62	.521 025	14
47	.460 527	6.98	.981 095	.63	.479 432	7.62	.520 568	13
48	.460 946	6.97	.981 057	.63	.479 889	7.60	.520 111	12
49	.461 364	6.97	.981 019	.63	.480 345	7.60	.519 655	11
50	9.461 782	6.95	9.980 981	.65	9.480 801	7.60	0.519 199	10
51	.462 199	6.95	.980 942	.63	.481 257	7.58	.518 743	9
52	.462 616	6.93	.980 904	.63	.481 712	7.58	.518 288	8
53	.463 032	6.93	.980 866	.65	.482 167	7.57	.517 833	7
54	.463 448	6.93	.980 827	.63	.482 621	7.57	.517 379	6
55	9.463 864	6.92	9.980 789	.65	9.483 075	7.57	0.516 925	5
56	.464 279	6.92	.980 750	.63	.483 529	7.55	.516 471	4
57	.464 694	6.90	.980 712	.65	.483 982	7.55	.516 018	3
58	.465 108	6.90	.980 673	.63	.484 435	7.53	.515 565	2
59	.465 522	6.88	.980 635	.65	.484 887	7.53	.515 113	1
60	9.465 935		9.980 596		9.485 339		0.514 661	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

73°

17°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.465 935	6.88	9.980 596	.63	9.485 339	7.53	0.514 661	60
1	.466 348	6.88	.980 558	.65	.485 791	7.52	.514 209	59
2	.466 761	6.87	.980 519	.65	.486 242	7.52	.513 758	58
3	.467 173	6.87	.980 480	.63	.486 693	7.50	.513 307	57
4	.467 585	6.85	.980 442	.65	.487 143	7.50	.512 857	56
5	9.467 996	6.85	9.980 403	.65	9.487 593	7.50	0.512 407	55
6	.468 407	6.83	.980 364	.65	.488 043	7.48	.511 957	54
7	.468 817	6.83	.980 325	.65	.488 492	7.48	.511 508	53
8	.469 227	6.83	.980 286	.65	.488 941	7.48	.511 059	52
9	.469 637	6.82	.980 247	.65	.489 390	7.47	.510 610	51
10	9.470 046	6.82	9.980 208	.65	9.489 838	7.47	0.510 162	50
11	.470 455	6.80	.980 169	.65	.490 286	7.45	.509 714	49
12	.470 863	6.80	.980 130	.65	.490 733	7.45	.509 267	48
13	.471 271	6.80	.980 091	.65	.491 180	7.45	.508 820	47
14	.471 679	6.78	.980 052	.67	.491 627	7.43	.508 373	46
15	9.472 086	6.77	9.980 012	.65	9.492 073	7.43	0.507 927	45
16	.472 492	6.77	.979 973	.65	.492 519	7.43	.507 481	44
17	.472 898	6.77	.979 934	.65	.492 965	7.42	.507 035	43
18	.473 304	6.77	.979 895	.67	.493 410	7.40	.506 590	42
19	.473 710	6.75	.979 855	.65	.493 854	7.42	.506 146	41
20	9.474 115	6.73	9.979 816	.67	9.494 299	7.40	0.505 701	40
21	.474 519	6.73	.979 776	.65	.494 743	7.38	.505 257	39
22	.474 923	6.73	.979 737	.67	.495 186	7.40	.504 814	38
23	.475 327	6.72	.979 697	.65	.495 630	7.38	.504 370	37
24	.475 730	6.72	.979 658	.67	.496 073	7.37	.503 927	36
25	9.476 133	6.72	9.979 618	.65	9.496 515	7.37	0.503 485	35
26	.476 536	6.70	.979 579	.67	.496 957	7.37	.503 043	34
27	.476 938	6.70	.979 539	.67	.497 399	7.37	.502 601	33
28	.477 340	6.68	.979 499	.67	.497 841	7.35	.502 159	32
29	.477 741	6.68	.979 459	.65	.498 282	7.33	.501 718	31
30	9.478 142	6.67	9.979 420	.67	9.498 722	7.35	0.501 278	30
31	.478 542	6.67	.979 380	.67	.499 163	7.33	.500 837	29
32	.478 942	6.67	.979 340	.67	.499 603	7.32	.500 397	28
33	.479 342	6.65	.979 300	.67	.500 042	7.32	.499 958	27
34	.479 741	6.65	.979 260	.67	.500 481	7.32	.499 519	26
35	9.480 140	6.65	9.979 220	.67	9.500 920	7.32	0.499 080	25
36	.480 539	6.63	.979 180	.67	.501 359	7.30	.498 641	24
37	.480 937	6.62	.979 140	.67	.501 797	7.30	.498 203	23
38	.481 334	6.62	.979 100	.68	.502 235	7.28	.497 765	22
39	.481 731	6.62	.979 059	.67	.502 672	7.28	.497 328	21
40	9.482 128	6.62	9.979 019	.67	9.503 109	7.28	0.496 891	20
41	.482 525	6.60	.978 979	.67	.503 546	7.27	.496 454	19
42	.482 921	6.58	.978 939	.68	.503 982	7.27	.496 018	18
43	.483 316	6.60	.978 898	.67	.504 418	7.27	.495 582	17
44	.483 712	6.58	.978 858	.68	.504 854	7.25	.495 146	16
45	9.484 107	6.57	9.978 817	.67	9.505 289	7.25	0.494 711	15
46	.484 501	6.57	.978 777	.67	.505 724	7.25	.494 276	14
47	.484 895	6.57	.978 737	.68	.506 159	7.23	.493 841	13
48	.485 289	6.55	.978 696	.68	.506 593	7.23	.493 407	12
49	.485 682	6.55	.978 655	.67	.507 027	7.22	.492 973	11
50	9.486 075	6.53	9.978 615	.68	9.507 460	7.22	0.492 540	10
51	.486 467	6.55	.978 574	.68	.507 893	7.22	.492 107	9
52	.486 860	6.52	.978 533	.67	.508 326	7.22	.491 674	8
53	.487 251	6.53	.978 493	.68	.508 759	7.20	.491 241	7
54	.487 643	6.52	.978 452	.68	.509 191	7.18	.490 809	6
55	9.488 034	6.50	9.978 411	.68	9.509 622	7.20	0.490 378	5
56	.488 424	6.50	.978 370	.68	.510 054	7.18	.489 946	4
57	.488 814	6.50	.978 329	.68	.510 485	7.18	.489 515	3
58	.489 204	6.48	.978 288	.68	.510 916	7.17	.489 084	2
59	.489 593	6.48	.978 247	.68	.511 346	7.17	.488 654	1
60	9.489 982		9.978 206		9.511 776		0.488 224	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

72°

18°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.489 982	6.48	9.978 206	.68	9.511 776	7.17	0.488 224	60
1	.490 371	6.47	.978 165	.68	.512 206	7.15	.487 794	59
2	.490 759	6.47	.978 124	.68	.512 635	7.15	.487 365	58
3	.491 147	6.47	.978 083	.68	.513 064	7.15	.486 936	57
4	.491 535	6.45	.978 042	.68	.513 493	7.13	.486 507	56
5	9.491 922	6.43	9.978 001	.70	9.513 921	7.13	0.486 079	55
6	.492 308	6.45	.977 959	.68	.514 349	7.13	.485 651	54
7	.492 695	6.43	.977 918	.68	.514 777	7.12	.485 223	53
8	.493 081	6.42	.977 877	.70	.515 204	7.12	.484 796	52
9	.493 466	6.42	.977 835	.68	.515 631	7.10	.484 369	51
10	9.493 851	6.42	9.977 794	.70	9.516 057	7.12	0.483 943	50
11	.494 236	6.42	.977 752	.68	.516 484	7.10	.483 516	49
12	.494 621	6.40	.977 711	.70	.516 910	7.08	.483 090	48
13	.495 005	6.38	.977 669	.68	.517 335	7.10	.482 665	47
14	.495 388	6.40	.977 628	.70	.517 761	7.08	.482 239	46
15	9.495 772	6.37	9.977 586	.70	9.518 186	7.07	0.481 814	45
16	.496 154	6.38	.977 544	.68	.518 610	7.07	.481 390	44
17	.496 537	6.37	.977 503	.70	.519 034	7.07	.480 966	43
18	.496 919	6.37	.977 461	.70	.519 458	7.07	.480 542	42
19	.497 301	6.35	.977 419	.70	.519 882	7.05	.480 118	41
20	9.497 682	6.37	9.977 377	.70	9.520 305	7.05	0.479 695	40
21	.498 064	6.33	.977 335	.70	.520 728	7.05	.479 272	39
22	.498 444	6.35	.977 293	.70	.521 151	7.03	.478 849	38
23	.498 825	6.32	.977 251	.70	.521 573	7.03	.478 427	37
24	.499 204	6.33	.977 209	.70	.521 995	7.03	.478 005	36
25	9.499 584	6.32	9.977 167	.70	9.522 417	7.02	0.477 583	35
26	.499 963	6.32	.977 125	.70	.522 838	7.02	.477 162	34
27	.500 342	6.32	.977 083	.70	.523 259	7.02	.476 741	33
28	.500 721	6.30	.977 041	.70	.523 680	7.00	.476 320	32
29	.501 099	6.28	.976 999	.70	.524 100	7.00	.475 900	31
30	9.501 476	6.30	9.976 957	.72	9.524 520	7.00	0.475 480	30
31	.501 854	6.28	.976 914	.70	.524 940	6.98	.475 060	29
32	.502 231	6.27	.976 872	.70	.525 359	6.98	.474 641	28
33	.502 607	6.28	.976 830	.72	.525 778	6.98	.474 222	27
34	.502 984	6.27	.976 787	.70	.526 197	6.97	.473 803	26
35	9.503 360	6.25	9.976 745	.72	9.526 615	6.97	0.473 385	25
36	.503 735	6.25	.976 702	.70	.527 033	6.97	.472 967	24
37	.504 110	6.25	.976 660	.72	.527 451	6.95	.472 549	23
38	.504 485	6.25	.976 617	.72	.527 868	6.95	.472 132	22
39	.504 860	6.23	.976 574	.70	.528 285	6.95	.471 715	21
40	9.505 234	6.23	9.976 532	.72	9.528 702	6.95	0.471 298	20
41	.505 608	6.22	.976 489	.72	.529 119	6.93	.470 881	19
42	.505 981	6.22	.976 446	.70	.529 535	6.93	.470 465	18
43	.506 354	6.22	.976 404	.72	.529 951	6.92	.470 049	17
44	.506 727	6.20	.976 361	.72	.530 366	6.92	.469 634	16
45	9.507 099	6.20	9.976 318	.72	9.530 781	6.92	0.469 219	15
46	.507 471	6.20	.976 275	.72	.531 196	6.92	.468 804	14
47	.507 843	6.18	.976 232	.72	.531 611	6.90	.468 389	13
48	.508 214	6.18	.976 189	.72	.532 025	6.90	.467 975	12
49	.508 585	6.18	.976 146	.72	.532 439	6.90	.467 561	11
50	9.508 956	6.17	9.976 103	.72	9.532 853	6.88	0.467 147	10
51	.509 326	6.17	.976 060	.72	.533 266	6.88	.466 734	9
52	.509 696	6.15	.976 017	.72	.533 679	6.88	.466 321	8
53	.510 065	6.15	.975 974	.73	.534 092	6.87	.465 908	7
54	.510 434	6.15	.975 930	.72	.534 504	6.87	.465 496	6
55	9.510 803	6.15	9.975 887	.72	9.534 916	6.87	0.465 084	5
56	.511 172	6.13	.975 844	.73	.535 328	6.85	.464 672	4
57	.511 540	6.12	.975 800	.72	.535 739	6.85	.464 261	3
58	.511 907	6.13	.975 757	.72	.536 150	6.85	.463 850	2
59	.512 275	6.12	.975 714	.73	.536 561	6.85	.463 439	1
60	9.512 642		9.975 670		9.536 972		0.463 028	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

71°

19°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.512 642	6.12	9.975 670	.72	9.536 972	6.83	0.463 028	60
1	.513 009	6.10	.975 627	.73	.537 382	6.83	.462 618	59
2	.513 375	6.10	.975 583	.73	.537 792	6.83	.462 208	58
3	.513 741	6.10	.975 539	.72	.538 202	6.82	.461 798	57
4	.514 107	6.08	.975 496	.73	.538 611	6.82	.461 389	56
5	9.514 472	6.08	9.975 452	.73	9.539 020	6.82	0.460 980	55
6	.514 837	6.08	.975 408	.72	.539 429	6.80	.460 571	54
7	.515 202	6.07	.975 365	.73	.539 837	6.80	.460 163	53
8	.515 566	6.07	.975 321	.73	.540 245	6.80	.459 755	52
9	.515 930	6.07	.975 277	.73	.540 653	6.80	.459 347	51
10	9.516 294	6.05	9.975 233	.73	9.541 061	6.78	0.458 939	50
11	.516 657	6.05	.975 189	.73	.541 468	6.78	.458 532	49
12	.517 020	6.03	.975 145	.73	.541 875	6.77	.458 125	48
13	.517 382	6.05	.975 101	.73	.542 281	6.78	.457 719	47
14	.517 745	6.03	.975 057	.73	.542 688	6.77	.457 312	46
15	9.518 107	6.02	9.975 013	.73	9.543 094	6.75	0.456 906	45
16	.518 468	6.02	.974 969	.73	.543 499	6.77	.456 501	44
17	.518 829	6.02	.974 925	.75	.543 905	6.75	.456 095	43
18	.519 190	6.02	.974 880	.73	.544 310	6.75	.455 690	42
19	.519 551	6.00	.974 836	.73	.544 715	6.73	.455 285	41
20	9.519 911	6.00	9.974 792	.73	9.545 119	6.75	0.454 881	40
21	.520 271	6.00	.974 748	.75	.545 524	6.73	.454 476	39
22	.520 631	5.98	.974 703	.73	.545 928	6.72	.454 072	38
23	.520 990	5.98	.974 659	.75	.546 331	6.73	.453 669	37
24	.521 349	5.97	.974 614	.73	.546 735	6.72	.453 265	36
25	9.521 707	5.98	9.974 570	.75	9.547 138	6.70	0.452 862	35
26	.522 066	5.97	.974 525	.73	.547 540	6.72	.452 460	34
27	.522 424	5.95	.974 481	.75	.547 943	6.70	.452 057	33
28	.522 781	5.95	.974 436	.75	.548 345	6.70	.451 655	32
29	.523 138	5.95	.974 391	.73	.548 747	6.70	.451 253	31
30	9.523 495	5.95	9.974 347	.75	9.549 149	6.68	0.450 851	30
31	.523 852	5.93	.974 302	.75	.549 550	6.68	.450 450	29
32	.524 208	5.93	.974 257	.75	.549 951	6.68	.450 049	28
33	.524 564	5.93	.974 212	.75	.550 352	6.67	.449 648	27
34	.524 920	5.92	.974 167	.75	.550 752	6.68	.449 248	26
35	9.525 275	5.92	9.974 122	.75	9.551 153	6.65	0.448 847	25
36	.525 630	5.90	.974 077	.75	.551 552	6.67	.448 448	24
37	.525 984	5.92	.974 032	.75	.551 952	6.65	.448 048	23
38	.526 339	5.90	.973 987	.75	.552 351	6.65	.447 649	22
39	.526 693	5.88	.973 942	.75	.552 750	6.65	.447 250	21
40	9.527 046	5.90	9.973 897	.75	9.553 149	6.65	0.446 851	20
41	.527 400	5.88	.973 852	.75	.553 548	6.63	.446 452	19
42	.527 753	5.87	.973 807	.77	.553 946	6.63	.446 054	18
43	.528 105	5.88	.973 761	.75	.554 344	6.62	.445 656	17
44	.528 458	5.87	.973 716	.75	.554 741	6.63	.445 259	16
45	9.528 810	5.85	9.973 671	.77	9.555 139	6.62	0.444 861	15
46	.529 161	5.87	.973 625	.75	.555 536	6.62	.444 464	14
47	.529 513	5.85	.973 580	.75	.555 933	6.60	.444 067	13
48	.529 864	5.85	.973 535	.77	.556 329	6.60	.443 671	12
49	.530 215	5.83	.973 489	.75	.556 725	6.60	.443 275	11
50	9.530 565	5.83	9.973 444	.77	9.557 121	6.60	0.442 879	10
51	.530 915	5.83	.973 398	.77	.557 517	6.60	.442 483	9
52	.531 265	5.82	.973 352	.75	.557 913	6.58	.442 087	8
53	.531 614	5.82	.973 307	.77	.558 308	6.58	.441 692	7
54	.531 963	5.82	.973 261	.77	.558 703	6.57	.441 297	6
55	9.532 312	5.82	9.973 215	.77	9.559 097	6.57	0.440 903	5
56	.532 661	5.80	.973 169	.75	.559 491	6.57	.440 509	4
57	.533 009	5.80	.973 124	.77	.559 885	6.57	.440 115	3
58	.533 357	5.78	.973 078	.77	.560 279	6.57	.439 721	2
59	.533 704	5.80	.973 032	.77	.560 673	6.55	.439 327	1
60	9.534 052		9.972 986		9.561 066		0.438 934	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

70°

18°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.489 982	6.48	9.978 206	.68	9.511 776	7.17	0.488 224	60
1	.490 371	6.47	.978 165	.68	.512 206	7.15	.487 794	59
2	.490 759	6.47	.978 124	.68	.512 635	7.15	.487 365	58
3	.491 147	6.47	.978 083	.68	.513 064	7.15	.486 936	57
4	.491 535	6.45	.978 042	.68	.513 493	7.13	.486 507	56
5	9.491 922	6.43	9.978 001	.70	9.513 921	7.13	0.486 079	55
6	.492 308	6.45	.977 959	.68	.514 349	7.13	.485 651	54
7	.492 695	6.43	.977 918	.68	.514 777	7.12	.485 223	53
8	.493 081	6.42	.977 877	.70	.515 204	7.12	.484 796	52
9	.493 466	6.42	.977 835	.68	.515 631	7.10	.484 369	51
10	9.493 851	6.42	9.977 794	.70	9.516 057	7.12	0.483 943	50
11	.494 236	6.42	.977 752	.68	.516 484	7.10	.483 516	49
12	.494 621	6.40	.977 711	.70	.516 910	7.08	.483 090	48
13	.495 005	6.38	.977 669	.68	.517 335	7.10	.482 665	47
14	.495 388	6.40	.977 628	.70	.517 761	7.08	.482 239	46
15	9.495 772	6.37	9.977 586	.70	9.518 186	7.07	0.481 814	45
16	.496 154	6.38	.977 544	.68	.518 610	7.07	.481 390	44
17	.496 537	6.37	.977 503	.70	.519 034	7.07	.480 966	43
18	.496 919	6.37	.977 461	.70	.519 458	7.07	.480 542	42
19	.497 301	6.35	.977 419	.70	.519 882	7.05	.480 118	41
20	9.497 682	6.37	9.977 377	.70	9.520 305	7.05	0.479 695	40
21	.498 064	6.33	.977 335	.70	.520 728	7.05	.479 272	39
22	.498 444	6.35	.977 293	.70	.521 151	7.03	.478 849	38
23	.498 825	6.32	.977 251	.70	.521 573	7.03	.478 427	37
24	.499 204	6.33	.977 209	.70	.521 995	7.03	.478 005	36
25	9.499 584	6.32	9.977 167	.70	9.522 417	7.02	0.477 583	35
26	.499 963	6.32	.977 125	.70	.522 838	7.02	.477 162	34
27	.500 342	6.32	.977 083	.70	.523 259	7.02	.476 741	33
28	.500 721	6.30	.977 041	.70	.523 680	7.00	.476 320	32
29	.501 099	6.28	.976 999	.70	.524 100	7.00	.475 900	31
30	9.501 476	6.30	9.976 957	.72	9.524 520	7.00	0.475 480	30
31	.501 854	6.28	.976 914	.70	.524 940	6.98	.475 060	29
32	.502 231	6.27	.976 872	.70	.525 359	6.98	.474 641	28
33	.502 607	6.28	.976 830	.72	.525 778	6.98	.474 222	27
34	.502 984	6.27	.976 787	.70	.526 197	6.97	.473 803	26
35	9.503 360	6.25	9.976 745	.72	9.526 615	6.97	0.473 385	25
36	.503 735	6.25	.976 702	.70	.527 033	6.97	.472 967	24
37	.504 110	6.25	.976 660	.72	.527 451	6.95	.472 549	23
38	.504 485	6.25	.976 617	.72	.527 868	6.95	.472 132	22
39	.504 860	6.23	.976 574	.70	.528 285	6.95	.471 715	21
40	9.505 234	6.23	9.976 532	.72	9.528 702	6.95	0.471 298	20
41	.505 608	6.22	.976 489	.72	.529 119	6.93	.470 881	19
42	.505 981	6.22	.976 446	.70	.529 535	6.93	.470 465	18
43	.506 354	6.22	.976 404	.72	.529 951	6.92	.470 049	17
44	.506 727	6.20	.976 361	.72	.530 366	6.92	.469 634	16
45	9.507 099	6.20	9.976 318	.72	9.530 781	6.92	0.469 219	15
46	.507 471	6.20	.976 275	.72	.531 196	6.92	.468 804	14
47	.507 843	6.18	.976 232	.72	.531 611	6.90	.468 389	13
48	.508 214	6.18	.976 189	.72	.532 025	6.90	.467 975	12
49	.508 585	6.18	.976 146	.72	.532 439	6.90	.467 561	11
50	9.508 956	6.17	9.976 103	.72	9.532 853	6.88	0.467 147	10
51	.509 326	6.17	.976 060	.72	.533 266	6.88	.466 734	9
52	.509 696	6.15	.976 017	.72	.533 679	6.88	.466 321	8
53	.510 065	6.15	.975 974	.73	.534 092	6.87	.465 908	7
54	.510 434	6.15	.975 930	.72	.534 504	6.87	.465 496	6
55	9.510 803	6.15	9.975 887	.72	9.534 916	6.87	0.465 084	5
56	.511 172	6.13	.975 844	.73	.535 328	6.85	.464 672	4
57	.511 540	6.12	.975 800	.72	.535 739	6.85	.464 261	3
58	.511 907	6.13	.975 757	.72	.536 150	6.85	.463 850	2
59	.512 275	6.12	.975 714	.73	.536 561	6.85	.463 439	1
60	9.512 642		9.975 670		9.536 972		0.463 028	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

71°

21°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.554 329	5.48	9.970 152	.82	9.584 177	6.30	0.415 823	60
1	.554 658	5.48	.970 103	.80	.584 555	6.28	.415 445	59
2	.554 987	5.47	.970 055	.82	.584 932	6.28	.415 068	58
3	.555 315	5.47	.970 006	.82	.585 309	6.28	.414 691	57
4	.555 643	5.47	.969 957	.80	.585 686	6.27	.414 314	56
5	9.555 971	5.47	9.969 909	.82	9.586 062	6.28	0.413 938	55
6	.556 299	5.45	.969 860	.82	.586 439	6.27	.413 561	54
7	.556 626	5.45	.969 811	.82	.586 815	6.25	.413 185	53
8	.556 953	5.45	.969 762	.80	.587 190	6.27	.412 810	52
9	.557 280	5.43	.969 714	.82	.587 566	6.25	.412 434	51
10	9.557 606	5.43	9.969 665	.82	9.587 941	6.25	0.412 059	50
11	.557 932	5.43	.969 616	.82	.588 316	6.25	.411 684	49
12	.558 258	5.42	.969 567	.82	.588 691	6.25	.411 309	48
13	.558 583	5.43	.969 518	.82	.589 066	6.23	.410 934	47
14	.558 909	5.42	.969 469	.82	.589 440	6.23	.410 560	46
15	9.559 234	5.40	9.969 420	.83	9.589 814	6.23	0.410 186	45
16	.559 558	5.42	.969 370	.82	.590 188	6.23	.409 812	44
17	.559 883	5.40	.969 321	.82	.590 562	6.22	.409 438	43
18	.560 207	5.40	.969 272	.82	.590 935	6.22	.409 065	42
19	.560 531	5.40	.969 223	.83	.591 308	6.22	.408 692	41
20	9.560 855	5.38	9.969 173	.82	9.591 681	6.22	0.408 319	40
21	.561 178	5.38	.969 124	.82	.592 054	6.20	.407 946	39
22	.561 501	5.38	.969 075	.83	.592 426	6.22	.407 574	38
23	.561 824	5.37	.969 025	.82	.592 799	6.20	.407 201	37
24	.562 146	5.37	.968 976	.83	.593 171	6.18	.406 829	36
25	9.562 468	5.37	9.968 926	.82	9.593 542	6.20	0.406 458	35
26	.562 790	5.37	.968 877	.83	.593 914	6.18	.406 086	34
27	.563 112	5.35	.968 827	.83	.594 285	6.18	.405 715	33
28	.563 433	5.37	.968 777	.82	.594 656	6.18	.405 344	32
29	.563 755	5.33	.968 728	.83	.595 027	6.18	.404 973	31
30	9.564 075	5.35	9.968 678	.83	9.595 398	6.17	0.404 602	30
31	.564 396	5.33	.968 628	.83	.595 768	6.17	.404 232	29
32	.564 716	5.33	.968 578	.83	.596 138	6.17	.403 862	28
33	.565 036	5.33	.968 528	.82	.596 508	6.17	.403 492	27
34	.565 356	5.33	.968 479	.83	.596 878	6.15	.403 122	26
35	9.565 676	5.32	9.968 429	.83	9.597 247	6.15	0.402 753	25
36	.565 995	5.32	.968 379	.83	.597 616	6.15	.402 384	24
37	.566 314	5.30	.968 329	.85	.597 985	6.15	.402 015	23
38	.566 632	5.32	.968 278	.83	.598 354	6.13	.401 646	22
39	.566 951	5.30	.968 228	.83	.598 722	6.15	.401 278	21
40	9.567 269	5.30	9.968 178	.83	9.599 091	6.13	0.400 909	20
41	.567 587	5.28	.968 128	.83	.599 459	6.13	.400 541	19
42	.567 904	5.30	.968 078	.85	.599 827	6.12	.400 173	18
43	.568 222	5.28	.968 027	.83	.600 194	6.13	.399 806	17
44	.568 539	5.28	.967 977	.83	.600 562	6.12	.399 438	16
45	9.568 856	5.27	9.967 927	.85	9.600 929	6.12	0.399 071	15
46	.569 172	5.27	.967 876	.83	.601 296	6.12	.398 704	14
47	.569 488	5.27	.967 826	.85	.601 663	6.10	.398 337	13
48	.569 804	5.27	.967 775	.83	.602 029	6.10	.397 971	12
49	.570 120	5.25	.967 725	.85	.602 395	6.10	.397 605	11
50	9.570 435	5.27	9.967 674	.83	9.602 761	6.10	0.397 239	10
51	.570 751	5.25	.967 624	.85	.603 127	6.10	.396 873	9
52	.571 066	5.23	.967 573	.85	.603 493	6.08	.396 507	8
53	.571 380	5.25	.967 522	.85	.603 858	6.08	.396 142	7
54	.571 695	5.23	.967 471	.83	.604 223	6.08	.395 777	6
55	9.572 009	5.23	9.967 421	.85	9.604 588	6.08	0.395 412	5
56	.572 323	5.22	.967 370	.85	.604 953	6.07	.395 047	4
57	.572 636	5.23	.967 319	.85	.605 317	6.08	.394 683	3
58	.572 950	5.22	.967 268	.85	.605 682	6.07	.394 318	2
59	.573 263	5.20	.967 217	.85	.606 046	6.07	.393 954	1
60	9.573 575		9.967 166		9.606 410		0.393 590	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

68°

20°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.534 052	5.78	9.972 986	.77	9.561 066	6.55	0.438 934	60
1	.534 399	5.77	.972 940	.77	.561 459	6.53	.438 541	59
2	.534 745	5.78	.972 894	.77	.561 851	6.55	.438 149	58
3	.535 092	5.77	.972 848	.77	.562 244	6.53	.437 756	57
4	.535 438	5.75	.972 802	.78	.562 636	6.53	.437 364	56
5	9.535 783	5.77	9.972 755	.77	9.563 028	6.52	0.436 972	55
6	.536 129	5.75	.972 709	.77	.563 419	6.53	.436 581	54
7	.536 474	5.73	.972 663	.77	.563 811	6.52	.436 189	53
8	.536 818	5.75	.972 617	.78	.564 202	6.52	.435 798	52
9	.537 163	5.73	.972 570	.77	.564 593	6.50	.435 407	51
10	9.537 507	5.73	9.972 524	.77	9.564 983	6.50	0.435 017	50
11	.537 851	5.72	.972 478	.78	.565 373	6.50	.434 627	49
12	.538 194	5.73	.972 431	.77	.565 763	6.50	.434 237	48
13	.538 538	5.70	.972 385	.78	.566 153	6.48	.433 847	47
14	.538 880	5.72	.972 338	.78	.566 542	6.50	.433 458	46
15	9.539 223	5.70	9.972 291	.77	9.566 932	6.47	0.433 068	45
16	.539 565	5.70	.972 245	.78	.567 320	6.48	.432 680	44
17	.539 907	5.70	.972 198	.78	.567 709	6.48	.432 291	43
18	.540 249	5.68	.972 151	.77	.568 098	6.47	431 902	42
19	.540 590	5.68	.972 105	.78	.568 486	6.45	.431 514	41
20	9.540 931	5.68	9.972 058	.78	9.568 873	6.47	0.431 127	40
21	.541 272	5.68	.972 011	.78	.569 261	6.45	.430 739	39
22	.541 613	5.67	.971 964	.78	.569 648	6.45	.430 352	38
23	.541 953	5.67	.971 917	.78	.570 035	6.45	.429 965	37
24	.542 293	5.65	.971 870	.78	.570 422	6.45	.429 578	36
25	9.542 632	5.65	9.971 823	.78	9.570 809	6.43	0.429 191	35
26	.542 971	5.65	.971 776	.78	.571 195	6.43	.428 805	34
27	.543 310	5.65	.971 729	.78	.571 581	6.43	.428 419	33
28	.543 649	5.63	.971 682	.78	.571 967	6.42	.428 033	32
29	.543 987	5.63	.971 635	.78	.572 352	6.43	.427 648	31
30	9.544 325	5.63	9.971 588	.80	9.572 738	6.42	0.427 262	30
31	.544 663	5.62	.971 540	.78	.573 123	6.40	.426 877	29
32	.545 000	5.63	.971 493	.78	.573 507	6.42	.426 493	28
33	.545 338	5.60	.971 446	.80	.573 892	6.40	.426 108	27
34	.545 674	5.62	.971 398	.78	.574 276	6.40	.425 724	26
35	9.546 011	5.60	9.971 351	.80	9.574 660	6.40	0.425 340	25
36	.546 347	5.60	.971 303	.78	.575 044	6.38	.424 956	24
37	.546 683	5.60	.971 256	.80	.575 427	6.38	.424 573	23
38	.547 019	5.58	.971 208	.78	.575 810	6.38	.424 190	22
39	.547 354	5.58	.971 161	.80	.576 193	6.38	.423 807	21
40	9.547 689	5.58	9.971 113	.78	9.576 576	6.38	0.423 424	20
41	.548 024	5.58	.971 066	.80	.576 959	6.37	.423 041	19
42	.548 359	5.57	.971 018	.80	.577 341	6.37	.422 659	18
43	.548 693	5.57	.970 970	.80	.577 723	6.35	.422 277	17
44	.549 027	5.55	.970 922	.80	.578 104	6.37	.421 896	16
45	9.549 360	5.55	9.970 874	.78	9.578 486	6.35	0.421 514	15
46	.549 693	5.55	.970 827	.80	.578 867	6.35	.421 133	14
47	.550 026	5.55	.970 779	.80	.579 248	6.35	.420 752	13
48	.550 359	5.55	.970 731	.80	.579 629	6.33	.420 371	12
49	.550 692	5.53	.970 683	.80	.580 009	6.33	.419 991	11
50	9.551 024	5.53	9.970 635	.82	9.580 389	6.33	0.419 611	10
51	.551 356	5.52	.970 586	.80	.580 769	6.33	.419 231	9
52	.551 687	5.52	.970 538	.80	.581 149	6.32	.418 851	8
53	.552 018	5.52	.970 490	.80	.581 528	6.32	.418 472	7
54	.552 349	5.52	.970 442	.80	.581 907	6.32	.418 093	6
55	9.552 680	5.50	9.970 394	.82	9.582 286	6.32	0.417 714	5
56	.553 010	5.52	.970 345	.80	.582 665	6.32	.417 335	4
57	.553 341	5.48	.970 297	.80	.583 044	6.30	.416 956	3
58	.553 670	5.50	.970 249	.82	.583 422	6.30	.416 578	2
59	.554 000	5.48	.970 200	.80	.583 800	6.28	.416 200	1
60	9.554 329		9.970 152		9.584 177		0.415 823	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

69°

21°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.554 329	5.48	9.970 152	.82	9.584 177	6.30	0.415 823	60
1	.554 658	5.48	.970 103	.80	.584 555	6.28	.415 445	59
2	.554 987	5.47	.970 055	.82	.584 932	6.28	.415 068	58
3	.555 315	5.47	.970 006	.82	.585 309	6.28	.414 691	57
4	.555 643	5.47	.969 957	.80	.585 686	6.27	.414 314	56
5	9.555 971	5.47	9.969 909	.82	9.586 062	6.28	0.413 938	55
6	.556 299	5.45	.969 860	.82	.586 439	6.27	.413 561	54
7	.556 626	5.45	.969 811	.82	.586 815	6.25	.413 185	53
8	.556 953	5.45	.969 762	.80	.587 190	6.27	.412 810	52
9	.557 280	5.43	.969 714	.82	.587 566	6.25	.412 434	51
10	9.557 606	5.43	9.969 665	.82	9.587 941	6.25	0.412 059	50
11	.557 932	5.43	.969 616	.82	.588 316	6.25	.411 684	49
12	.558 258	5.42	.969 567	.82	.588 691	6.25	.411 309	48
13	.558 583	5.43	.969 518	.82	.589 066	6.23	.410 934	47
14	.558 909	5.42	.969 469	.82	.589 440	6.23	.410 560	46
15	9.559 234	5.40	9.969 420	.83	9.589 814	6.23	0.410 186	45
16	.559 558	5.42	.969 370	.82	.590 188	6.23	.409 812	44
17	.559 883	5.40	.969 321	.82	.590 562	6.22	.409 438	43
18	.560 207	5.40	.969 272	.82	.590 935	6.22	.409 065	42
19	.560 531	5.40	.969 223	.83	.591 308	6.22	.408 692	41
20	9.560 855	5.38	9.969 173	.82	9.591 681	6.22	0.408 319	40
21	.561 178	5.38	.969 124	.82	.592 054	6.20	.407 946	39
22	.561 501	5.38	.969 075	.83	.592 426	6.22	.407 574	38
23	.561 824	5.37	.969 025	.82	.592 799	6.20	.407 201	37
24	.562 146	5.37	.968 976	.83	.593 171	6.18	.406 829	36
25	9.562 468	5.37	9.968 926	.82	9.593 542	6.20	0.406 458	35
26	.562 790	5.37	.968 877	.83	.593 914	6.18	.406 086	34
27	.563 112	5.35	.968 827	.83	.594 285	6.18	.405 715	33
28	.563 433	5.37	.968 777	.82	.594 656	6.18	.405 344	32
29	.563 755	5.33	.968 728	.83	.595 027	6.18	.404 973	31
30	9.564 075	5.35	9.968 678	.83	9.595 398	6.17	0.404 602	30
31	.564 396	5.33	.968 628	.83	.595 768	6.17	.404 232	29
32	.564 716	5.33	.968 578	.83	.596 138	6.17	.403 862	28
33	.565 036	5.33	.968 528	.82	.596 508	6.17	.403 492	27
34	.565 356	5.33	.968 479	.83	.596 878	6.15	.403 122	26
35	9.565 676	5.32	9.968 429	.83	9.597 247	6.15	0.402 753	25
36	.565 995	5.32	.968 379	.83	.597 616	6.15	.402 384	24
37	.566 314	5.30	.968 329	.85	.597 985	6.15	.402 015	23
38	.566 632	5.32	.968 278	.83	.598 354	6.13	.401 646	22
39	.566 951	5.30	.968 228	.83	.598 722	6.15	.401 278	21
40	9.567 269	5.30	9.968 178	.83	9.599 091	6.13	0.400 909	20
41	.567 587	5.28	.968 128	.83	.599 459	6.13	.400 541	19
42	.567 904	5.30	.968 078	.85	.599 827	6.12	.400 173	18
43	.568 222	5.28	.968 027	.83	.600 194	6.13	.399 806	17
44	.568 539	5.28	.967 977	.83	.600 562	6.12	.399 438	16
45	9.568 856	5.27	9.967 927	.85	9.600 929	6.12	0.399 071	15
46	.569 172	5.27	.967 876	.83	.601 296	6.12	.398 704	14
47	.569 488	5.27	.967 826	.85	.601 663	6.10	.398 337	13
48	.569 804	5.27	.967 775	.83	.602 029	6.10	.397 971	12
49	.570 120	5.25	.967 725	.85	.602 395	6.10	.397 605	11
50	9.570 435	5.27	9.967 674	.83	9.602 761	6.10	0.397 239	10
51	.570 751	5.25	.967 624	.85	.603 127	6.10	.396 873	9
52	.571 066	5.23	.967 573	.85	.603 493	6.08	.396 507	8
53	.571 380	5.25	.967 522	.85	.603 858	6.08	.396 142	7
54	.571 695	5.23	.967 471	.83	.604 223	6.08	.395 777	6
55	9.572 009	5.23	9.967 421	.85	9.604 588	6.08	0.395 412	5
56	.572 323	5.22	.967 370	.85	.604 953	6.07	.395 047	4
57	.572 636	5.23	.967 319	.85	.605 317	6.08	.394 683	3
58	.572 950	5.22	.967 268	.85	.605 682	6.07	.394 318	2
59	.573 263	5.20	.967 217	.85	.606 046	6.07	.393 954	1
60	9.573 575		9.967 166		9.606 410		0.393 590	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

68°

22°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.573 575	5.22	9.967 166	.85	9.606 410	6.05	0.393 590	60
1	.573 888	5.20	.967 115	.85	.606 773	6.07	.393 227	59
2	.574 200	5.20	.967 064	.85	.607 137	6.05	.392 863	58
3	.574 512	5.20	.967 013	.87	.607 500	6.05	.392 500	57
4	.574 824	5.20	.966 961	.85	.607 863	6.03	.392 137	56
5	9.575 136	5.18	9.966 910	.85	9.608 225	6.05	0.391 775	55
6	.575 447	5.18	.966 859	.85	.608 588	6.03	.391 412	54
7	.575 758	5.18	.966 808	.87	.608 950	6.03	.391 050	53
8	.576 069	5.17	.966 756	.85	.609 312	6.03	.390 688	52
9	.576 379	5.17	.966 705	.87	.609 674	6.03	.390 326	51
10	9.576 689	5.17	9.966 653	.85	9.610 036	6.02	0.389 964	50
11	.576 999	5.17	.966 602	.87	.610 397	6.03	.389 603	49
12	.577 309	5.15	.966 550	.85	.610 759	6.02	.389 241	48
13	.577 618	5.15	.966 499	.87	.611 120	6.00	.388 880	47
14	.577 927	5.15	.966 447	.87	.611 480	6.02	.388 520	46
15	9.578 236	5.15	9.966 395	.85	9.611 841	6.00	0.388 159	45
16	.578 545	5.13	.966 344	.87	.612 201	6.00	.387 799	44
17	.578 853	5.15	.966 292	.87	.612 561	6.00	.387 439	43
18	.579 162	5.13	.966 240	.87	.612 921	6.00	.387 079	42
19	.579 470	5.12	.966 188	.87	.613 281	6.00	.386 719	41
20	9.579 777	5.13	9.966 136	.85	9.613 641	5.98	0.386 359	40
21	.580 085	5.12	.966 085	.87	.614 000	5.98	.386 000	39
22	.580 392	5.12	.966 033	.87	.614 359	5.98	.385 641	38
23	.580 699	5.10	.965 981	.87	.614 718	5.98	.385 282	37
24	.581 005	5.12	.965 929	.88	.615 077	5.97	.384 923	36
25	9.581 312	5.10	9.965 876	.87	9.615 435	5.97	0.384 565	35
26	.581 618	5.10	.965 824	.87	.615 793	5.97	.384 207	34
27	.581 924	5.08	.965 772	.87	.616 151	5.97	.383 849	33
28	.582 229	5.10	.965 720	.87	.616 509	5.97	.383 491	32
29	.582 535	5.08	.965 668	.88	.616 867	5.95	.383 133	31
30	9.582 840	5.08	9.965 615	.87	9.617 224	5.97	0.382 776	30
31	.583 145	5.07	.965 563	.87	.617 582	5.95	.382 418	29
32	.583 449	5.08	.965 511	.88	.617 939	5.93	.382 061	28
33	.583 754	5.07	.965 458	.87	.618 295	5.95	.381 705	27
34	.584 058	5.05	.965 406	.88	.618 652	5.93	.381 348	26
35	9.584 361	5.07	9.965 353	.87	9.619 008	5.93	0.380 992	25
36	.584 665	5.05	.965 301	.88	.619 364	5.93	.380 636	24
37	.584 968	5.07	.965 248	.88	.619 720	5.93	.380 280	23
38	.585 272	5.03	.965 195	.87	.620 076	5.93	.379 924	22
39	.585 574	5.05	.965 143	.88	.620 432	5.92	.379 568	21
40	9.585 877	5.03	9.965 090	.88	9.620 787	5.92	0.379 213	20
41	.586 179	5.05	.965 037	.88	.621 142	5.92	.378 858	19
42	.586 482	5.02	.964 984	.88	.621 497	5.92	.378 503	18
43	.586 783	5.03	.964 931	.87	.621 852	5.92	.378 148	17
44	.587 085	5.02	.964 879	.88	.622 207	5.90	.377 793	16
45	9.587 386	5.03	9.964 826	.88	9.622 561	5.90	0.377 439	15
46	.587 688	5.02	.964 773	.88	.622 915	5.90	.377 085	14
47	.587 989	5.00	.964 720	.90	.623 269	5.90	.376 731	13
48	.588 289	5.02	.964 666	.88	.623 623	5.88	.376 377	12
49	.588 590	5.00	.964 613	.88	.623 976	5.90	.376 024	11
50	9.588 890	5.00	9.964 560	.88	9.624 330	5.88	0.375 670	10
51	.589 190	4.98	.964 507	.88	.624 683	5.88	.375 317	9
52	.589 489	5.00	.964 454	.90	.625 036	5.87	.374 964	8
53	.589 789	4.98	.964 400	.88	.625 388	5.88	.374 612	7
54	.590 088	4.98	.964 347	.88	.625 741	5.87	.374 259	6
55	9.590 387	4.98	9.964 294	.90	9.626 093	5.87	0.373 907	5
56	.590 686	4.97	.964 240	.88	.626 445	5.87	.373 555	4
57	.590 984	4.97	.964 187	.90	.626 797	5.87	.373 203	3
58	.591 282	4.97	.964 133	.88	.627 149	5.87	.372 851	2
59	.591 580	4.97	.964 080	.90	.627 501	5.85	.372 499	1
60	9.591 878		9.964 026		9.627 852		0.372 148	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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23°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.591 878	4.97	9.964 026	.90	9.627 852	5.85	0.372 148	60
1	.592 176	4.95	.963 972	.88	.628 203	5.85	.371 797	59
2	.592 473	4.95	.963 919	.90	.628 554	5.85	.371 446	58
3	.592 770	4.95	.963 865	.90	.628 905	5.83	.371 095	57
4	.593 067	4.93	.963 811	.90	.629 255	5.85	.370 745	56
5	9.593 363	4.93	9.963 757	.88	9.629 606	5.83	0.370 394	55
6	.593 659	4.93	.963 704	.90	.629 956	5.83	.370 044	54
7	.593 955	4.93	.963 650	.90	.630 306	5.83	.369 694	53
8	.594 251	4.93	.963 596	.90	.630 656	5.82	.369 344	52
9	.594 547	4.92	.963 542	.90	.631 005	5.83	.368 995	51
10	9.594 842	4.92	9.963 488	.90	9.631 355	5.82	0.368 645	50
11	.595 137	4.92	.963 434	.92	.631 704	5.82	.368 296	49
12	.595 432	4.92	.963 379	.90	.632 053	5.82	.367 947	48
13	.595 727	4.90	.963 325	.90	.632 402	5.80	.367 598	47
14	.596 021	4.90	.963 271	.90	.632 750	5.82	.367 250	46
15	9.596 315	4.90	9.963 217	.90	9.633 099	5.80	0.366 901	45
16	.596 609	4.90	.963 163	.92	.633 447	5.80	.366 553	44
17	.596 903	4.88	.963 108	.90	.633 795	5.80	.366 205	43
18	.597 196	4.90	.963 054	.92	.634 143	5.78	.365 857	42
19	.597 490	4.88	.962 999	.90	.634 490	5.80	.365 510	41
20	9.597 783	4.87	9.962 945	.92	9.634 838	5.78	0.365 162	40
21	.598 075	4.88	.962 890	.90	.635 185	5.78	.364 815	39
22	.598 368	4.87	.962 836	.92	.635 532	5.78	.364 468	38
23	.598 660	4.87	.962 781	.90	.635 879	5.78	.364 121	37
24	.598 952	4.87	.962 727	.92	.636 226	5.77	.363 774	36
25	9.599 244	4.87	9.962 672	.92	9.636 572	5.78	0.363 428	35
26	.599 536	4.85	.962 617	.92	.636 919	5.77	.363 081	34
27	.599 827	4.85	.962 562	.90	.637 265	5.77	.362 735	33
28	.600 118	4.85	.962 508	.92	.637 611	5.75	.362 389	32
29	.600 409	4.85	.962 453	.92	.637 956	5.77	.362 044	31
30	9.600 700	4.83	9.962 398	.92	9.638 302	5.75	0.361 698	30
31	.600 990	4.83	.962 343	.92	.638 647	5.75	.361 353	29
32	.601 280	4.83	.962 288	.92	.638 992	5.75	.361 008	28
33	.601 570	4.83	.962 233	.92	.639 337	5.75	.360 663	27
34	.601 860	4.83	.962 178	.92	.639 682	5.75	.360 318	26
35	9.602 150	4.82	9.962 123	.93	9.640 027	5.73	0.359 973	25
36	.602 439	4.82	.962 067	.92	.640 371	5.75	.359 629	24
37	.602 728	4.82	.962 012	.92	.640 716	5.73	.359 284	23
38	.603 017	4.80	.961 957	.92	.641 060	5.73	.358 940	22
39	.603 305	4.82	.961 902	.93	.641 404	5.72	.358 596	21
40	9.603 594	4.80	9.961 846	.92	9.641 747	5.73	0.358 253	20
41	.603 882	4.80	.961 791	.93	.642 091	5.72	.357 909	19
42	.604 170	4.78	.961 735	.92	.642 434	5.72	.357 566	18
43	.604 457	4.80	.961 680	.93	.642 777	5.72	.357 223	17
44	.604 745	4.78	.961 624	.92	.643 120	5.72	.356 880	16
45	9.605 032	4.78	9.961 569	.93	9.643 463	5.72	0.356 537	15
46	.605 319	4.78	.961 513	.92	.643 806	5.70	.356 194	14
47	.605 606	4.77	.961 458	.93	.644 148	5.70	.355 852	13
48	.605 892	4.78	.961 402	.93	.644 490	5.70	.355 510	12
49	.606 179	4.77	.961 346	.93	.644 832	5.70	.355 168	11
50	9.606 465	4.77	9.961 290	.92	9.645 174	5.70	0.354 826	10
51	.606 751	4.75	.961 235	.93	.645 516	5.68	.354 484	9
52	.607 036	4.77	.961 179	.93	.645 857	5.70	.354 143	8
53	.607 322	4.75	.961 123	.93	.646 199	5.68	.353 801	7
54	.607 607	4.75	.961 067	.93	.646 540	5.68	.353 460	6
55	9.607 892	4.75	9.961 011	.93	9.646 881	5.68	0.353 119	5
56	.608 177	4.73	.960 955	.93	.647 222	5.67	.352 778	4
57	.608 461	4.73	.960 899	.93	.647 562	5.68	.352 438	3
58	.608 745	4.73	.960 843	.95	.647 903	5.67	.352 097	2
59	.609 029	4.73	.960 786	.93	.648 243	5.67	.351 757	1
60	9.609 313		9.960 730		9.648 583		0.351 417	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

66°

24°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.609 313	4.73	9.960 730	.93	9.648 583	5.67	0.351 417	60
1	.609 597	4.72	.960 674	.93	.648 923	5.67	.351 077	59
2	.609 880	4.73	.960 618	.95	.649 263	5.65	.350 737	58
3	.610 164	4.72	.960 561	.93	.649 602	5.67	.350 398	57
4	.610 447	4.70	.960 505	.95	.649 942	5.65	.350 058	56
5	9.610 729	4.72	9.960 448	.93	9.650 281	5.65	0.349 719	55
6	.611 012	4.70	.960 392	.95	.650 620	5.65	.349 380	54
7	.611 294	4.70	.960 335	.93	.650 959	5.63	.349 041	53
8	.611 576	4.70	.960 279	.95	.651 297	5.65	.348 703	52
9	.611 858	4.70	.960 222	.95	.651 636	5.63	.348 364	51
10	9.612 140	4.68	9.960 165	.93	9.651 974	5.63	0.348 026	50
11	.612 421	4.68	.960 109	.95	.652 312	5.63	.347 688	49
12	.612 702	4.68	.960 052	.95	.652 650	5.63	.347 350	48
13	.612 983	4.68	.959 995	.95	.652 988	5.63	.347 012	47
14	.613 264	4.68	.959 938	.93	.653 326	5.62	.346 674	46
15	9.613 545	4.67	9.959 882	.95	9.653 663	5.62	0.346 337	45
16	.613 825	4.67	.959 825	.95	.654 000	5.62	.346 000	44
17	.614 105	4.67	.959 768	.95	.654 337	5.62	.345 663	43
18	.614 385	4.67	.959 711	.95	.654 674	5.62	.345 326	42
19	.614 665	4.65	.959 654	.97	.655 011	5.62	.344 989	41
20	9.614 944	4.65	9.959 596	.95	9.655 348	5.60	0.344 652	40
21	.615 223	4.65	.959 539	.95	.655 684	5.60	.344 316	39
22	.615 502	4.65	.959 482	.95	.656 020	5.60	.343 980	38
23	.615 781	4.65	.959 425	.95	.656 356	5.60	.343 644	37
24	.616 060	4.63	.959 368	.97	.656 692	5.60	.343 308	36
25	9.616 338	4.63	9.959 310	.95	9.657 028	5.60	0.342 972	35
26	.616 616	4.63	.959 253	.97	.657 364	5.58	.342 636	34
27	.616 894	4.63	.959 195	.95	.657 699	5.58	.342 301	33
28	.617 172	4.63	.959 138	.97	.658 034	5.58	.341 966	32
29	.617 450	4.62	.959 080	.95	.658 369	5.58	.341 631	31
30	9.617 727	4.62	9.959 023	.97	9.658 704	5.58	0.341 296	30
31	.618 004	4.62	.958 965	.95	.659 039	5.57	.340 961	29
32	.618 281	4.62	.958 908	.97	.659 373	5.58	.340 627	28
33	.618 558	4.60	.958 850	.97	.659 708	5.57	.340 292	27
34	.618 834	4.60	.958 792	.97	.660 042	5.57	.339 958	26
35	9.619 110	4.60	9.958 734	.95	9.660 376	5.57	0.339 624	25
36	.619 386	4.60	.958 677	.97	.660 710	5.55	.339 290	24
37	.619 662	4.60	.958 619	.97	.661 043	5.57	.338 957	23
38	.619 938	4.58	.958 561	.97	.661 377	5.55	.338 623	22
39	.620 213	4.58	.958 503	.97	.661 710	5.55	.338 290	21
40	9.620 488	4.58	9.958 445	.97	9.662 043	5.55	0.337 957	20
41	.620 763	4.58	.958 387	.97	.662 376	5.55	.337 624	19
42	.621 038	4.58	.958 329	.97	.662 709	5.55	.337 291	18
43	.621 313	4.57	.958 271	.97	.663 042	5.55	.336 958	17
44	.621 587	4.57	.958 213	.98	.663 375	5.53	.336 625	16
45	9.621 861	4.57	9.958 154	.97	9.663 707	5.53	0.336 293	15
46	.622 135	4.57	.958 096	.97	.664 039	5.53	.335 961	14
47	.622 409	4.55	.958 038	.98	.664 371	5.53	.335 629	13
48	.622 682	4.57	.957 979	.97	.664 703	5.53	.335 297	12
49	.622 956	4.55	.957 921	.97	.665 035	5.52	.334 965	11
50	9.623 229	4.55	9.957 863	.98	9.665 366	5.53	0.334 634	10
51	.623 502	4.53	.957 804	.97	.665 698	5.52	.334 302	9
52	.623 774	4.55	.957 746	.98	.666 029	5.52	.333 971	8
53	.624 047	4.53	.957 687	.98	.666 360	5.52	.333 640	7
54	.624 319	4.53	.957 628	.97	.666 691	5.50	.333 309	6
55	9.624 591	4.53	9.957 570	.98	9.667 021	5.52	0.332 979	5
56	.624 863	4.53	.957 511	.98	.667 352	5.50	.332 648	4
57	.625 135	4.52	.957 452	.98	.667 682	5.52	.332 318	3
58	.625 406	4.52	.957 393	.97	.668 013	5.50	.331 987	2
59	.625 677	4.52	.957 335	.98	.668 343	5.50	.331 657	1
60	9.625 948		9.957 276		9.668 673		0.331 327	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

65°

25°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.625 948	4.52	9.957 276	.98	9.668 673	5.48	0.331 327	60
1	.626 219	4.52	.957 217	.98	.669 002	5.50	.330 998	59
2	.626 490	4.50	.957 158	.98	.669 332	5.48	.330 668	58
3	.626 760	4.50	.957 099	.98	.669 661	5.50	.330 339	57
4	.627 030	4.50	.957 040	.98	.669 991	5.48	.330 009	56
5	9.627 300	4.50	9.956 981	1.00	9.670 320	5.48	0.329 680	55
6	.627 570	4.50	.956 921	.98	.670 649	5.47	.329 351	54
7	.627 840	4.48	.956 862	.98	.670 977	5.48	.329 023	53
8	.628 109	4.48	.956 803	.98	.671 306	5.48	.328 694	52
9	.628 378	4.48	.956 744	1.00	.671 635	5.47	.328 365	51
10	9.628 647	4.48	9.956 684	.98	9.671 963	5.47	0.328 037	50
11	.628 916	4.48	.956 625	.98	.672 291	5.47	.327 709	49
12	.629 185	4.47	.956 566	1.00	.672 619	5.47	.327 381	48
13	.629 453	4.47	.956 506	.98	.672 947	5.45	.327 053	47
14	.629 721	4.47	.956 447	1.00	.673 274	5.47	.326 726	46
15	9.629 989	4.47	9.956 387	1.00	9.673 602	5.45	0.326 398	45
16	.630 257	4.45	.956 327	.98	.673 929	5.47	.326 071	44
17	.630 524	4.47	.956 268	1.00	.674 257	5.45	.325 743	43
18	.630 792	4.45	.956 208	1.00	.674 584	5.45	.325 416	42
19	.631 059	4.45	.956 148	.98	.674 911	5.43	.325 089	41
20	9.631 326	4.45	9.956 089	1.00	9.675 237	5.45	0.324 763	40
21	.631 593	4.43	.956 029	1.00	.675 564	5.43	.324 436	39
22	.631 859	4.43	.955 969	1.00	.675 890	5.45	.324 110	38
23	.632 125	4.45	.955 909	1.00	.676 217	5.43	.323 783	37
24	.632 392	4.43	.955 849	1.00	.676 543	5.43	.323 457	36
25	9.632 658	4.42	9.955 789	1.00	9.676 869	5.42	0.323 131	35
26	.632 923	4.43	.955 729	1.00	.677 194	5.43	.322 806	34
27	.633 189	4.42	.955 669	1.00	.677 520	5.43	.322 480	33
28	.633 454	4.42	.955 609	1.02	.677 846	5.42	.322 154	32
29	.633 719	4.42	.955 548	1.00	.678 171	5.42	.321 829	31
30	9.633 984	4.42	9.955 488	1.00	9.678 496	5.42	0.321 504	30
31	.634 249	4.42	.955 428	1.00	.678 821	5.42	.321 179	29
32	.634 514	4.40	.955 368	1.02	.679 146	5.42	.320 854	28
33	.634 778	4.40	.955 307	1.00	.679 471	5.40	.320 529	27
34	.635 042	4.40	.955 247	1.02	.679 795	5.42	.320 205	26
35	9.635 306	4.40	9.955 186	1.00	9.680 120	5.40	0.319 880	25
36	.635 570	4.40	.955 126	1.02	.680 444	5.40	.319 556	24
37	.635 834	4.38	.955 065	1.00	.680 768	5.40	.319 232	23
38	.636 097	4.38	.955 005	1.02	.681 092	5.40	.318 908	22
39	.636 360	4.38	.954 944	1.02	.681 416	5.40	.318 584	21
40	9.636 623	4.38	9.954 883	1.00	9.681 740	5.38	0.318 260	20
41	.636 886	4.37	.954 823	1.02	.682 063	5.40	.317 937	19
42	.637 148	4.38	.954 762	1.02	.682 387	5.38	.317 613	18
43	.637 411	4.37	.954 701	1.02	.682 710	5.38	.317 290	17
44	.637 673	4.37	.954 640	1.02	.683 033	5.38	.316 967	16
45	9.637 935	4.37	9.954 579	1.02	9.683 356	5.38	0.316 644	15
46	.638 197	4.35	.954 518	1.02	.683 679	5.37	.316 321	14
47	.638 458	4.37	.954 457	1.02	.684 001	5.38	.315 999	13
48	.638 720	4.35	.954 396	1.02	.684 324	5.37	.315 676	12
49	.638 981	4.35	.954 335	1.02	.684 646	5.37	.315 354	11
50	9.639 242	4.35	9.954 274	1.02	9.684 968	5.37	0.315 032	10
51	.639 503	4.35	.954 213	1.02	.685 290	5.37	.314 710	9
52	.639 764	4.33	.954 152	1.03	.685 612	5.37	.314 388	8
53	.640 024	4.33	.954 090	1.02	.685 934	5.35	.314 066	7
54	.640 284	4.33	.954 029	1.02	.686 255	5.37	.313 745	6
55	9.640 544	4.33	9.953 968	1.03	9.686 577	5.35	0.313 423	5
56	.640 804	4.33	.953 906	1.02	.686 898	5.35	.313 102	4
57	.641 064	4.33	.953 845	1.03	.687 219	5.35	.312 781	3
58	.641 324	4.32	.953 783	1.02	.687 540	5.35	.312 460	2
59	.641 583	4.32	.953 722	1.03	.687 861	5.35	.312 139	1
60	9.641 842		9.953 660		9.688 182		0.311 818	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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26°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.641 842	4.32	9.953 660	1.02	9.688 182	5.33	0.311 818	60
1	.642 101	4.32	.953 599	1.03	.688 502	5.35	.311 498	59
2	.642 360	4.30	.953 537	1.03	.688 823	5.33	.311 177	58
3	.642 618	4.32	.953 475	1.03	.689 143	5.33	.310 857	57
4	.642 877	4.30	.953 413	1.02	.689 463	5.33	.310 537	56
5	9.643 135	4.30	9.953 352	1.03	9.689 783	5.33	0.310 217	55
6	.643 393	4.28	.953 290	1.03	.690 103	5.33	.309 897	54
7	.643 650	4.30	.953 228	1.03	.690 423	5.32	.309 577	53
8	.643 908	4.28	.953 166	1.03	.690 742	5.33	.309 258	52
9	.644 165	4.30	.953 104	1.03	.691 062	5.32	.308 938	51
10	9.644 423	4.28	9.953 042	1.03	9.691 381	5.32	0.308 619	50
11	.644 680	4.27	.952 980	1.03	.691 700	5.32	.308 300	49
12	.644 936	4.28	.952 918	1.05	.692 019	5.32	.307 981	48
13	.645 193	4.28	.952 855	1.03	.692 338	5.30	.307 662	47
14	.645 450	4.27	.952 793	1.03	.692 656	5.32	.307 344	46
15	9.645 706	4.27	9.952 731	1.03	9.692 975	5.30	0.307 025	45
16	.645 962	4.27	.952 669	1.05	.693 293	5.32	.306 707	44
17	.646 218	4.27	.952 606	1.03	.693 612	5.30	.306 388	43
18	.646 474	4.25	.952 544	1.05	.693 930	5.30	.306 070	42
19	.646 729	4.25	.952 481	1.03	.694 248	5.30	.305 752	41
20	9.646 984	4.27	9.952 419	1.05	9.694 566	5.28	0.305 434	40
21	.647 240	4.23	.952 356	1.03	.694 883	5.30	.305 117	39
22	.647 494	4.25	.952 294	1.05	.695 201	5.28	.304 799	38
23	.647 749	4.25	.952 231	1.05	.695 518	5.30	.304 482	37
24	.648 004	4.23	.952 168	1.03	.695 836	5.28	.304 164	36
25	9.648 258	4.23	9.952 106	1.05	9.696 153	5.28	0.303 847	35
26	.648 512	4.23	.952 043	1.05	.696 470	5.28	.303 530	34
27	.648 766	4.23	.951 980	1.05	.696 787	5.27	.303 213	33
28	.649 020	4.23	.951 917	1.05	.697 103	5.28	.302 897	32
29	.649 274	4.22	.951 854	1.05	.697 420	5.27	.302 580	31
30	9.649 527	4.23	9.951 791	1.05	9.697 736	5.28	0.302 264	30
31	.649 781	4.22	.951 728	1.05	.698 053	5.27	.301 947	29
32	.650 034	4.22	.951 665	1.05	.698 369	5.27	.301 631	28
33	.650 287	4.20	.951 602	1.05	.698 685	5.27	.301 315	27
34	.650 539	4.22	.951 539	1.05	.699 001	5.25	.300 999	26
35	9.650 792	4.20	9.951 476	1.07	9.699 316	5.27	0.300 684	25
36	.651 044	4.22	.951 412	1.05	.699 632	5.25	.300 368	24
37	.651 297	4.20	.951 349	1.05	.699 947	5.27	.300 053	23
38	.651 549	4.18	.951 286	1.07	.700 263	5.25	.299 737	22
39	.651 800	4.20	.951 222	1.05	.700 578	5.25	.299 422	21
40	9.652 052	4.20	9.951 159	1.05	9.700 893	5.25	0.299 107	20
41	.652 304	4.18	.951 096	1.07	.701 208	5.25	.298 792	19
42	.652 555	4.18	.951 032	1.07	.701 523	5.23	.298 477	18
43	.652 806	4.18	.950 968	1.05	.701 837	5.25	.298 163	17
44	.653 057	4.18	.950 905	1.07	.702 152	5.23	.297 848	16
45	9.653 308	4.17	9.950 841	1.05	9.702 466	5.25	0.297 534	15
46	.653 558	4.17	.950 778	1.07	.702 781	5.23	.297 219	14
47	.653 808	4.18	.950 714	1.07	.703 095	5.23	.296 905	13
48	.654 059	4.17	.950 650	1.07	.703 409	5.22	.296 591	12
49	.654 309	4.15	.950 586	1.07	.703 722	5.23	.296 278	11
50	9.654 558	4.17	9.950 522	1.07	9.704 036	5.23	0.295 964	10
51	.654 808	4.17	.950 458	1.07	.704 350	5.22	.295 650	9
52	.655 058	4.15	.950 394	1.07	.704 663	5.22	.295 337	8
53	.655 307	4.15	.950 330	1.07	.704 976	5.23	.295 024	7
54	.655 556	4.15	.950 266	1.07	.705 290	5.22	.294 710	6
55	9.655 805	4.15	9.950 202	1.07	9.705 603	5.22	0.294 397	5
56	.656 054	4.13	.950 138	1.07	.705 916	5.20	.294 084	4
57	.656 302	4.15	.950 074	1.07	.706 228	5.22	.293 772	3
58	.656 551	4.13	.950 010	1.08	.706 541	5.22	.293 459	2
59	.656 799	4.13	.949 945	1.07	.706 854	5.20	.293 146	1
60	9.657 047		9.949 881		9.707 166		0.292 834	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

63°

27°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.657 047	4.13	9.949 881	1.08	9.707 166	5.20	0.292 834	60
1	.657 295	4.12	.949 816	1.07	.707 478	5.20	.292 522	59
2	.657 542	4.13	.949 752	1.07	.707 790	5.20	.292 210	58
3	.657 790	4.12	.949 688	1.08	.708 102	5.20	.291 898	57
4	.658 037	4.12	.949 623	1.08	.708 414	5.20	.291 586	56
5	9.658 284	4.12	9.949 558	1.07	9.708 726	5.18	0.291 274	55
6	.658 531	4.12	.949 494	1.08	.709 037	5.20	.290 963	54
7	.658 778	4.12	.949 429	1.08	.709 349	5.18	.290 651	53
8	.659 025	4.10	.949 364	1.07	.709 660	5.18	.290 340	52
9	.659 271	4.10	.949 300	1.08	.709 971	5.18	.290 029	51
10	9.659 517	4.10	9.949 235	1.08	9.710 282	5.18	0.289 718	50
11	.659 763	4.10	.949 170	1.08	.710 593	5.18	.289 407	49
12	.660 009	4.10	.949 105	1.08	.710 904	5.18	.289 096	48
13	.660 255	4.10	.949 040	1.08	.711 215	5.17	.288 785	47
14	.660 501	4.08	.948 975	1.08	.711 525	5.18	.288 475	46
15	9.660 746	4.08	9.948 910	1.08	9.711 836	5.17	0.288 164	45
16	.660 991	4.08	.948 845	1.08	.712 146	5.17	.287 854	44
17	.661 236	4.08	.948 780	1.08	.712 456	5.17	.287 544	43
18	.661 481	4.08	.948 715	1.08	.712 766	5.17	.287 234	42
19	.661 726	4.07	.948 650	1.10	.713 076	5.17	.286 924	41
20	9.661 970	4.07	9.948 584	1.08	9.713 386	5.17	0.286 614	40
21	.662 214	4.08	.948 519	1.08	.713 696	5.15	.286 304	39
22	.662 459	4.07	.948 454	1.10	.714 005	5.15	.285 995	38
23	.662 703	4.05	.948 388	1.08	.714 314	5.17	.285 686	37
24	.662 946	4.07	.948 323	1.10	.714 624	5.15	.285 376	36
25	9.663 190	4.05	9.948 257	1.08	9.714 933	5.15	0.285 067	35
26	.663 433	4.07	.948 192	1.10	.715 242	5.15	.284 758	34
27	.663 677	4.05	.948 126	1.10	.715 551	5.15	.284 449	33
28	.663 920	4.05	.948 060	1.08	.715 860	5.13	.284 140	32
29	.664 163	4.05	.947 995	1.10	.716 168	5.15	.283 832	31
30	9.664 406	4.03	9.947 929	1.10	9.716 477	5.13	0.283 523	30
31	.664 648	4.05	.947 863	1.10	.716 785	5.13	.283 215	29
32	.664 891	4.03	.947 797	1.10	.717 093	5.13	.282 907	28
33	.665 133	4.03	.947 731	1.10	.717 401	5.13	.282 599	27
34	.665 375	4.03	.947 665	1.08	.717 709	5.13	.282 291	26
35	9.665 617	4.03	9.947 600	1.12	9.718 017	5.13	0.281 983	25
36	.665 859	4.02	.947 533	1.10	.718 325	5.13	.281 675	24
37	.666 100	4.03	.947 467	1.10	.718 633	5.12	.281 367	23
38	.666 342	4.02	.947 401	1.10	.718 940	5.13	.281 060	22
39	.666 583	4.02	.947 335	1.10	.719 248	5.12	.280 752	21
40	9.666 824	4.02	9.947 269	1.10	9.719 555	5.12	0.280 445	20
41	.667 065	4.00	.947 203	1.12	.719 862	5.12	.280 138	19
42	.667 305	4.02	.947 136	1.10	.720 169	5.12	.279 831	18
43	.667 546	4.00	.947 070	1.10	.720 476	5.12	.279 524	17
44	.667 786	4.02	.947 004	1.12	.720 783	5.10	.279 217	16
45	9.668 027	4.00	9.946 937	1.10	9.721 089	5.12	0.278 911	15
46	.668 267	3.98	.946 871	1.12	.721 396	5.10	.278 604	14
47	.668 506	4.00	.946 804	1.10	.721 702	5.12	.278 298	13
48	.668 746	4.00	.946 738	1.12	.722 009	5.10	.277 991	12
49	.668 986	3.98	.946 671	1.12	.722 315	5.10	.277 685	11
50	9.669 225	3.98	9.946 604	1.10	9.722 621	5.10	0.277 379	10
51	.669 464	3.98	.946 538	1.12	.722 927	5.08	.277 073	9
52	.669 703	3.98	.946 471	1.12	.723 232	5.10	.276 768	8
53	.669 942	3.98	.946 404	1.12	.723 538	5.10	.276 462	7
54	.670 181	3.97	.946 337	1.12	.723 844	5.08	.276 156	6
55	9.670 419	3.98	9.946 270	1.12	9.724 149	5.08	0.275 851	5
56	.670 658	3.97	.946 203	1.12	.724 454	5.10	.275 546	4
57	.670 896	3.97	.946 136	1.12	.724 760	5.08	.275 240	3
58	.671 134	3.97	.946 069	1.12	.725 065	5.08	.274 935	2
59	.671 372	3.95	.946 002	1.12	.725 370	5.07	.274 630	1
60	9.671 609		9.945 935		9.725 674		0.274 326	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

62°

28°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.671 609	3.97	9.945 935	1.12	9.725 674	5.08	0.274 326	60
1	.671 847	3.95	.945 868	1.13	.725 979	5.08	.274 021	59
2	.672 084	3.95	.945 800	1.12	.726 284	5.07	.273 716	58
3	.672 321	3.95	.945 733	1.12	.726 588	5.07	.273 412	57
4	.672 558	3.95	.945 666	1.13	.726 892	5.08	.273 108	56
5	9.672 795	3.95	9.945 598	1.12	9.727 197	5.07	0.272 803	55
6	.673 032	3.93	.945 531	1.12	.727 501	5.07	.272 499	54
7	.673 268	3.95	.945 464	1.13	.727 805	5.07	.272 195	53
8	.673 505	3.93	.945 396	1.13	.728 109	5.05	.271 891	52
9	.673 741	3.93	.945 328	1.12	.728 412	5.07	.271 588	51
10	9.673 977	3.93	9.945 261	1.13	9.728 716	5.07	0.271 284	50
11	.674 213	3.92	.945 193	1.13	.729 020	5.05	.270 980	49
12	.674 448	3.93	.945 125	1.12	.729 323	5.05	.270 677	48
13	.674 684	3.92	.945 058	1.13	.729 626	5.05	.270 374	47
14	.674 919	3.93	.944 990	1.13	.729 929	5.07	.270 071	46
15	9.675 155	3.92	9.944 922	1.13	9.730 233	5.03	0.269 767	45
16	.675 390	3.90	.944 854	1.13	.730 535	5.05	.269 465	44
17	.675 624	3.92	.944 786	1.13	.730 838	5.05	.269 162	43
18	.675 859	3.92	.944 718	1.13	.731 141	5.05	.268 859	42
19	.676 094	3.90	.944 650	1.13	.731 444	5.03	.268 556	41
20	9.676 328	3.90	9.944 582	1.13	9.731 746	5.03	0.268 254	40
21	.676 562	3.90	.944 514	1.13	.732 048	5.05	.267 952	39
22	.676 796	3.90	.944 446	1.15	.732 351	5.03	.267 649	38
23	.677 030	3.90	.944 377	1.13	.732 653	5.03	.267 347	37
24	.677 264	3.90	.944 309	1.13	.732 955	5.03	.267 045	36
25	9.677 498	3.88	9.944 241	1.15	9.733 257	5.02	0.266 743	35
26	.677 731	3.88	.944 172	1.13	.733 558	5.03	.266 442	34
27	.677 964	3.88	.944 104	1.13	.733 860	5.03	.266 140	33
28	.678 197	3.88	.944 036	1.15	.734 162	5.02	.265 838	32
29	.678 430	3.88	.943 967	1.13	.734 463	5.02	.265 537	31
30	9.678 663	3.87	9.943 899	1.15	9.734 764	5.03	0.265 236	30
31	.678 895	3.88	.943 830	1.15	.735 066	5.02	.264 934	29
32	.679 128	3.87	.943 761	1.13	.735 367	5.02	.264 633	28
33	.679 360	3.87	.943 693	1.15	.735 668	5.02	.264 332	27
34	.679 592	3.87	.943 624	1.15	.735 969	5.00	.264 031	26
35	9.679 824	3.87	9.943 555	1.15	9.736 269	5.02	0.263 731	25
36	.680 056	3.87	.943 486	1.15	.736 570	5.00	.263 430	24
37	.680 288	3.85	.943 417	1.15	.736 870	5.02	.263 130	23
38	.680 519	3.85	.943 348	1.15	.737 171	5.00	.262 829	22
39	.680 750	3.87	.943 279	1.15	.737 471	5.00	.262 529	21
40	9.680 982	3.85	9.943 210	1.15	9.737 771	5.00	0.262 229	20
41	.681 213	3.83	.943 141	1.15	.738 071	5.00	.261 929	19
42	.681 443	3.85	.943 072	1.15	.738 371	5.00	.261 629	18
43	.681 674	3.85	.943 003	1.15	.738 671	5.00	.261 329	17
44	.681 905	3.83	.942 934	1.17	.738 971	5.00	.261 029	16
45	9.682 135	3.83	9.942 864	1.15	9.739 271	4.98	0.260 729	15
46	.682 365	3.83	.942 795	1.15	.739 570	5.00	.260 430	14
47	.682 595	3.83	.942 726	1.17	.739 870	4.98	.260 130	13
48	.682 825	3.83	.942 656	1.15	.740 169	4.98	.259 831	12
49	.683 055	3.82	.942 587	1.17	.740 468	4.98	.259 532	11
50	9.683 284	3.83	9.942 517	1.15	9.740 767	4.98	0.259 233	10
51	.683 514	3.82	.942 448	1.17	.741 066	4.98	.258 934	9
52	.683 743	3.82	.942 378	1.17	.741 365	4.98	.258 635	8
53	.683 972	3.82	.942 308	1.15	.741 664	4.97	.258 336	7
54	.684 201	3.82	.942 239	1.17	.741 962	4.98	.258 038	6
55	9.684 430	3.80	9.942 169	1.17	9.742 261	4.97	0.257 739	5
56	.684 658	3.82	.942 099	1.17	.742 559	4.98	.257 441	4
57	.684 887	3.80	.942 029	1.17	.742 858	4.97	.257 142	3
58	.685 115	3.80	.941 959	1.17	.743 156	4.97	.256 844	2
59	.685 343	3.80	.941 889	1.17	.743 454	4.97	.256 546	1
60	9.685 571		9.941 819		9.743 752		0.256 248	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

61°

29°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.685 571	3.80	9.941 819	1.17	9.743 752	4.97	0.256 248	60
1	.685 799	3.80	.941 749	1.17	.744 050	4.97	.255 950	59
2	.686 027	3.78	.941 679	1.17	.744 348	4.95	.255 652	58
3	.686 254	3.80	.941 609	1.17	.744 645	4.97	.255 355	57
4	.686 482	3.78	.941 539	1.17	.744 943	4.95	.255 057	56
5	9.686 709	3.78	9.941 469	1.18	9.745 240	4.97	0.254 760	55
6	.686 936	3.78	.941 398	1.17	.745 538	4.95	.254 462	54
7	.687 163	3.77	.941 328	1.17	.745 835	4.95	.254 165	53
8	.687 389	3.78	.941 258	1.18	.746 132	4.95	.253 868	52
9	.687 616	3.78	.941 187	1.17	.746 429	4.95	.253 571	51
10	9.687 843	3.77	9.941 117	1.18	9.746 726	4.95	0.253 274	50
11	.688 069	3.77	.941 046	1.18	.747 023	4.93	.252 977	49
12	.688 295	3.77	.940 975	1.17	.747 319	4.95	.252 681	48
13	.688 521	3.77	.940 905	1.18	.747 616	4.95	.252 384	47
14	.688 747	3.75	.940 834	1.18	.747 913	4.93	.252 087	46
15	9.688 972	3.77	9.940 763	1.17	9.748 209	4.93	0.251 791	45
16	.689 198	3.75	.940 693	1.18	.748 505	4.93	.251 495	44
17	.689 423	3.75	.940 622	1.18	.748 801	4.93	.251 199	43
18	.689 648	3.75	.940 551	1.18	.749 097	4.93	.250 903	42
19	.689 873	3.75	.940 480	1.18	.749 393	4.93	.250 607	41
20	9.690 098	3.75	9.940 409	1.18	9.749 689	4.93	0.250 311	40
21	.690 323	3.75	.940 338	1.18	.749 985	4.93	.250 015	39
22	.690 548	3.73	.940 267	1.18	.750 281	4.92	.249 719	38
23	.690 772	3.73	.940 196	1.18	.750 576	4.93	.249 424	37
24	.690 996	3.73	.940 125	1.18	.750 872	4.92	.249 128	36
25	9.691 220	3.73	9.940 054	1.20	9.751 167	4.92	0.248 833	35
26	.691 444	3.73	.939 982	1.18	.751 462	4.92	.248 538	34
27	.691 668	3.73	.939 911	1.18	.751 757	4.92	.248 243	33
28	.691 892	3.72	.939 840	1.20	.752 052	4.92	.247 948	32
29	.692 115	3.73	.939 768	1.18	.752 347	4.92	.247 653	31
30	9.692 339	3.72	9.939 697	1.20	9.752 642	4.92	0.247 358	30
31	.692 562	3.72	.939 625	1.18	.752 937	4.90	.247 063	29
32	.692 785	3.72	.939 554	1.20	.753 231	4.92	.246 769	28
33	.693 008	3.72	.939 482	1.20	.753 526	4.90	.246 474	27
34	.693 231	3.70	.939 410	1.18	.753 820	4.92	.246 180	26
35	9.693 453	3.72	9.939 339	1.20	9.754 115	4.90	0.245 885	25
36	.693 676	3.70	.939 267	1.20	.754 409	4.90	.245 591	24
37	.693 898	3.70	.939 195	1.20	.754 703	4.90	.245 297	23
38	.694 120	3.70	.939 123	1.18	.754 997	4.90	.245 003	22
39	.694 342	3.70	.939 052	1.20	.755 291	4.90	.244 709	21
40	9.694 564	3.70	9.938 980	1.20	9.755 585	4.88	0.244 415	20
41	.694 786	3.68	.938 908	1.20	.755 878	4.90	.244 122	19
42	.695 007	3.70	.938 836	1.22	.756 172	4.88	.243 828	18
43	.695 229	3.68	.938 763	1.20	.756 465	4.90	.243 535	17
44	.695 450	3.68	.938 691	1.20	.756 759	4.88	.243 241	16
45	9.695 671	3.68	9.938 619	1.20	9.757 052	4.88	0.242 948	15
46	.695 892	3.68	.938 547	1.20	.757 345	4.88	.242 655	14
47	.696 113	3.68	.938 475	1.22	.757 638	4.88	.242 362	13
48	.696 334	3.67	.938 402	1.20	.757 931	4.88	.242 069	12
49	.696 554	3.68	.938 330	1.20	.758 224	4.88	.241 776	11
50	9.696 775	3.67	9.938 258	1.22	9.758 517	4.88	0.241 483	10
51	.696 995	3.67	.938 185	1.20	.758 810	4.87	.241 190	9
52	.697 215	3.67	.938 113	1.22	.759 102	4.88	.240 898	8
53	.697 435	3.65	.938 040	1.22	.759 395	4.87	.240 605	7
54	.697 654	3.67	.937 967	1.20	.759 687	4.87	.240 313	6
55	9.697 874	3.67	9.937 895	1.22	9.759 979	4.88	0.240 021	5
56	.698 094	3.65	.937 822	1.22	.760 272	4.87	.239 728	4
57	.698 313	3.65	.937 749	1.22	.760 564	4.87	.239 436	3
58	.698 532	3.65	.937 676	1.20	.760 856	4.87	.239 144	2
59	.698 751	3.65	.937 604	1.22	.761 148	4.85	.238 852	1
60	9.698 970		9.937 531		9.761 439		0.238 561	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

60°

30°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.698 970	3.65	9.937 531	1.22	9.761 439	4.87	0.238 561	60
1	.699 189	3.63	.937 458	1.22	.761 731	4.87	.238 269	59
2	.699 407	3.65	.937 385	1.22	.762 023	4.85	.237 977	58
3	.699 626	3.63	.937 312	1.23	.762 314	4.87	.237 686	57
4	.699 844	3.63	.937 238	1.22	.762 606	4.85	.237 394	56
5	9.700 062	3.63	9.937 165	1.22	9.762 897	4.85	0.237 103	55
6	.700 280	3.63	.937 092	1.22	.763 188	4.85	.236 812	54
7	.700 498	3.63	.937 019	1.22	.763 479	4.85	.236 521	53
8	.700 716	3.62	.936 946	1.23	.763 770	4.85	.236 230	52
9	.700 933	3.63	.936 872	1.22	.764 061	4.85	.235 939	51
10	9.701 151	3.62	9.936 799	1.23	9.764 352	4.85	0.235 648	50
11	.701 368	3.62	.936 725	1.22	.764 643	4.83	.235 357	49
12	.701 585	3.62	.936 652	1.23	.764 933	4.85	.235 067	48
13	.701 802	3.62	.936 578	1.22	.765 224	4.83	.234 776	47
14	.702 019	3.62	.936 505	1.23	.765 514	4.85	.234 486	46
15	9.702 236	3.60	9.936 431	1.23	9.765 805	4.83	0.234 195	45
16	.702 452	3.62	.936 357	1.22	.766 095	4.83	.233 905	44
17	.702 669	3.60	.936 284	1.23	.766 385	4.83	.233 615	43
18	.702 885	3.60	.936 210	1.23	.766 675	4.83	.233 325	42
19	.703 101	3.60	.936 136	1.23	.766 965	4.83	.233 035	41
20	9.703 317	3.60	9.936 062	1.23	9.767 255	4.83	0.232 745	40
21	.703 533	3.60	.935 988	1.23	.767 545	4.82	.232 455	39
22	.703 749	3.58	.935 914	1.23	.767 834	4.83	.232 166	38
23	.703 964	3.58	.935 840	1.23	.768 124	4.83	.231 876	37
24	.704 179	3.60	.935 766	1.23	.768 414	4.82	.231 586	36
25	9.704 395	3.58	9.935 692	1.23	9.768 703	4.82	0.231 297	35
26	.704 610	3.58	.935 618	1.25	.768 992	4.82	.231 008	34
27	.704 825	3.58	.935 543	1.23	.769 281	4.83	.230 719	33
28	.705 040	3.57	.935 469	1.23	.769 571	4.82	.230 429	32
29	.705 254	3.58	.935 395	1.25	.769 860	4.80	.230 140	31
30	9.705 469	3.57	9.935 320	1.23	9.770 148	4.82	0.229 852	30
31	.705 683	3.58	.935 246	1.25	.770 437	4.82	.229 563	29
32	.705 898	3.57	.935 171	1.23	.770 726	4.82	.229 274	28
33	.706 112	3.57	.935 097	1.25	.771 015	4.80	.228 985	27
34	.706 326	3.55	.935 022	1.23	.771 303	4.82	.228 697	26
35	9.706 539	3.57	9.934 948	1.25	9.771 592	4.80	0.228 408	25
36	.706 753	3.57	.934 873	1.25	.771 880	4.80	.228 120	24
37	.706 967	3.55	.934 798	1.25	.772 168	4.82	.227 832	23
38	.707 180	3.55	.934 723	1.23	.772 457	4.80	.227 543	22
39	.707 393	3.55	.934 649	1.25	.772 745	4.80	.227 255	21
40	9.707 606	3.55	9.934 574	1.25	9.773 033	4.80	0.226 967	20
41	.707 819	3.55	.934 499	1.25	.773 321	4.78	.226 679	19
42	.708 032	3.55	.934 424	1.25	.773 608	4.80	.226 392	18
43	.708 245	3.55	.934 349	1.25	.773 896	4.80	.226 104	17
44	.708 458	3.53	.934 274	1.25	.774 184	4.78	.225 816	16
45	9.708 670	3.53	9.934 199	1.27	9.774 471	4.80	0.225 529	15
46	.708 882	3.53	.934 123	1.25	.774 759	4.78	.225 241	14
47	.709 094	3.53	.934 048	1.25	.775 046	4.78	.224 954	13
48	.709 306	3.53	.933 973	1.25	.775 333	4.80	.224 667	12
49	.709 518	3.53	.933 898	1.27	.775 621	4.78	.224 379	11
50	9.709 730	3.52	9.933 822	1.25	9.775 908	4.78	0.224 092	10
51	.709 941	3.53	.933 747	1.27	.776 195	4.78	.223 805	9
52	.710 153	3.52	.933 671	1.25	.776 482	4.77	.223 518	8
53	.710 364	3.52	.933 596	1.27	.776 768	4.78	.223 232	7
54	.710 575	3.52	.933 520	1.25	.777 055	4.78	.222 945	6
55	9.710 786	3.52	9.933 445	1.27	9.777 342	4.77	0.222 658	5
56	.710 997	3.52	.933 369	1.27	.777 628	4.78	.222 372	4
57	.711 208	3.52	.933 293	1.27	.777 915	4.77	.222 085	3
58	.711 419	3.50	.933 217	1.27	.778 201	4.78	.221 799	2
59	.711 629	3.50	.933 141	1.25	.778 488	4.77	.221 512	1
60	9.711 839		9.933 066		9.778 774		0.221 226	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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31°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.711 839	3.52	9.933 066	1.27	9.778 774	4.77	0.221 226	60
1	.712 050	3.50	.932 990	1.27	.779 060	4.77	.220 940	59
2	.712 260	3.48	.932 914	1.27	.779 346	4.77	.220 654	58
3	.712 469	3.50	.932 838	1.27	.779 632	4.77	.220 368	57
4	.712 679	3.50	.932 762	1.28	.779 918	4.75	.220 082	56
5	9.712 889	3.48	9.932 685	1.27	9.780 203	4.77	0.219 797	55
6	.713 098	3.50	.932 609	1.27	.780 489	4.77	.219 511	54
7	.713 308	3.48	.932 533	1.27	.780 775	4.75	.219 225	53
8	.713 517	3.48	.932 457	1.28	.781 060	4.77	.218 940	52
9	.713 726	3.48	.932 380	1.27	.781 346	4.75	.218 654	51
10	9.713 935	3.48	9.932 304	1.27	9.781 631	4.75	0.218 369	50
11	.714 144	3.47	.932 228	1.28	.781 916	4.75	.218 084	49
12	.714 352	3.48	.932 151	1.27	.782 201	4.75	.217 799	48
13	.714 561	3.47	.932 075	1.28	.782 486	4.75	.217 514	47
14	.714 769	3.48	.931 998	1.28	.782 771	4.75	.217 229	46
15	9.714 978	3.47	9.931 921	1.27	9.783 056	4.75	0.216 944	45
16	.715 186	3.47	.931 845	1.28	.783 341	4.75	.216 659	44
17	.715 394	3.47	.931 768	1.28	.783 626	4.73	.216 374	43
18	.715 602	3.45	.931 691	1.28	.783 910	4.75	.216 090	42
19	.715 809	3.47	.931 614	1.28	.784 195	4.73	.215 805	41
20	9.716 017	3.45	9.931 537	1.28	9.784 479	4.75	0.215 521	40
21	.716 224	3.47	.931 460	1.28	.784 764	4.73	.215 236	39
22	.716 432	3.45	.931 383	1.28	.785 048	4.73	.214 952	38
23	.716 639	3.45	.931 306	1.28	.785 332	4.73	.214 668	37
24	.716 846	3.45	.931 229	1.28	.785 616	4.73	.214 384	36
25	9.717 053	3.43	9.931 152	1.28	9.785 900	4.73	0.214 100	35
26	.717 259	3.45	.931 075	1.28	.786 184	4.73	.213 816	34
27	.717 466	3.45	.930 998	1.28	.786 468	4.73	.213 532	33
28	.717 673	3.43	.930 921	1.30	.786 752	4.73	.213 248	32
29	.717 879	3.43	.930 843	1.28	.787 036	4.72	.212 964	31
30	9.718 085	3.43	9.930 766	1.30	9.787 319	4.73	0.212 681	30
31	.718 291	3.43	.930 688	1.28	.787 603	4.72	.212 397	29
32	.718 497	3.43	.930 611	1.30	.787 886	4.73	.212 114	28
33	.718 703	3.43	.930 533	1.28	.788 170	4.72	.211 830	27
34	.718 909	3.42	.930 456	1.30	.788 453	4.72	.211 547	26
35	9.719 114	3.43	9.930 378	1.30	9.788 736	4.72	0.211 264	25
36	.719 320	3.42	.930 300	1.28	.789 019	4.72	.210 981	24
37	.719 525	3.42	.930 223	1.30	.789 302	4.72	.210 698	23
38	.719 730	3.42	.930 145	1.30	.789 585	4.72	.210 415	22
39	.719 935	3.42	.930 067	1.30	.789 868	4.72	.210 132	21
40	9.720 140	3.42	9.929 989	1.30	9.790 151	4.72	0.209 849	20
41	.720 345	3.40	.929 911	1.30	.790 434	4.70	.209 566	19
42	.720 549	3.42	.929 833	1.30	.790 716	4.72	.209 284	18
43	.720 754	3.40	.929 755	1.30	.790 999	4.70	.209 001	17
44	.720 958	3.40	.929 677	1.30	.791 281	4.70	.208 719	16
45	9.721 162	3.40	9.929 599	1.30	9.791 563	4.72	0.208 437	15
46	.721 366	3.40	.929 521	1.32	.791 846	4.70	.208 154	14
47	.721 570	3.40	.929 442	1.30	.792 128	4.70	.207 872	13
48	.721 774	3.40	.929 364	1.30	.792 410	4.70	.207 590	12
49	.721 978	3.38	.929 286	1.32	.792 692	4.70	.207 308	11
50	9.722 181	3.40	9.929 207	1.30	9.792 974	4.70	0.207 026	10
51	.722 385	3.38	.929 129	1.32	.793 256	4.70	.206 744	9
52	.722 588	3.38	.929 050	1.30	.793 538	4.68	.206 462	8
53	.722 791	3.38	.928 972	1.32	.793 819	4.70	.206 181	7
54	.722 994	3.38	.928 893	1.30	.794 101	4.70	.205 899	6
55	9.723 197	3.38	9.928 815	1.32	9.794 383	4.68	0.205 617	5
56	.723 400	3.38	.928 736	1.32	.794 664	4.70	.205 336	4
57	.723 603	3.37	.928 657	1.32	.794 946	4.68	.205 054	3
58	.723 805	3.37	.928 578	1.32	.795 227	4.68	.204 773	2
59	.724 007	3.38	.928 499	1.32	.795 508	4.68	.204 492	1
60	9.724 210		9.928 420		9.795 789		0.204 211	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

58°

32°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.724 210	3.37	9.928 420	1.30	9.795 789	4.68	0.204 211	60
1	.724 412	3.37	.928 342	1.32	.796 070	4.68	.203 930	59
2	.724 614	3.37	.928 263	1.33	.796 351	4.68	.203 649	58
3	.724 816	3.35	.928 183	1.32	.796 632	4.68	.203 368	57
4	.725 017	3.37	.928 104	1.32	.796 913	4.68	.203 087	56
5	9.725 219	3.35	9.928 025	1.32	9.797 194	4.67	0.202 806	55
6	.725 420	3.37	.927 946	1.32	.797 474	4.68	.202 526	54
7	.725 622	3.35	.927 867	1.33	.797 755	4.68	.202 245	53
8	.725 823	3.35	.927 787	1.32	.798 036	4.67	.201 964	52
9	.726 024	3.35	.927 708	1.32	.798 316	4.67	.201 684	51
10	9.726 225	3.35	9.927 629	1.33	9.798 596	4.68	0.201 404	50
11	.726 426	3.33	.927 549	1.32	.798 877	4.67	.201 123	49
12	.726 626	3.35	.927 470	1.33	.799 157	4.67	.200 843	48
13	.726 827	3.33	.927 390	1.33	.799 437	4.67	.200 563	47
14	.727 027	3.35	.927 310	1.32	.799 717	4.67	.200 283	46
15	9.727 228	3.33	9.927 231	1.33	9.799 997	4.67	0.200 003	45
16	.727 428	3.33	.927 151	1.33	.800 277	4.67	.199 723	44
17	.727 628	3.33	.927 071	1.33	.800 557	4.65	.199 443	43
18	.727 828	3.32	.926 991	1.33	.800 836	4.67	.199 164	42
19	.728 027	3.33	.926 911	1.33	.801 116	4.67	.198 884	41
20	9.728 227	3.33	9.926 831	1.33	9.801 396	4.65	0.198 604	40
21	.728 427	3.32	.926 751	1.33	.801 675	4.67	.198 325	39
22	.728 626	3.32	.926 671	1.33	.801 955	4.65	.198 045	38
23	.728 825	3.32	.926 591	1.33	.802 234	4.65	.197 766	37
24	.729 024	3.32	.926 511	1.33	.802 513	4.65	.197 487	36
25	9.729 223	3.32	9.926 431	1.33	9.802 792	4.67	0.197 208	35
26	.729 422	3.32	.926 351	1.35	.803 072	4.65	.196 928	34
27	.729 621	3.32	.926 270	1.33	.803 351	4.65	.196 649	33
28	.729 820	3.30	.926 190	1.33	.803 630	4.65	.196 370	32
29	.730 018	3.32	.926 110	1.35	.803 909	4.63	.196 091	31
30	9.730 217	3.30	9.926 029	1.33	9.804 187	4.65	0.195 813	30
31	.730 415	3.30	.925 949	1.35	.804 466	4.65	.195 534	29
32	.730 613	3.30	.925 868	1.33	.804 745	4.63	.195 255	28
33	.730 811	3.30	.925 788	1.35	.805 023	4.65	.194 977	27
34	.731 009	3.28	.925 707	1.35	.805 302	4.63	.194 698	26
35	9.731 206	3.30	9.925 626	1.35	9.805 580	4.65	0.194 420	25
36	.731 404	3.30	.925 545	1.33	.805 859	4.63	.194 141	24
37	.731 602	3.28	.925 465	1.35	.806 137	4.63	.193 863	23
38	.731 799	3.28	.925 384	1.35	.806 415	4.63	.193 585	22
39	.731 996	3.28	.925 303	1.35	.806 693	4.63	.193 307	21
40	9.732 193	3.28	9.925 222	1.35	9.806 971	4.63	0.193 029	20
41	.732 390	3.28	.925 141	1.35	.807 249	4.63	.192 751	19
42	.732 587	3.28	.925 060	1.35	.807 527	4.63	.192 473	18
43	.732 784	3.27	.924 979	1.37	.807 805	4.63	.192 195	17
44	.732 980	3.28	.924 897	1.35	.808 083	4.63	.191 917	16
45	9.733 177	3.27	9.924 816	1.35	9.808 361	4.62	0.191 639	15
46	.733 373	3.27	.924 735	1.35	.808 638	4.63	.191 362	14
47	.733 569	3.27	.924 654	1.37	.808 916	4.62	.191 084	13
48	.733 765	3.27	.924 572	1.35	.809 193	4.63	.190 807	12
49	.733 961	3.27	.924 491	1.37	.809 471	4.62	.190 529	11
50	9.734 157	3.27	9.924 409	1.35	9.809 748	4.62	0.190 252	10
51	.734 353	3.27	.924 328	1.37	.810 025	4.62	.189 975	9
52	.734 549	3.25	.924 246	1.37	.810 302	4.63	.189 698	8
53	.734 744	3.25	.924 164	1.35	.810 580	4.62	.189 420	7
54	.734 939	3.27	.924 083	1.37	.810 857	4.62	.189 143	6
55	9.735 135	3.25	9.924 001	1.37	9.811 134	4.60	0.188 866	5
56	.735 330	3.25	.923 919	1.37	.811 410	4.62	.188 590	4
57	.735 525	3.23	.923 837	1.37	.811 687	4.62	.188 313	3
58	.735 719	3.25	.923 755	1.37	.811 964	4.62	.188 036	2
59	.735 914	3.25	.923 673	1.37	.812 241	4.60	.187 759	1
60	9.736 109		9.923 591		9.812 517		0.187 483	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

57°

33°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.736 109	3.23	9.923 591	1.37	9.812 517	4.62	0.187 483	60
1	.736 303	3.25	.923 509	1.37	.812 794	4.60	.187 206	59
2	.736 498	3.23	.923 427	1.37	.813 070	4.62	.186 930	58
3	.736 692	3.23	.923 345	1.37	.813 347	4.60	.186 653	57
4	.736 886	3.23	.923 263	1.37	.813 623	4.60	.186 377	56
5	9.737 080	3.23	9.923 181	1.38	9.813 899	4.62	0.186 101	55
6	.737 274	3.22	.923 098	1.37	.814 176	4.60	.185 824	54
7	.737 467	3.23	.923 016	1.38	.814 452	4.60	.185 548	53
8	.737 661	3.23	.922 933	1.37	.814 728	4.60	.185 272	52
9	.737 855	3.22	.922 851	1.38	.815 004	4.60	.184 996	51
10	9.738 048	3.22	9.922 768	1.37	9.815 280	4.58	0.184 720	50
11	.738 241	3.22	.922 686	1.38	.815 555	4.60	.184 445	49
12	.738 434	3.22	.922 603	1.38	.815 831	4.60	.184 169	48
13	.738 627	3.22	.922 520	1.37	.816 107	4.58	.183 893	47
14	.738 820	3.22	.922 438	1.38	.816 382	4.60	.183 618	46
15	9.739 013	3.22	9.922 355	1.38	9.816 658	4.58	0.183 342	45
16	.739 206	3.20	.922 272	1.38	.816 933	4.60	.183 067	44
17	.739 398	3.20	.922 189	1.38	.817 209	4.58	.182 791	43
18	.739 590	3.22	.922 106	1.38	.817 484	4.58	.182 516	42
19	.739 783	3.20	.922 023	1.38	.817 759	4.60	.182 241	41
20	9.739 975	3.20	9.921 940	1.38	9.818 035	4.58	0.181 965	40
21	.740 167	3.20	.921 857	1.38	.818 310	4.58	.181 690	39
22	.740 359	3.18	.921 774	1.38	.818 585	4.58	.181 415	38
23	.740 550	3.20	.921 691	1.40	.818 860	4.58	.181 140	37
24	.740 742	3.20	.921 607	1.38	.819 135	4.58	.180 865	36
25	9.740 934	3.18	9.921 524	1.38	9.819 410	4.57	0.180 590	35
26	.741 125	3.18	.921 441	1.40	.819 684	4.58	.180 316	34
27	.741 316	3.20	.921 357	1.38	.819 959	4.58	.180 041	33
28	.741 508	3.18	.921 274	1.40	.820 234	4.57	.179 766	32
29	.741 699	3.17	.921 190	1.38	.820 508	4.58	.179 492	31
30	9.741 889	3.18	9.921 107	1.40	9.820 783	4.57	0.179 217	30
31	.742 080	3.18	.921 023	1.40	.821 057	4.58	.178 943	29
32	.742 271	3.18	.920 939	1.38	.821 332	4.57	.178 668	28
33	.742 462	3.17	.920 856	1.40	.821 606	4.57	.178 394	27
34	.742 652	3.17	.920 772	1.40	.821 880	4.57	.178 120	26
35	9.742 842	3.18	9.920 688	1.40	9.822 154	4.58	0.177 846	25
36	.743 033	3.17	.920 604	1.40	.822 429	4.57	.177 571	24
37	.743 223	3.17	.920 520	1.40	.822 703	4.57	.177 297	23
38	.743 413	3.15	.920 436	1.40	.822 977	4.57	.177 023	22
39	.743 602	3.17	.920 352	1.40	.823 251	4.55	.176 749	21
40	9.743 792	3.17	9.920 268	1.40	9.823 524	4.57	0.176 476	20
41	.743 982	3.15	.920 184	1.42	.823 798	4.57	.176 202	19
42	.744 171	3.17	.920 099	1.40	.824 072	4.55	.175 928	18
43	.744 361	3.15	.920 015	1.40	.824 345	4.57	.175 655	17
44	.744 550	3.15	.919 931	1.42	.824 619	4.57	.175 381	16
45	9.744 739	3.15	9.919 846	1.40	9.824 893	4.55	0.175 107	15
46	.744 928	3.15	.919 762	1.42	.825 166	4.55	.174 834	14
47	.745 117	3.15	.919 677	1.40	.825 439	4.57	.174 561	13
48	.745 306	3.13	.919 593	1.42	.825 713	4.55	.174 287	12
49	.745 494	3.15	.919 508	1.40	.825 986	4.55	.174 014	11
50	9.745 683	3.13	9.919 424	1.42	9.826 259	4.55	0.173 741	10
51	.745 871	3.15	.919 339	1.42	.826 532	4.55	.173 468	9
52	.746 060	3.13	.919 254	1.42	.826 805	4.55	.173 195	8
53	.746 248	3.13	.919 169	1.40	.827 078	4.55	.172 922	7
54	.746 436	3.13	.919 085	1.42	.827 351	4.55	.172 649	6
55	9.746 624	3.13	9.919 000	1.42	9.827 624	4.55	0.172 376	5
56	.746 812	3.12	.918 915	1.42	.827 897	4.55	.172 103	4
57	.746 999	3.13	.918 830	1.42	.828 170	4.53	.171 830	3
58	.747 187	3.12	.918 745	1.43	.828 442	4.55	.171 558	2
59	.747 374	3.13	.918 659	1.42	.828 715	4.53	.171 285	1
60	9.747 562		9.918 574		9.828 987		0.171 013	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

56°

34°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.747 562	3.12	9.918 574	1.42	9.828 987	4.55	0.171 013	60
1	·747 749	3.12	.918 489	1.42	.829 260	4.53	.170 740	59
2	·747 936	3.12	.918 404	1.43	.829 532	4.55	.170 468	58
3	·748 123	3.12	.918 318	1.42	.829 805	4.53	.170 195	57
4	·748 310	3.12	.918 233	1.43	.830 077	4.53	.169 923	56
5	9.748 497	3.10	9.918 147	1.42	9.830 349	4.53	0.169 651	55
6	·748 683	3.12	.918 062	1.43	.830 621	4.53	.169 379	54
7	·748 870	3.10	.917 976	1.42	.830 893	4.53	.169 107	53
8	·749 056	3.12	.917 891	1.43	.831 165	4.53	.168 835	52
9	·749 243	3.10	.917 805	1.43	.831 437	4.53	.168 563	51
10	9.749 429	3.10	9.917 719	1.42	9.831 709	4.53	0.168 291	50
11	·749 615	3.10	.917 634	1.43	.831 981	4.53	.168 019	49
12	·749 801	3.10	.917 548	1.43	.832 253	4.53	.167 747	48
13	·749 987	3.08	.917 462	1.43	.832 525	4.52	.167 475	47
14	·750 172	3.10	.917 376	1.43	.832 796	4.53	.167 204	46
15	9.750 358	3.08	9.917 290	1.43	9.833 068	4.52	0.166 932	45
16	·750 543	3.10	.917 204	1.43	.833 339	4.53	.166 661	44
17	·750 729	3.08	.917 118	1.43	.833 611	4.52	.166 389	43
18	·750 914	3.08	.917 032	1.43	.833 882	4.53	.166 118	42
19	·751 099	3.08	.916 946	1.45	.834 154	4.52	.165 846	41
20	9.751 284	3.08	9.916 859	1.43	9.834 425	4.52	0.165 575	40
21	·751 469	3.08	.916 773	1.43	.834 696	4.52	.165 304	39
22	·751 654	3.08	.916 687	1.45	.834 967	4.52	.165 033	38
23	·751 839	3.07	.916 600	1.43	.835 238	4.52	.164 762	37
24	·752 023	3.08	.916 514	1.45	.835 509	4.52	.164 491	36
25	9.752 208	3.07	9.916 427	1.43	9.835 780	4.52	0.164 220	35
26	·752 392	3.07	.916 341	1.45	.836 051	4.52	.163 949	34
27	·752 576	3.07	.916 254	1.45	.836 322	4.52	.163 678	33
28	·752 760	3.07	.916 167	1.43	.836 593	4.52	.163 407	32
29	·752 944	3.07	.916 081	1.45	.836 864	4.50	.163 136	31
30	9.753 128	3.07	9.915 994	1.45	9.837 134	4.52	0.162 866	30
31	·753 312	3.05	.915 907	1.45	.837 405	4.50	.162 595	29
32	·753 495	3.07	.915 820	1.45	.837 675	4.52	.162 325	28
33	·753 679	3.05	.915 733	1.45	.837 946	4.50	.162 054	27
34	·753 862	3.07	.915 646	1.45	.838 216	4.52	.161 784	26
35	9.754 046	3.05	9.915 559	1.45	9.838 487	4.50	0.161 513	25
36	·754 229	3.05	.915 472	1.45	.838 757	4.50	.161 243	24
37	·754 412	3.05	.915 385	1.47	.839 027	4.50	.160 973	23
38	·754 595	3.05	.915 297	1.45	.839 297	4.52	.160 703	22
39	·754 778	3.03	.915 210	1.45	.839 568	4.50	.160 432	21
40	9.754 960	3.05	9.915 123	1.47	9.839 838	4.50	0.160 162	20
41	·755 143	3.05	.915 035	1.45	.840 108	4.50	.159 892	19
42	·755 326	3.03	.914 948	1.47	.840 378	4.50	.159 622	18
43	·755 508	3.03	.914 860	1.45	.840 648	4.48	.159 352	17
44	·755 690	3.03	.914 773	1.47	.840 917	4.50	.159 083	16
45	9.755 872	3.03	9.914 685	1.45	9.841 187	4.50	0.158 813	15
46	·756 054	3.03	.914 598	1.47	.841 457	4.50	.158 543	14
47	·756 236	3.03	.914 510	1.47	.841 727	4.48	.158 273	13
48	·756 418	3.03	.914 422	1.47	.841 996	4.50	.158 004	12
49	·756 600	3.03	.914 334	1.47	.842 266	4.48	.157 734	11
50	9.756 782	3.02	9.914 246	1.47	9.842 535	4.50	0.157 465	10
51	·756 963	3.02	.914 158	1.47	.842 805	4.48	.157 195	9
52	·757 144	3.03	.914 070	1.47	.843 074	4.48	.156 926	8
53	·757 326	3.02	.913 982	1.47	.843 343	4.48	.156 657	7
54	·757 507	3.02	.913 894	1.47	.843 612	4.50	.156 388	6
55	9.757 688	3.02	9.913 806	1.47	9.843 882	4.48	0.156 118	5
56	·757 869	3.02	.913 718	1.47	.844 151	4.48	.155 849	4
57	·758 050	3.00	.913 630	1.48	.844 420	4.48	.155 580	3
58	·758 230	3.02	.913 541	1.47	.844 689	4.48	.155 311	2
59	·758 411	3.00	.913 453	1.47	.844 958	4.48	.155 042	1
60	9.758 591		9.913 365		9.845 227		0.154 773	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

55°

35°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.758 591	3.02	9.913 365	1.48	9.845 227	4.48	0.154 773	60
1	.758 772	3.00	.913 276	1.48	.845 496	4.47	.154 504	59
2	.758 952	3.00	.913 187	1.47	.845 764	4.48	.154 236	58
3	.759 132	3.00	.913 099	1.48	.846 033	4.48	.153 967	57
4	.759 312	3.00	.913 010	1.47	.846 302	4.47	.153 698	56
5	9.759 492	3.00	9.912 922	1.48	9.846 570	4.48	0.153 430	55
6	.759 672	3.00	.912 833	1.48	.846 839	4.48	.153 161	54
7	.759 852	2.98	.912 744	1.48	.847 108	4.47	.152 892	53
8	.760 031	3.00	.912 655	1.48	.847 376	4.47	.152 624	52
9	.760 211	2.98	.912 566	1.48	.847 644	4.48	.152 356	51
10	9.760 390	2.98	9.912 477	1.48	9.847 913	4.47	0.152 087	50
11	.760 569	2.98	.912 388	1.48	.848 181	4.47	.151 819	49
12	.760 748	2.98	.912 299	1.48	.848 449	4.47	.151 551	48
13	.760 927	2.98	.912 210	1.48	.848 717	4.48	.151 283	47
14	.761 106	2.98	.912 121	1.50	.848 986	4.47	.151 014	46
15	9.761 285	2.98	9.912 031	1.48	9.849 254	4.47	0.150 746	45
16	.761 464	2.97	.911 942	1.48	.849 522	4.47	.150 478	44
17	.761 642	2.98	.911 853	1.50	.849 790	4.45	.150 210	43
18	.761 821	2.97	.911 763	1.48	.850 057	4.47	.149 943	42
19	.761 999	2.97	.911 674	1.50	.850 325	4.47	.149 675	41
20	9.762 177	2.98	9.911 584	1.48	9.850 593	4.47	0.149 407	40
21	.762 356	2.97	.911 495	1.50	.850 861	4.47	.149 139	39
22	.762 534	2.97	.911 405	1.50	.851 129	4.45	.148 871	38
23	.762 712	2.95	.911 315	1.48	.851 396	4.47	.148 604	37
24	.762 889	2.97	.911 226	1.50	.851 664	4.45	.148 336	36
25	9.763 067	2.97	9.911 136	1.50	9.851 931	4.47	0.148 069	35
26	.763 245	2.95	.911 046	1.50	.852 199	4.45	.147 801	34
27	.763 422	2.97	.910 956	1.50	.852 466	4.45	.147 534	33
28	.763 600	2.95	.910 866	1.50	.852 733	4.47	.147 267	32
29	.763 777	2.95	.910 776	1.50	.853 001	4.45	.146 999	31
30	9.763 954	2.95	9.910 686	1.50	9.853 268	4.45	0.146 732	30
31	.764 131	2.95	.910 596	1.50	.853 535	4.45	.146 465	29
32	.764 308	2.95	.910 506	1.52	.853 802	4.45	.146 198	28
33	.764 485	2.95	.910 415	1.50	.854 069	4.45	.145 931	27
34	.764 662	2.93	.910 325	1.50	.854 336	4.45	.145 664	26
35	9.764 838	2.95	9.910 235	1.52	9.854 603	4.45	0.145 397	25
36	.765 015	2.93	.910 144	1.50	.854 870	4.45	.145 130	24
37	.765 191	2.93	.910 054	1.52	.855 137	4.45	.144 863	23
38	.765 367	2.95	.909 963	1.50	.855 404	4.45	.144 596	22
39	.765 544	2.93	.909 873	1.52	.855 671	4.45	.144 329	21
40	9.765 720	2.93	9.909 782	1.52	9.855 938	4.43	0.144 062	20
41	.765 896	2.93	.909 691	1.50	.856 204	4.45	.143 796	19
42	.766 072	2.92	.909 601	1.52	.856 471	4.43	.143 529	18
43	.766 247	2.93	.909 510	1.52	.856 737	4.45	.143 263	17
44	.766 423	2.92	.909 419	1.52	.857 004	4.43	.142 996	16
45	9.766 598	2.93	9.909 328	1.52	9.857 270	4.45	0.142 730	15
46	.766 774	2.92	.909 237	1.52	.857 537	4.43	.142 463	14
47	.766 949	2.92	.909 146	1.52	.857 803	4.43	.142 197	13
48	.767 124	2.93	.909 055	1.52	.858 069	4.45	.141 931	12
49	.767 300	2.92	.908 964	1.52	.858 336	4.43	.141 664	11
50	9.767 475	2.90	9.908 873	1.53	9.858 602	4.43	0.141 398	10
51	.767 649	2.92	.908 781	1.52	.858 868	4.43	.141 132	9
52	.767 824	2.92	.908 690	1.52	.859 134	4.43	.140 866	8
53	.767 999	2.90	.908 599	1.53	.859 400	4.43	.140 600	7
54	.768 173	2.92	.908 507	1.52	.859 666	4.43	.140 334	6
55	9.768 348	2.90	9.908 416	1.53	9.859 932	4.43	0.140 068	5
56	.768 522	2.92	.908 324	1.52	.860 198	4.43	.139 802	4
57	.768 697	2.90	.908 233	1.53	.860 464	4.43	.139 536	3
58	.768 871	2.90	.908 141	1.53	.860 730	4.42	.139 270	2
59	.769 045	2.90	.908 049	1.52	.860 995	4.43	.139 005	1
60	9.769 219		9.907 958		9.861 261		0.138 739	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

54°

36°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.769 219	2.90	9.907 958	1.53	9.861 261	4.43	0.138 739	60
1	.769 393	2.88	.907 866	1.53	.861 527	4.42	.138 473	59
2	.769 566	2.90	.907 774	1.53	.861 792	4.43	.138 208	58
3	.769 740	2.88	.907 682	1.53	.862 058	4.42	.137 942	57
4	.769 913	2.90	.907 590	1.53	.862 323	4.43	.137 677	56
5	9.770 087	2.88	9.907 498	1.53	9.862 589	4.42	0.137 411	55
6	.770 260	2.88	.907 406	1.53	.862 854	4.42	.137 146	54
7	.770 433	2.88	.907 314	1.53	.863 119	4.43	.136 881	53
8	.770 606	2.88	.907 222	1.55	.863 385	4.42	.136 615	52
9	.770 779	2.88	.907 129	1.53	.863 650	4.42	.136 350	51
10	9.770 952	2.88	9.907 037	1.53	9.863 915	4.42	0.136 085	50
11	.771 125	2.88	.906 945	1.55	.864 180	4.42	.135 820	49
12	.771 298	2.87	.906 852	1.53	.864 445	4.42	.135 555	48
13	.771 470	2.88	.906 760	1.55	.864 710	4.42	.135 290	47
14	.771 643	2.87	.906 667	1.53	.864 975	4.42	.135 025	46
15	9.771 815	2.87	9.906 575	1.55	9.865 240	4.42	0.134 760	45
16	.771 987	2.87	.906 482	1.55	.865 505	4.42	.134 495	44
17	.772 159	2.87	.906 389	1.55	.865 770	4.42	.134 230	43
18	.772 331	2.87	.906 296	1.53	.866 035	4.42	.133 965	42
19	.772 503	2.87	.906 204	1.55	.866 300	4.40	.133 700	41
20	9.772 675	2.87	9.906 111	1.55	9.866 564	4.42	0.133 436	40
21	.772 847	2.85	.906 018	1.55	.866 829	4.42	.133 171	39
22	.773 018	2.87	.905 925	1.55	.867 094	4.40	.132 906	38
23	.773 190	2.85	.905 832	1.55	.867 358	4.42	.132 642	37
24	.773 361	2.87	.905 739	1.57	.867 623	4.40	.132 377	36
25	9.773 533	2.85	9.905 645	1.55	9.867 887	4.42	0.132 113	35
26	.773 704	2.85	.905 552	1.55	.868 152	4.40	.131 848	34
27	.773 875	2.85	.905 459	1.55	.868 416	4.40	.131 584	33
28	.774 046	2.85	.905 366	1.57	.868 680	4.42	.131 320	32
29	.774 217	2.85	.905 272	1.55	.868 945	4.40	.131 055	31
30	9.774 388	2.83	9.905 179	1.57	9.869 209	4.40	0.130 791	30
31	.774 558	2.85	.905 085	1.55	.869 473	4.40	.130 527	29
32	.774 729	2.83	.904 992	1.57	.869 737	4.40	.130 263	28
33	.774 899	2.85	.904 898	1.57	.870 001	4.40	.129 999	27
34	.775 070	2.83	.904 804	1.55	.870 265	4.40	.129 735	26
35	9.775 240	2.83	9.904 711	1.57	9.870 529	4.40	0.129 471	25
36	.775 410	2.83	.904 617	1.57	.870 793	4.40	.129 207	24
37	.775 580	2.83	.904 523	1.57	.871 057	4.40	.128 943	23
38	.775 750	2.83	.904 429	1.57	.871 321	4.40	.128 679	22
39	.775 920	2.83	.904 335	1.57	.871 585	4.40	.128 415	21
40	9.776 090	2.82	9.904 241	1.57	9.871 849	4.38	0.128 151	20
41	.776 259	2.83	.904 147	1.57	.872 112	4.40	.127 888	19
42	.776 429	2.82	.904 053	1.57	.872 376	4.40	.127 624	18
43	.776 598	2.83	.903 959	1.58	.872 640	4.38	.127 360	17
44	.776 768	2.82	.903 864	1.57	.872 903	4.40	.127 097	16
45	9.776 937	2.82	9.903 770	1.57	9.873 167	4.38	0.126 833	15
46	.777 106	2.82	.903 676	1.58	.873 430	4.40	.126 570	14
47	.777 275	2.82	.903 581	1.57	.873 694	4.38	.126 306	13
48	.777 444	2.82	.903 487	1.58	.873 957	4.38	.126 043	12
49	.777 613	2.80	.903 392	1.57	.874 220	4.40	.125 780	11
50	9.777 781	2.82	9.903 298	1.58	9.874 484	4.38	0.125 516	10
51	.777 950	2.82	.903 203	1.58	.874 747	4.38	.125 253	9
52	.778 119	2.80	.903 108	1.57	.875 010	4.38	.124 990	8
53	.778 287	2.80	.903 014	1.58	.875 273	4.40	.124 727	7
54	.778 455	2.82	.902 919	1.58	.875 537	4.38	.124 463	6
55	9.778 624	2.80	9.902 824	1.58	9.875 800	4.38	0.124 200	5
56	.778 792	2.80	.902 729	1.58	.876 063	4.38	.123 937	4
57	.778 960	2.80	.902 634	1.58	.876 326	4.38	.123 674	3
58	.779 128	2.78	.902 539	1.58	.876 589	4.38	.123 411	2
59	.779 295	2.80	.902 444	1.58	.876 852	4.37	.123 148	1
60	9.779 463		9.902 349		9.877 114		0.122 886	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

53°

37°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.779 463	2.80	9.902 349	1.60	9.877 114	4.38	0.122 886	60
1	.779 631	2.78	.902 253	1.58	.877 377	4.38	.122 623	59
2	.779 798	2.80	.902 158	1.58	.877 640	4.38	.122 360	58
3	.779 966	2.78	.902 063	1.60	.877 903	4.37	.122 097	57
4	.780 133	2.78	.901 967	1.58	.878 165	4.38	.121 835	56
5	9.780 300	2.78	9.901 872	1.60	9.878 428	4.38	0.121 572	55
6	.780 467	2.78	.901 776	1.58	.878 691	4.37	.121 309	54
7	.780 634	2.78	.901 681	1.60	.878 953	4.38	.121 047	53
8	.780 801	2.78	.901 585	1.58	.879 216	4.37	.120 784	52
9	.780 968	2.77	.901 490	1.60	.879 478	4.38	.120 522	51
10	9.781 134	2.78	9.901 394	1.60	9.879 741	4.37	0.120 259	50
11	.781 301	2.78	.901 298	1.60	.880 003	4.37	.119 997	49
12	.781 468	2.77	.901 202	1.60	.880 265	4.38	.119 735	48
13	.781 634	2.77	.901 106	1.60	.880 528	4.37	.119 472	47
14	.781 800	2.77	.901 010	1.60	.880 790	4.37	.119 210	46
15	9.781 966	2.77	9.900 914	1.60	9.881 052	4.37	0.118 948	45
16	.782 132	2.77	.900 818	1.60	.881 314	4.38	.118 686	44
17	.782 298	2.77	.900 722	1.60	.881 577	4.37	.118 423	43
18	.782 464	2.77	.900 626	1.62	.881 839	4.37	.118 161	42
19	.782 630	2.77	.900 529	1.60	.882 101	4.37	.117 899	41
20	9.782 796	2.75	9.900 433	1.60	9.882 363	4.37	0.117 637	40
21	.782 961	2.77	.900 337	1.62	.882 625	4.37	.117 375	39
22	.783 127	2.75	.900 240	1.60	.882 887	4.35	.117 113	38
23	.783 292	2.77	.900 144	1.62	.883 148	4.37	.116 852	37
24	.783 458	2.75	.900 047	1.60	.883 410	4.37	.116 590	36
25	9.783 623	2.75	9.899 951	1.62	9.883 672	4.37	0.116 328	35
26	.783 788	2.75	.899 854	1.62	.883 934	4.37	.116 066	34
27	.783 953	2.75	.899 757	1.62	.884 196	4.35	.115 804	33
28	.784 118	2.73	.899 660	1.60	.884 457	4.37	.115 543	32
29	.784 282	2.75	.899 564	1.62	.884 719	4.35	.115 281	31
30	9.784 447	2.75	9.899 467	1.62	9.884 980	4.37	0.115 020	30
31	.784 612	2.73	.899 370	1.62	.885 242	4.37	.114 758	29
32	.784 776	2.75	.899 273	1.62	.885 504	4.35	.114 496	28
33	.784 941	2.73	.899 176	1.63	.885 765	4.35	.114 235	27
34	.785 105	2.73	.899 078	1.62	.886 026	4.37	.113 974	26
35	9.785 269	2.73	9.898 981	1.62	9.886 288	4.35	0.113 712	25
36	.785 433	2.73	.898 884	1.62	.886 549	4.37	.113 451	24
37	.785 597	2.73	.898 787	1.63	.886 811	4.35	.113 189	23
38	.785 761	2.73	.898 689	1.62	.887 072	4.35	.112 928	22
39	.785 925	2.73	.898 592	1.63	.887 333	4.35	.112 667	21
40	9.786 089	2.72	9.898 494	1.62	9.887 594	4.35	0.112 406	20
41	.786 252	2.73	.898 397	1.63	.887 855	4.35	.112 145	19
42	.786 416	2.72	.898 299	1.62	.888 116	4.37	.111 884	18
43	.786 579	2.72	.898 202	1.63	.888 378	4.35	.111 622	17
44	.786 742	2.73	.898 104	1.63	.888 639	4.35	.111 361	16
45	9.786 906	2.72	9.898 006	1.63	9.888 900	4.35	0.111 100	15
46	.787 069	2.72	.897 908	1.63	.889 161	4.33	.110 839	14
47	.787 232	2.72	.897 810	1.63	.889 421	4.35	.110 579	13
48	.787 395	2.70	.897 712	1.63	.889 682	4.35	.110 318	12
49	.787 557	2.72	.897 614	1.63	.889 943	4.35	.110 057	11
50	9.787 720	2.72	9.897 516	1.63	9.890 204	4.35	0.109 796	10
51	.787 883	2.70	.897 418	1.63	.890 465	4.33	.109 535	9
52	.788 045	2.72	.897 320	1.63	.890 725	4.35	.109 275	8
53	.788 208	2.70	.897 222	1.65	.890 986	4.35	.109 014	7
54	.788 370	2.70	.897 123	1.63	.891 247	4.33	.108 753	6
55	9.788 532	2.70	9.897 025	1.65	9.891 507	4.35	0.108 493	5
56	.788 694	2.70	.896 926	1.63	.891 768	4.33	.108 232	4
57	.788 856	2.70	.896 828	1.65	.892 028	4.35	.107 972	3
58	.789 018	2.70	.896 729	1.63	.892 289	4.33	.107 711	2
59	.789 180	2.70	.896 631	1.65	.892 549	4.35	.107 451	1
60	9.789 342		9.896 532		9.892 810		0.107 190	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

52°

38°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.789 342	2.70	9.896 532	1.65	9.892 810	4.33	0.107 190	60
1	.789 504	2.68	.896 433	1.63	.893 070	4.35	.106 930	59
2	.789 665	2.70	.896 335	1.65	.893 331	4.33	.106 669	58
3	.789 827	2.68	.896 236	1.65	.893 591	4.33	.106 409	57
4	.789 988	2.68	.896 137	1.65	.893 851	4.33	.106 149	56
5	9.790 149	2.68	9.896 038	1.65	9.894 111	4.35	0.105 889	55
6	.790 310	2.68	.895 939	1.65	.894 372	4.33	.105 628	54
7	.790 471	2.68	.895 840	1.65	.894 632	4.33	.105 368	53
8	.790 632	2.68	.895 741	1.67	.894 892	4.33	.105 108	52
9	.790 793	2.68	.895 641	1.65	.895 152	4.33	.104 848	51
10	9.790 954	2.68	9.895 542	1.65	9.895 412	4.33	0.104 588	50
11	.791 115	2.67	.895 443	1.67	.895 672	4.33	.104 328	49
12	.791 275	2.68	.895 343	1.65	.895 932	4.33	.104 068	48
13	.791 436	2.67	.895 244	1.65	.896 192	4.33	.103 808	47
14	.791 596	2.68	.895 145	1.67	.896 452	4.33	.103 548	46
15	9.791 757	2.67	9.895 045	1.67	9.896 712	4.32	0.103 288	45
16	.791 917	2.67	.894 945	1.65	.896 971	4.33	.103 029	44
17	.792 077	2.67	.894 846	1.67	.897 231	4.33	.102 769	43
18	.792 237	2.67	.894 746	1.67	.897 491	4.33	.102 509	42
19	.792 397	2.67	.894 646	1.67	.897 751	4.32	.102 249	41
20	9.792 557	2.65	9.894 546	1.67	9.898 010	4.33	0.101 990	40
21	.792 716	2.67	.894 446	1.67	.898 270	4.33	.101 730	39
22	.792 876	2.65	.894 346	1.67	.898 530	4.32	.101 470	38
23	.793 035	2.67	.894 246	1.67	.898 789	4.33	.101 211	37
24	.793 195	2.65	.894 146	1.67	.899 049	4.32	.100 951	36
25	9.793 354	2.67	9.894 046	1.67	9.899 308	4.33	0.100 692	35
26	.793 514	2.65	.893 946	1.67	.899 568	4.32	.100 432	34
27	.793 673	2.65	.893 846	1.68	.899 827	4.33	.100 173	33
28	.793 832	2.65	.893 745	1.67	.900 087	4.32	.099 913	32
29	.793 991	2.65	.893 645	1.68	.900 346	4.32	.099 654	31
30	9.794 150	2.63	9.893 544	1.67	9.900 605	4.32	0.099 395	30
31	.794 308	2.65	.893 444	1.68	.900 864	4.33	.099 136	29
32	.794 467	2.65	.893 343	1.67	.901 124	4.32	.098 876	28
33	.794 626	2.63	.893 243	1.68	.901 383	4.32	.098 617	27
34	.794 784	2.63	.893 142	1.68	.901 642	4.32	.098 358	26
35	9.794 942	2.65	9.893 041	1.68	9.901 901	4.32	0.098 099	25
36	.795 101	2.63	.892 940	1.68	.902 160	4.33	.097 840	24
37	.795 259	2.63	.892 839	1.67	.902 420	4.32	.097 580	23
38	.795 417	2.63	.892 739	1.68	.902 679	4.32	.097 321	22
39	.795 575	2.63	.892 638	1.70	.902 938	4.32	.097 062	21
40	9.795 733	2.63	9.892 536	1.68	9.903 197	4.32	0.096 803	20
41	.795 891	2.63	.892 435	1.68	.903 456	4.30	.096 544	19
42	.796 049	2.62	.892 334	1.68	.903 714	4.32	.096 286	18
43	.796 206	2.63	.892 233	1.68	.903 973	4.32	.096 027	17
44	.796 364	2.62	.892 132	1.70	.904 232	4.32	.095 768	16
45	9.796 521	2.63	9.892 030	1.68	9.904 491	4.32	0.095 509	15
46	.796 679	2.62	.891 929	1.70	.904 750	4.30	.095 250	14
47	.796 836	2.62	.891 827	1.68	.905 008	4.32	.094 992	13
48	.796 993	2.62	.891 726	1.70	.905 267	4.32	.094 733	12
49	.797 150	2.62	.891 624	1.68	.905 526	4.32	.094 474	11
50	9.797 307	2.62	9.891 523	1.70	9.905 785	4.30	0.094 215	10
51	.797 464	2.62	.891 421	1.70	.906 043	4.32	.093 957	9
52	.797 621	2.60	.891 319	1.70	.906 302	4.30	.093 698	8
53	.797 777	2.62	.891 217	1.70	.906 560	4.32	.093 440	7
54	.797 934	2.62	.891 115	1.70	.906 819	4.30	.093 181	6
55	9.798 091	2.60	9.891 013	1.70	9.907 077	4.32	0.092 923	5
56	.798 247	2.60	.890 911	1.70	.907 336	4.30	.092 664	4
57	.798 403	2.62	.890 809	1.70	.907 594	4.32	.092 406	3
58	.798 560	2.60	.890 707	1.70	.907 853	4.30	.092 147	2
59	.798 716	2.60	.890 605	1.70	.908 111	4.30	.091 889	1
60	9.798 872		9.890 503		9.908 369		0.091 631	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

51°

39°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.798 872	2.60	9.890 503	1.72	9.908 369	4.32	0.091 631	60
1	.799 028	2.60	.890 400	1.70	.908 628	4.30	.091 372	59
2	.799 184	2.58	.890 298	1.72	.908 886	4.30	.091 114	58
3	.799 339	2.60	.890 195	1.70	.909 144	4.30	.090 856	57
4	.799 495	2.60	.890 093	1.72	.909 402	4.30	.090 598	56
5	9.799 651	2.58	9.889 990	1.70	9.909 660	4.30	0.090 340	55
6	.799 806	2.60	.889 888	1.72	.909 918	4.32	.090 082	54
7	.799 962	2.58	.889 785	1.72	.910 177	4.30	.089 823	53
8	.800 117	2.58	.889 682	1.72	.910 435	4.30	.089 565	52
9	.800 272	2.58	.889 579	1.70	.910 693	4.30	.089 307	51
10	9.800 427	2.58	9.889 477	1.72	9.910 951	4.30	0.089 049	50
11	.800 582	2.58	.889 374	1.72	.911 209	4.30	.088 791	49
12	.800 737	2.58	.889 271	1.72	.911 467	4.30	.088 533	48
13	.800 892	2.58	.889 168	1.73	.911 725	4.28	.088 275	47
14	.801 047	2.57	.889 064	1.72	.911 982	4.30	.088 018	46
15	9.801 201	2.58	9.888 961	1.72	9.912 240	4.30	0.087 760	45
16	.801 356	2.58	.888 858	1.72	.912 498	4.30	.087 502	44
17	.801 511	2.57	.888 755	1.73	.912 756	4.30	.087 244	43
18	.801 665	2.57	.888 651	1.72	.913 014	4.28	.086 986	42
19	.801 819	2.57	.888 548	1.73	.913 271	4.30	.086 729	41
20	9.801 973	2.58	9.888 444	1.72	9.913 529	4.30	0.086 471	40
21	.802 128	2.57	.888 341	1.73	.913 787	4.28	.086 213	39
22	.802 282	2.57	.888 237	1.72	.914 044	4.30	.085 956	38
23	.802 436	2.55	.888 134	1.73	.914 302	4.30	.085 698	37
24	.802 589	2.57	.888 030	1.73	.914 560	4.28	.085 440	36
25	9.802 743	2.57	9.887 926	1.73	9.914 817	4.30	0.085 183	35
26	.802 897	2.55	.887 822	1.73	.915 075	4.28	.084 925	34
27	.803 050	2.57	.887 718	1.73	.915 332	4.30	.084 668	33
28	.803 204	2.55	.887 614	1.73	.915 590	4.28	.084 410	32
29	.803 357	2.57	.887 510	1.73	.915 847	4.28	.084 153	31
30	9.803 511	2.55	9.887 406	1.73	9.916 104	4.30	0.083 896	30
31	.803 664	2.55	.887 302	1.73	.916 362	4.28	.083 638	29
32	.803 817	2.55	.887 198	1.75	.916 619	4.30	.083 381	28
33	.803 970	2.55	.887 093	1.73	.916 877	4.28	.083 123	27
34	.804 123	2.55	.886 989	1.73	.917 134	4.28	.082 866	26
35	9.804 276	2.53	9.886 885	1.75	9.917 391	4.28	0.082 609	25
36	.804 428	2.55	.886 780	1.73	.917 648	4.30	.082 352	24
37	.804 581	2.55	.886 676	1.75	.917 906	4.28	.082 094	23
38	.804 734	2.53	.886 571	1.75	.918 163	4.28	.081 837	22
39	.804 886	2.55	.886 466	1.73	.918 420	4.28	.081 580	21
40	9.805 039	2.53	9.886 362	1.75	9.918 677	4.28	0.081 323	20
41	.805 191	2.53	.886 257	1.75	.918 934	4.28	.081 066	19
42	.805 343	2.53	.886 152	1.75	.919 191	4.28	.080 809	18
43	.805 495	2.53	.886 047	1.75	.919 448	4.28	.080 552	17
44	.805 647	2.53	.885 942	1.75	.919 705	4.28	.080 295	16
45	9.805 799	2.53	9.885 837	1.75	9.919 962	4.28	0.080 038	15
46	.805 951	2.53	.885 732	1.75	.920 219	4.28	.079 781	14
47	.806 103	2.52	.885 627	1.75	.920 476	4.28	.079 524	13
48	.806 254	2.53	.885 522	1.77	.920 733	4.28	.079 267	12
49	.806 406	2.52	.885 416	1.75	.920 990	4.28	.079 010	11
50	9.806 557	2.53	9.885 311	1.77	9.921 247	4.27	0.078 753	10
51	.806 709	2.52	.885 205	1.75	.921 503	4.28	.078 497	9
52	.806 860	2.52	.885 100	1.77	.921 760	4.28	.078 240	8
53	.807 011	2.53	.884 994	1.75	.922 017	4.28	.077 983	7
54	.807 163	2.52	.884 889	1.77	.922 274	4.27	.077 726	6
55	9.807 314	2.52	9.884 783	1.77	9.922 530	4.28	0.077 470	5
56	.807 465	2.50	.884 677	1.75	.922 787	4.28	.077 213	4
57	.807 615	2.52	.884 572	1.77	.923 044	4.27	.076 956	3
58	.807 766	2.52	.884 466	1.77	.923 300	4.28	.076 700	2
59	.807 917	2.50	.884 360	1.77	.923 557	4.28	.076 443	1
60	9.808 067		9.884 254		9.923 814		0.076 186	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

50°

40°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.808 067	2.52	9.884 254	1.77	9.923 814	4.27	0.076 186	60
1	.808 218	2.50	.884 148	1.77	.924 070	4.28	.075 930	59
2	.808 368	2.52	.884 042	1.77	.924 327	4.27	.075 673	58
3	.808 519	2.50	.883 936	1.78	.924 583	4.28	.075 417	57
4	.808 669	2.50	.883 829	1.77	.924 840	4.27	.075 160	56
5	9.808 819	2.50	9.883 723	1.77	9.925 096	4.27	0.074 904	55
6	.808 969	2.50	.883 617	1.78	.925 352	4.28	.074 648	54
7	.809 119	2.50	.883 510	1.77	.925 609	4.27	.074 391	53
8	.809 269	2.50	.883 404	1.78	.925 865	4.28	.074 135	52
9	.809 419	2.50	.883 297	1.77	.926 122	4.27	.073 878	51
10	9.809 569	2.48	9.883 191	1.78	9.926 378	4.27	0.073 622	50
11	.809 718	2.50	.883 084	1.78	.926 634	4.27	.073 366	49
12	.809 868	2.48	.882 977	1.77	.926 890	4.28	.073 110	48
13	.810 017	2.50	.882 871	1.78	.927 147	4.27	.072 853	47
14	.810 167	2.48	.882 764	1.78	.927 403	4.27	.072 597	46
15	9.810 316	2.48	9.882 657	1.78	9.927 659	4.27	0.072 341	45
16	.810 465	2.48	.882 550	1.78	.927 915	4.27	.072 085	44
17	.810 614	2.48	.882 443	1.78	.928 171	4.27	.071 829	43
18	.810 763	2.48	.882 336	1.78	.928 427	4.28	.071 573	42
19	.810 912	2.48	.882 229	1.80	.928 684	4.27	.071 316	41
20	9.811 061	2.48	9.882 121	1.78	9.928 940	4.27	0.071 060	40
21	.811 210	2.47	.882 014	1.78	.929 196	4.27	.070 804	39
22	.811 358	2.48	.881 907	1.80	.929 452	4.27	.070 548	38
23	.811 507	2.47	.881 799	1.78	.929 708	4.27	.070 292	37
24	.811 655	2.48	.881 692	1.80	.929 964	4.27	.070 036	36
25	9.811 804	2.47	9.881 584	1.78	9.930 220	4.25	0.069 780	35
26	.811 952	2.47	.881 477	1.80	.930 475	4.27	.069 525	34
27	.812 100	2.47	.881 369	1.80	.930 731	4.27	.069 269	33
28	.812 248	2.47	.881 261	1.80	.930 987	4.27	.069 013	32
29	.812 396	2.47	.881 153	1.78	.931 243	4.27	.068 757	31
30	9.812 544	2.47	9.881 046	1.80	9.931 499	4.27	0.068 501	30
31	.812 692	2.47	.880 938	1.80	.931 755	4.25	.068 245	29
32	.812 840	2.47	.880 830	1.80	.932 010	4.27	.067 990	28
33	.812 988	2.45	.880 722	1.82	.932 266	4.27	.067 734	27
34	.813 135	2.47	.880 613	1.80	.932 522	4.27	.067 478	26
35	9.813 283	2.45	9.880 505	1.80	9.932 778	4.25	0.067 222	25
36	.813 430	2.47	.880 397	1.80	.933 033	4.27	.066 967	24
37	.813 578	2.45	.880 289	1.82	.933 289	4.27	.066 711	23
38	.813 725	2.45	.880 180	1.80	.933 545	4.25	.066 455	22
39	.813 872	2.45	.880 072	1.82	.933 800	4.27	.066 200	21
40	9.814 019	2.45	9.879 963	1.80	9.934 056	4.25	0.065 944	20
41	.814 166	2.45	.879 855	1.82	.934 311	4.27	.065 689	19
42	.814 313	2.45	.879 746	1.82	.934 567	4.25	.065 433	18
43	.814 460	2.45	.879 637	1.80	.934 822	4.27	.065 178	17
44	.814 607	2.43	.879 529	1.82	.935 078	4.25	.064 922	16
45	9.814 753	2.45	9.879 420	1.82	9.935 333	4.27	0.064 667	15
46	.814 900	2.43	.879 311	1.82	.935 589	4.25	.064 411	14
47	.815 046	2.45	.879 202	1.82	.935 844	4.27	.064 156	13
48	.815 193	2.43	.879 093	1.82	.936 100	4.25	.063 900	12
49	.815 339	2.43	.878 984	1.82	.936 355	4.27	.063 645	11
50	9.815 485	2.45	9.878 875	1.82	9.936 611	4.25	0.063 389	10
51	.815 632	2.43	.878 766	1.83	.936 866	4.25	.063 134	9
52	.815 778	2.43	.878 656	1.82	.937 121	4.27	.062 879	8
53	.815 924	2.42	.878 547	1.82	.937 377	4.25	.062 623	7
54	.816 069	2.43	.878 438	1.83	.937 632	4.25	.062 368	6
55	9.816 215	2.43	9.878 328	1.82	9.937 887	4.25	0.062 113	5
56	.816 361	2.43	.878 219	1.83	.938 142	4.27	.061 858	4
57	.816 507	2.42	.878 109	1.83	.938 398	4.25	.061 602	3
58	.816 652	2.43	.877 999	1.82	.938 653	4.25	.061 347	2
59	.816 798	2.42	.877 890	1.83	.938 908	4.25	.061 092	1
60	9.816 943		9.877 780		9.939 163		0.060 837	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

49°

41°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.816 943	2.42	9.877 780	1.83	9.939 163	4.25	0.060 837	60
1	.817 088	2.42	.877 670	1.83	.939 418	4.25	.060 582	59
2	.817 233	2.43	.877 560	1.83	.939 673	4.25	.060 327	58
3	.817 379	2.42	.877 450	1.83	.939 928	4.25	.060 072	57
4	.817 524	2.40	.877 340	1.83	.940 183	4.27	.059 817	56
5	9.817 668	2.42	9.877 230	1.83	9.940 439	4.25	0.059 561	55
6	.817 813	2.42	.877 120	1.83	.940 694	4.25	.059 306	54
7	.817 958	2.42	.877 010	1.85	.940 949	4.25	.059 051	53
8	.818 103	2.40	.876 899	1.83	.941 204	4.25	.058 796	52
9	.818 247	2.42	.876 789	1.85	.941 459	4.23	.058 541	51
10	9.818 392	2.40	9.876 678	1.83	9.941 713	4.25	0.058 287	50
11	.818 536	2.42	.876 568	1.85	.941 968	4.25	.058 032	49
12	.818 681	2.40	.876 457	1.83	.942 223	4.25	.057 777	48
13	.818 825	2.40	.876 347	1.85	.942 478	4.25	.057 522	47
14	.818 969	2.40	.876 236	1.85	.942 733	4.25	.057 267	46
15	9.819 113	2.40	9.876 125	1.85	9.942 988	4.25	0.057 012	45
16	.819 257	2.40	.876 014	1.83	.943 243	4.25	.056 757	44
17	.819 401	2.40	.875 904	1.85	.943 498	4.23	.056 502	43
18	.819 545	2.40	.875 793	1.85	.943 752	4.25	.056 248	42
19	.819 689	2.38	.875 682	1.85	.944 007	4.25	.055 993	41
20	9.819 832	2.40	9.875 571	1.87	9.944 262	4.25	0.055 738	40
21	.819 976	2.40	.875 459	1.85	.944 517	4.23	.055 483	39
22	.820 120	2.38	.875 348	1.85	.944 771	4.25	.055 229	38
23	.820 263	2.38	.875 237	1.85	.945 026	4.25	.054 974	37
24	.820 406	2.40	.875 126	1.87	.945 281	4.23	.054 719	36
25	9.820 550	2.38	9.875 014	1.85	9.945 535	4.25	0.054 465	35
26	.820 693	2.38	.874 903	1.87	.945 790	4.25	.054 210	34
27	.820 836	2.38	.874 791	1.85	.946 045	4.23	.053 955	33
28	.820 979	2.38	.874 680	1.87	.946 299	4.25	.053 701	32
29	.821 122	2.38	.874 568	1.87	.946 554	4.23	.053 446	31
30	9.821 265	2.37	9.874 456	1.87	9.946 808	4.25	0.053 192	30
31	.821 407	2.38	.874 344	1.87	.947 063	4.25	.052 937	29
32	.821 550	2.38	.874 232	1.85	.947 318	4.23	.052 682	28
33	.821 693	2.37	.874 121	1.87	.947 572	4.25	.052 428	27
34	.821 835	2.37	.874 009	1.88	.947 827	4.23	.052 173	26
35	9.821 977	2.38	9.873 896	1.87	9.948 081	4.23	0.051 919	25
36	.822 120	2.37	.873 784	1.87	.948 335	4.25	.051 665	24
37	.822 262	2.37	.873 672	1.87	.948 590	4.23	.051 410	23
38	.822 404	2.37	.873 560	1.87	.948 844	4.25	.051 156	22
39	.822 546	2.37	.873 448	1.88	.949 099	4.23	.050 901	21
40	9.822 688	2.37	9.873 335	1.87	9.949 353	4.25	0.050 647	20
41	.822 830	2.37	.873 223	1.88	.949 608	4.23	.050 392	19
42	.822 972	2.37	.873 110	1.87	.949 862	4.23	.050 138	18
43	.823 114	2.35	.872 998	1.88	.950 116	4.25	.049 884	17
44	.823 255	2.37	.872 885	1.88	.950 371	4.23	.049 629	16
45	9.823 397	2.37	9.872 772	1.88	9.950 625	4.23	0.049 375	15
46	.823 539	2.35	.872 659	1.87	.950 879	4.23	.049 121	14
47	.823 680	2.35	.872 547	1.88	.951 133	4.25	.048 867	13
48	.823 821	2.37	.872 434	1.88	.951 388	4.23	.048 612	12
49	.823 963	2.35	.872 321	1.88	.951 642	4.23	.048 358	11
50	9.824 104	2.35	9.872 208	1.88	9.951 896	4.23	0.048 104	10
51	.824 245	2.35	.872 095	1.90	.952 150	4.25	.047 850	9
52	.824 386	2.35	.871 981	1.88	.952 405	4.23	.047 595	8
53	.824 527	2.35	.871 868	1.88	.952 659	4.23	.047 341	7
54	.824 668	2.33	.871 755	1.90	.952 913	4.23	.047 087	6
55	9.824 808	2.35	9.871 641	1.88	9.953 167	4.23	0.046 833	5
56	.824 949	2.35	.871 528	1.90	.953 421	4.23	.046 579	4
57	.825 090	2.33	.871 414	1.88	.953 675	4.23	.046 325	3
58	.825 230	2.35	.871 301	1.90	.953 929	4.23	.046 071	2
59	.825 371	2.33	.871 187	1.90	.954 183	4.23	.045 817	1
60	9.825 511		9.871 073		9.954 437		0.045 563	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

48°

42°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.825 511	2.33	9.871 073	1.88	9.954 437	4.23	0.045 563	60
1	.825 651	2.33	.870 960	1.90	.954 691	4.25	.045 309	59
2	.825 791	2.33	.870 846	1.90	.954 946	4.23	.045 054	58
3	.825 931	2.33	.870 732	1.90	.955 200	4.23	.044 800	57
4	.826 071	2.33	.870 618	1.90	.955 454	4.23	.044 546	56
5	9.826 211	2.33	9.870 504	1.90	9.955 708	4.22	0.044 292	55
6	.826 351	2.33	.870 390	1.90	.955 961	4.23	.044 039	54
7	.826 491	2.33	.870 276	1.92	.956 215	4.23	.043 785	53
8	.826 631	2.32	.870 161	1.90	.956 469	4.23	.043 531	52
9	.826 770	2.33	.870 047	1.90	.956 723	4.23	.043 277	51
10	9.826 910	2.32	9.869 933	1.92	9.956 977	4.23	0.043 023	50
11	.827 049	2.33	.869 818	1.90	.957 231	4.23	.042 769	49
12	.827 189	2.32	.869 704	1.92	.957 485	4.23	.042 515	48
13	.827 328	2.32	.869 589	1.92	.957 739	4.23	.042 261	47
14	.827 467	2.32	.869 474	1.90	.957 993	4.23	.042 007	46
15	9.827 606	2.32	9.869 360	1.92	9.958 247	4.22	0.041 753	45
16	.827 745	2.32	.869 245	1.92	.958 500	4.23	.041 500	44
17	.827 884	2.32	.869 130	1.92	.958 754	4.23	.041 246	43
18	.828 023	2.32	.869 015	1.92	.959 008	4.23	.040 992	42
19	.828 162	2.32	.868 900	1.92	.959 262	4.23	.040 738	41
20	9.828 301	2.30	9.868 785	1.92	9.959 516	4.22	0.040 484	40
21	.828 439	2.32	.868 670	1.92	.959 769	4.23	.040 231	39
22	.828 578	2.30	.868 555	1.92	.960 023	4.23	.039 977	38
23	.828 716	2.32	.868 440	1.93	.960 277	4.22	.039 723	37
24	.828 855	2.30	.868 324	1.92	.960 530	4.23	.039 470	36
25	9.828 993	2.30	9.868 209	1.93	9.960 784	4.23	0.039 216	35
26	.829 131	2.30	.868 093	1.92	.961 038	4.23	.038 962	34
27	.829 269	2.30	.867 978	1.93	.961 292	4.22	.038 708	33
28	.829 407	2.30	.867 862	1.92	.961 545	4.23	.038 455	32
29	.829 545	2.30	.867 747	1.93	.961 799	4.22	.038 201	31
30	9.829 683	2.30	9.867 631	1.93	9.962 052	4.23	0.037 948	30
31	.829 821	2.30	.867 515	1.93	.962 306	4.23	.037 694	29
32	.829 959	2.30	.867 399	1.93	.962 560	4.22	.037 440	28
33	.830 097	2.28	.867 283	1.93	.962 813	4.23	.037 187	27
34	.830 234	2.30	.867 167	1.93	.963 067	4.22	.036 933	26
35	9.830 372	2.28	9.867 051	1.93	9.963 320	4.23	0.036 680	25
36	.830 509	2.28	.866 935	1.93	.963 574	4.23	.036 426	24
37	.830 646	2.30	.866 819	1.93	.963 828	4.22	.036 172	23
38	.830 784	2.28	.866 703	1.95	.964 081	4.23	.035 919	22
39	.830 921	2.28	.866 586	1.93	.964 335	4.22	.035 665	21
40	9.831 058	2.28	9.866 470	1.95	9.964 588	4.23	0.035 412	20
41	.831 195	2.28	.866 353	1.93	.964 842	4.22	.035 158	19
42	.831 332	2.28	.866 237	1.95	.965 095	4.23	.034 905	18
43	.831 469	2.28	.866 120	1.93	.965 349	4.22	.034 651	17
44	.831 606	2.27	.866 004	1.95	.965 602	4.22	.034 398	16
45	9.831 742	2.28	9.865 887	1.95	9.965 855	4.23	0.034 145	15
46	.831 879	2.27	.865 770	1.95	.966 109	4.22	.033 891	14
47	.832 015	2.28	.865 653	1.95	.966 362	4.23	.033 638	13
48	.832 152	2.27	.865 536	1.95	.966 616	4.22	.033 384	12
49	.832 288	2.28	.865 419	1.95	.966 869	4.23	.033 131	11
50	9.832 425	2.27	9.865 302	1.95	9.967 123	4.22	0.032 877	10
51	.832 561	2.27	.865 185	1.95	.967 376	4.22	.032 624	9
52	.832 697	2.27	.865 068	1.97	.967 629	4.23	.032 371	8
53	.832 833	2.27	.864 950	1.95	.967 883	4.22	.032 117	7
54	.832 969	2.27	.864 833	1.95	.968 136	4.22	.031 864	6
55	9.833 105	2.27	9.864 716	1.97	9.968 389	4.23	0.031 611	5
56	.833 241	2.27	.864 598	1.95	.968 643	4.22	.031 357	4
57	.833 377	2.25	.864 481	1.97	.968 896	4.22	.031 104	3
58	.833 512	2.27	.864 363	1.97	.969 149	4.23	.030 851	2
59	.833 648	2.25	.864 245	1.97	.969 403	4.22	.030 597	1
60	9.833 783		9.864 127		9.969 656		0.030 344	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

47°

43°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.833 783	2.27	9.864 127	1.95	9.969 656	4.22	0.030 344	60
1	.833 919	2.25	.864 010	1.97	.969 909	4.22	.030 091	59
2	.834 054	2.25	.863 892	1.97	.970 162	4.23	.029 838	58
3	.834 189	2.27	.863 774	1.97	.970 416	4.22	.029 584	57
4	.834 325	2.25	.863 656	1.97	.970 669	4.22	.029 331	56
5	9.834 460	2.25	9.863 538	1.98	9.970 922	4.22	0.029 078	55
6	.834 595	2.25	.863 419	1.97	.971 175	4.23	.028 825	54
7	.834 730	2.25	.863 301	1.97	.971 429	4.22	.028 571	53
8	.834 865	2.23	.863 183	1.98	.971 682	4.22	.028 318	52
9	.834 999	2.25	.863 064	1.97	.971 935	4.22	.028 065	51
10	9.835 134	2.25	9.862 946	1.98	9.972 188	4.22	0.027 812	50
11	.835 269	2.23	.862 827	1.97	.972 441	4.23	.027 559	49
12	.835 403	2.25	.862 709	1.98	.972 695	4.22	.027 305	48
13	.835 538	2.23	.862 590	1.98	.972 948	4.22	.027 052	47
14	.835 672	2.25	.862 471	1.97	.973 201	4.22	.026 799	46
15	9.835 807	2.23	9.862 353	1.98	9.973 454	4.22	0.026 546	45
16	.835 941	2.23	.862 234	1.98	.973 707	4.22	.026 293	44
17	.836 075	2.23	.862 115	1.98	.973 960	4.22	.026 040	43
18	.836 209	2.23	.861 996	1.98	.974 213	4.22	.025 787	42
19	.836 343	2.23	.861 877	1.98	.974 466	4.23	.025 534	41
20	9.836 477	2.23	9.861 758	2.00	9.974 720	4.22	0.025 280	40
21	.836 611	2.23	.861 638	1.98	.974 973	4.22	.025 027	39
22	.836 745	2.22	.861 519	1.98	.975 226	4.22	.024 774	38
23	.836 878	2.23	.861 400	2.00	.975 479	4.22	.024 521	37
24	.837 012	2.23	.861 280	1.98	.975 732	4.22	.024 268	36
25	9.837 146	2.22	9.861 161	2.00	9.975 985	4.22	0.024 015	35
26	.837 279	2.22	.861 041	1.98	.976 238	4.22	.023 762	34
27	.837 412	2.23	.860 922	2.00	.976 491	4.22	.023 509	33
28	.837 546	2.22	.860 802	2.00	.976 744	4.22	.023 256	32
29	.837 679	2.22	.860 682	2.00	.976 997	4.22	.023 003	31
30	9.837 812	2.22	9.860 562	2.00	9.977 250	4.22	0.022 750	30
31	.837 945	2.22	.860 442	2.00	.977 503	4.22	.022 497	29
32	.838 078	2.22	.860 322	2.00	.977 756	4.22	.022 244	28
33	.838 211	2.22	.860 202	2.00	.978 009	4.22	.021 991	27
34	.838 344	2.22	.860 082	2.00	.978 262	4.22	.021 738	26
35	9.838 477	2.22	9.859 962	2.00	9.978 515	4.22	0.021 485	25
36	.838 610	2.20	.859 842	2.02	.978 768	4.22	.021 232	24
37	.838 742	2.22	.859 721	2.00	.979 021	4.22	.020 979	23
38	.838 875	2.20	.859 601	2.02	.979 274	4.22	.020 726	22
39	.839 007	2.22	.859 480	2.00	.979 527	4.22	.020 473	21
40	9.839 140	2.20	9.859 360	2.02	9.979 780	4.22	0.020 220	20
41	.839 272	2.20	.859 239	2.00	.980 033	4.22	.019 967	19
42	.839 404	2.20	.859 119	2.02	.980 286	4.20	.019 714	18
43	.839 536	2.20	.858 998	2.02	.980 538	4.22	.019 462	17
44	.839 668	2.20	.858 877	2.02	.980 791	4.22	.019 209	16
45	9.839 800	2.20	9.858 756	2.02	9.981 044	4.22	0.018 956	15
46	.839 932	2.20	.858 635	2.02	.981 297	4.22	.018 703	14
47	.840 064	2.20	.858 514	2.02	.981 550	4.22	.018 450	13
48	.840 196	2.20	.858 393	2.02	.981 803	4.22	.018 197	12
49	.840 328	2.18	.858 272	2.02	.982 056	4.22	.017 944	11
50	9.840 459	2.20	9.858 151	2.03	9.982 309	4.22	0.017 691	10
51	.840 591	2.18	.858 029	2.02	.982 562	4.20	.017 438	9
52	.840 722	2.20	.857 908	2.03	.982 814	4.22	.017 186	8
53	.840 854	2.18	.857 786	2.02	.983 067	4.22	.016 933	7
54	.840 985	2.18	.857 665	2.03	.983 320	4.22	.016 680	6
55	9.841 116	2.18	9.857 543	2.02	9.983 573	4.22	0.016 427	5
56	.841 247	2.18	.857 422	2.03	.983 826	4.22	.016 174	4
57	.841 378	2.18	.857 300	2.03	.984 079	4.22	.015 921	3
58	.841 509	2.18	.857 178	2.03	.984 332	4.20	.015 668	2
59	.841 640	2.18	.857 056	2.03	.984 584	4.22	.015 416	1
60	9.841 771		9.856 934		9.984 837		0.015 163	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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44°

M.	Sin.	D. 1″.	Cos.	D. 1″.	Tan.	D. 1″.	Cot.	
0	9.841 771	2.18	9.856 934	2.03	9.984 837	4.22	0.015 163	60
1	.841 902	2.18	.856 812	2.03	.985 090	4.22	.014 910	59
2	.842 033	2.17	.856 690	2.03	.985 343	4.22	.014 657	58
3	.842 163	2.18	.856 568	2.03	.985 596	4.20	.014 404	57
4	.842 294	2.17	.856 446	2.05	.985 848	4.22	.014 152	56
5	9.842 424	2.18	9.856 323	2.03	9.986 101	4.22	0.013 899	55
6	.842 555	2.17	.856 201	2.05	.986 354	4.22	.013 646	54
7	.842 685	2.17	.856 078	2.03	.986 607	4.22	.013 393	53
8	.842 815	2.18	.855 956	2.05	.986 860	4.20	.013 140	52
9	.842 946	2.17	.855 833	2.03	.987 112	4.22	.012 888	51
10	9.843 076	2.17	9.855 711	2.05	9.987 365	4.22	0.012 635	50
11	.843 206	2.17	.855 588	2.05	.987 618	4.22	.012 382	49
12	.843 336	2.17	.855 465	2.05	.987 871	4.20	.012 129	48
13	.843 466	2.15	.855 342	2.05	.988 123	4.22	.011 877	47
14	.843 595	2.17	.855 219	2.05	.988 376	4.22	.011 624	46
15	9.843 725	2.17	9.855 096	2.05	9.988 629	4.22	0.011 371	45
16	.843 855	2.15	.854 973	2.05	.988 882	4.20	.011 118	44
17	.843 984	2.17	.854 850	2.05	.989 134	4.22	.010 866	43
18	.844 114	2.15	.854 727	2.07	.989 387	4.22	.010 613	42
19	.844 243	2.15	.854 603	2.05	.989 640	4.22	.010 360	41
20	9.844 372	2.17	9.854 480	2.07	9.989 893	4.20	0.010 107	40
21	.844 502	2.15	.854 356	2.05	.990 145	4.22	.009 855	39
22	.844 631	2.15	.854 233	2.07	.990 398	4.22	.009 602	38
23	.844 760	2.15	.854 109	2.05	.990 651	4.20	.009 349	37
24	.844 889	2.15	.853 986	2.07	.990 903	4.22	.009 097	36
25	9.845 018	2.15	9.853 862	2.07	9.991 156	4.22	0.008 844	35
26	.845 147	2.15	.853 738	2.07	.991 409	4.22	.008 591	34
27	.845 276	2.15	.853 614	2.07	.991 662	4.20	.008 338	33
28	.845 405	2.13	.853 490	2.07	.991 914	4.22	.008 086	32
29	.845 533	2.15	.853 366	2.07	.992 167	4.22	.007 833	31
30	9.845 662	2.13	9.853 242	2.07	9.992 420	4.20	0.007 580	30
31	.845 790	2.15	.853 118	2.07	.992 672	4.22	.007 328	29
32	.845 919	2.13	.852 994	2.08	.992 925	4.22	.007 075	28
33	.846 047	2.13	.852 869	2.07	.993 178	4.22	.006 822	27
34	.846 175	2.15	.852 745	2.08	.993 431	4.20	.006 569	26
35	9.846 304	2.13	9.852 620	2.07	9.993 683	4.22	0.006 317	25
36	.846 432	2.13	.852 496	2.08	.993 936	4.22	.006 064	24
37	.846 560	2.13	.852 371	2.07	.994 189	4.20	.005 811	23
38	.846 688	2.13	.852 247	2.08	.994 441	4.22	.005 559	22
39	.846 816	2.13	.852 122	2.08	.994 694	4.22	.005 306	21
40	9.846 944	2.12	9.851 997	2.08	9.994 947	4.20	0.005 053	20
41	.847 071	2.13	.851 872	2.08	.995 199	4.22	.004 801	19
42	.847 199	2.13	.851 747	2.08	.995 452	4.22	.004 548	18
43	.847 327	2.12	.851 622	2.08	.995 705	4.20	.004 295	17
44	.847 454	2.13	.851 497	2.08	.995 957	4.22	.004 043	16
45	9.847 582	2.12	9.851 372	2.10	9.996 210	4.22	0.003 790	15
46	.847 709	2.12	.851 246	2.08	.996 463	4.20	.003 537	14
47	.847 836	2.13	.851 121	2.08	.996 715	4.22	.003 285	13
48	.847 964	2.12	.850 996	2.10	.996 968	4.22	.003 032	12
49	.848 091	2.12	.850 870	2.08	.997 221	4.20	.002 779	11
50	9.848 218	2.12	9.850 745	2.10	9.997 473	4.22	0.002 527	10
51	.848 345	2.12	.850 619	2.10	.997 726	4.22	.002 274	9
52	.848 472	2.12	.850 493	2.08	.997 979	4.20	.002 021	8
53	.848 599	2.12	.850 368	2.10	.998 231	4.22	.001 769	7
54	.848 726	2.10	.850 242	2.10	.998 484	4.22	.001 516	6
55	9.848 852	2.12	9.850 116	2.10	9.998 737	4.20	0.001 263	5
56	.848 979	2.12	.849 990	2.10	.998 989	4.22	.001 011	4
57	.849 106	2.10	.849 864	2.10	.999 242	4.22	.000 758	3
58	.849 232	2.12	.849 738	2.12	.999 495	4.20	.000 505	2
59	.849 359	2.10	.849 611	2.10	.999 747	4.22	.000 253	1
60	9.849 485		9.849 485		0.000 000		0.000 000	0
	Cos.	D. 1″.	Sin.	D. 1″.	Cot.	D. 1″.	Tan.	M.

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A TABLE

OF THE

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS,

FOR EVERY

DEGREE AND MINUTE FROM 0° TO 90°.

	0°				1°				2°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.00000	1.0000	.00000	∞	.01745	.99985	.01746	57.290	.03490	.99939	.03492	28.636	60
1	029	000	029	3437.7	774	984	775	56.351	519	938	521	.399	59
2	058	000	058	1718.9	803	984	804	55.442	548	937	550	.166	58
3	087	000	087	1145.9	832	983	833	54.561	577	936	579	27.937	57
4	116	000	116	859.44	862	983	862	53.709	606	935	609	.712	56
5	.00145	1.0000	.00145	687.55	.01891	.99982	.01891	52.882	.03635	.99934	.03638	27.490	55
6	175	000	175	572.96	920	982	920	.081	664	933	667	.271	54
7	204	000	204	491.11	949	981	949	51.303	693	932	696	.057	53
8	233	000	233	429.72	978	980	978	50.549	723	931	725	26.845	52
9	262	000	262	381.97	.02007	980	.02007	49.816	752	930	754	.637	51
10	.00291	1.0000	.00291	343.77	.02036	.99979	.02036	49.104	.03781	.99929	.03783	26.432	50
11	320	.99999	320	312.52	065	979	066	48.412	810	927	812	.230	49
12	349	999	349	286.48	094	978	095	47.740	839	926	842	.031	48
13	378	999	378	264.44	123	977	124	.085	868	925	871	25.835	47
14	407	999	407	245.55	152	977	153	46.449	897	924	900	.642	46
15	.00436	.99999	.00436	229.18	.02181	.99976	.02182	45.829	.03926	.99923	.03929	25.452	45
16	465	999	465	214.86	211	976	211	.226	955	922	958	.264	44
17	495	999	495	202.22	240	975	240	44.639	984	921	987	.080	43
18	524	999	524	190.98	269	974	269	.066	.04013	919	.04016	24.898	42
19	553	998	553	180.93	298	974	298	43.508	042	918	046	.719	41
20	.00582	.99998	.00582	171.89	.02327	.99973	.02328	42.964	.04071	.99917	.04075	24.542	40
21	611	998	611	163.70	356	972	357	.433	100	916	104	.368	39
22	640	998	640	156.26	385	972	386	41.916	129	915	133	.196	38
23	669	998	669	149.47	414	971	415	.411	159	913	162	.026	37
24	698	998	698	143.24	443	970	444	40.917	188	912	191	23.859	36
25	.00727	.99997	.00727	137.51	.02472	.99969	.02473	40.436	.04217	.99911	.04220	23.695	35
26	756	997	756	132.22	501	969	502	39.965	246	910	250	.532	34
27	785	997	785	127.32	530	968	531	.506	275	909	279	.372	33
28	814	997	815	122.77	560	967	560	.057	304	907	308	.214	32
29	844	996	844	118.54	589	966	589	38.618	333	906	337	.058	31
30	.00873	.99996	.00873	114.59	.02618	.99966	.02619	38.188	.04362	.99905	.04366	22.904	30
31	902	996	902	110.89	647	965	648	37.769	391	904	395	.752	29
32	931	996	931	107.43	676	964	677	.358	420	902	424	.602	28
33	960	995	960	104.17	705	963	706	36.956	449	901	454	.454	27
34	989	995	989	101.11	734	963	735	.563	478	900	483	.308	26
35	.01018	.99995	.01018	98.218	.02763	.99962	.02764	36.178	.04507	.99898	.04512	22.164	25
36	047	995	047	95.489	792	961	793	35.801	536	897	541	.022	24
37	076	994	076	92.908	821	960	822	.431	565	896	570	21.881	23
38	105	994	105	90.463	850	959	851	.070	594	894	599	.743	22
39	134	994	135	88.144	879	959	881	34.715	623	893	628	.606	21
40	.01164	.99993	.01164	85.940	.02908	.99958	.02910	34.368	.04653	.99892	.04658	21.470	20
41	193	993	193	83.844	938	957	939	.027	682	890	687	.337	19
42	222	993	222	81.847	967	956	968	33.694	711	889	716	.205	18
43	251	992	251	79.943	996	955	997	.366	740	888	745	.075	17
44	280	992	280	78.126	.03025	954	.03026	.045	769	886	774	20.946	16
45	.01309	.99991	.01309	76.390	.03054	.99953	.03055	32.730	.04798	.99885	.04803	20.819	15
46	338	991	338	74.729	083	952	084	.421	827	883	833	.693	14
47	367	991	367	73.139	112	952	114	.118	856	882	862	.569	13
48	396	990	396	71.615	141	951	143	31.821	885	881	891	.446	12
49	425	990	425	70.153	170	950	172	.528	914	879	920	.325	11
50	.01454	.99989	.01455	68.750	.03199	.99949	.03201	31.242	.04943	.99878	.04949	20.206	10
51	483	989	484	67.402	228	948	230	30.960	972	876	978	.087	9
52	513	989	513	66.105	257	947	259	.683	.05001	875	.05007	19.970	8
53	542	988	542	64.858	286	946	288	.412	030	873	037	.855	7
54	571	988	571	63.657	316	945	317	.145	059	872	066	.740	6
55	.01600	.99987	.01600	62.499	.03345	.99944	.03346	29.882	.05088	.99870	.05095	19.627	5
56	629	987	629	61.383	374	943	376	.624	117	869	124	.516	4
57	658	986	658	60.306	403	942	405	.371	146	867	153	.405	3
58	687	986	687	59.266	432	941	434	.122	175	866	182	.296	2
59	716	985	716	58.261	461	940	463	28.877	205	864	212	.188	1
60	.01745	.99985	.01746	57.290	.03490	.99939	.03492	28.636	.05234	.99863	.05241	19.081	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	89°				88°				87°				

	3°				4°				5°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.05234	.99863	.05241	19.081	.06976	.99756	.06993	14.301	.08716	.99619	.08749	11.430	60
1	263	861	270	18.976	.07005	754	.07022	.241	745	617	778	.392	59
2	292	860	299	.871	034	752	051	.182	774	614	807	.354	58
3	321	858	328	.768	063	750	080	.124	803	612	837	.316	57
4	350	857	357	.666	092	748	110	.065	831	609	866	.279	56
5	.05379	.99855	.05387	18.564	.07121	.99746	.07139	14.008	.08860	.99607	.08895	11.242	55
6	408	854	416	.464	150	744	168	13.951	889	604	925	.205	54
7	437	852	445	.366	179	742	197	.894	918	602	954	.168	53
8	466	851	474	.268	208	740	227	.838	947	599	983	.132	52
9	495	849	503	.171	237	738	256	.782	976	596	.09013	.095	51
10	.05524	.99847	.05533	18.075	.07266	.99736	.07285	13.727	.09005	.99594	.09042	11.059	50
11	553	846	562	17.980	295	734	314	.672	034	591	071	.024	49
12	582	844	591	.886	324	731	344	.617	063	588	101	10.988	48
13	611	842	620	.793	353	729	373	.563	092	586	130	.953	47
14	640	841	649	.702	382	727	402	.510	121	583	159	.918	46
15	.05669	.99839	.05678	17.611	.07411	.99725	.07431	13.457	.09150	.99580	.09189	10.883	45
16	698	838	708	.521	440	723	461	.404	179	578	218	.848	44
17	727	836	737	.431	469	721	490	.352	208	575	247	.814	43
18	756	834	766	.343	498	719	519	.300	237	572	277	.780	42
19	785	833	795	.256	527	716	548	.248	266	570	306	.746	41
20	.05814	.99831	.05824	17.169	.07556	.99714	.07578	13.197	.09295	.99567	.09335	10.712	40
21	844	829	854	.084	585	712	607	.146	324	564	365	.678	39
22	873	827	883	16.999	614	710	636	.096	353	562	394	.645	38
23	902	826	912	.915	643	708	665	.046	382	559	423	.612	37
24	931	824	941	.832	672	705	695	12.996	411	556	453	.579	36
25	.05960	.99822	.05970	16.750	.07701	.99703	.07724	12.947	.09440	.99553	.09482	10.546	35
26	989	821	999	.668	730	701	753	.898	469	551	511	.514	34
27	.06018	819	.06029	.587	759	699	782	.850	498	548	541	.481	33
28	047	817	058	.507	788	696	812	.801	527	545	570	.449	32
29	076	815	087	.428	817	694	841	.754	556	542	600	.417	31
30	.06105	.99813	.06116	16.350	.07846	.99692	.07870	12.706	.09585	.99540	.09629	10.385	30
31	134	812	145	.272	875	689	899	.659	614	537	658	.354	29
32	163	810	175	.195	904	687	929	.612	642	534	688	.322	28
33	192	808	204	.119	933	685	958	.566	671	531	717	.291	27
34	221	806	233	.043	962	683	987	.520	700	528	746	.260	26
35	.06250	.99804	.06262	15.969	.07991	.99680	.08017	12.474	.09729	.99526	.09776	10.229	25
36	279	803	291	.895	.08020	678	046	.429	758	523	805	.199	24
37	308	801	321	.821	049	676	075	.384	787	520	834	.168	23
38	337	799	350	.748	078	673	104	.339	816	517	864	.138	22
39	366	797	379	.676	107	671	134	.295	845	514	893	.108	21
40	.06395	.99795	.06408	15.605	.08136	.99668	.08163	12.251	.09874	.99511	.09923	10.078	20
41	424	793	438	.534	165	666	192	.207	903	508	952	.048	19
42	453	792	467	.464	194	664	221	.163	932	506	981	.019	18
43	482	790	496	.394	223	661	251	.120	961	503	.10011	9.9893	17
44	511	788	525	.325	252	659	280	.077	990	500	040	.9601	16
45	.06540	.99786	.06554	15.257	.08281	.99657	.08309	12.035	.10019	.99497	.10069	9.9310	15
46	569	784	584	.189	310	654	339	11.992	048	494	099	.9021	14
47	598	782	613	.122	339	652	368	.950	077	491	128	.8734	13
48	627	780	642	.056	368	649	397	.909	106	488	158	.8448	12
49	656	778	671	14.990	397	647	427	.867	135	485	187	.8164	11
50	.06685	.99776	.06700	14.924	.08426	.99644	.08456	11.826	.10164	.99482	.10216	9.7882	10
51	714	774	730	.860	455	642	485	.785	192	479	246	.7601	9
52	743	772	759	.795	484	639	514	.745	221	476	275	.7322	8
53	773	770	788	.732	513	637	544	.705	250	473	305	.7044	7
54	802	768	817	.669	542	635	573	.664	279	470	334	.6768	6
55	.06831	.99766	.06847	14.606	.08571	.99632	.08602	11.625	.10308	.99467	.10363	9.6493	5
56	860	764	876	.544	600	630	632	.585	337	464	393	.6220	4
57	889	762	905	.482	629	627	661	.546	366	461	422	.5949	3
58	918	760	934	.421	658	625	690	.507	395	458	452	.5679	2
59	947	758	963	.361	687	622	720	.468	424	455	481	.5411	1
60	.06976	.99756	.06993	14.301	.08716	.99619	.08749	11.430	.10453	.99452	.10510	9.5144	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	86°				85°				84°				

	6°				7°				8°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.10453	.99452	.10510	9.5144	.12187	.99255	.12278	8.1443	.13917	.99027	.14054	7.1154	60
1	482	449	540	.4878	216	251	308	.1248	946	023	084	.1004	59
2	511	446	569	.4614	245	248	338	.1054	975	019	113	.0855	58
3	540	443	599	.4352	274	244	367	.0860	.14004	015	143	.0706	57
4	569	440	628	.4090	302	240	397	.0667	033	011	173	.0558	56
5	.10597	.99437	.10657	9.3831	.12331	.99237	.12426	8.0476	.14061	.99006	.14202	7.0410	55
6	626	434	687	.3572	360	233	456	.0285	090	002	232	.0264	54
7	655	431	716	.3315	389	230	485	.0095	119	.98998	262	.0117	53
8	684	428	746	.3060	418	226	515	7.9906	148	994	291	6.9972	52
9	713	424	775	.2806	447	222	544	.9718	177	990	321	.9827	51
10	.10742	.99421	.10805	9.2553	.12476	.99219	.12574	7.9530	.14205	.98986	.14351	6.9682	50
11	771	418	834	.2302	504	215	603	.9344	234	982	381	.9538	49
12	800	415	863	.2052	533	211	633	.9158	263	978	410	.9395	48
13	829	412	893	.1803	562	208	662	.8973	292	973	440	.9252	47
14	858	409	922	.1555	591	204	692	.8789	320	969	470	.9110	46
15	.10887	.99406	.10952	9.1309	.12620	.99200	.12722	7.8606	.14349	.98965	.14499	6.8969	45
16	916	402	981	.1065	649	197	751	.8424	378	961	529	.8828	44
17	945	399	.11011	.0821	678	193	781	.8243	407	957	559	.8687	43
18	973	396	040	.0579	706	189	810	.8062	436	953	588	.8548	42
19	.11002	393	070	.0338	735	186	840	.7882	464	948	618	.8408	41
20	.11031	.99390	.11099	9.0098	.12764	.99182	.12869	7.7704	.14493	.98944	.14648	6.8269	40
21	060	386	128	8.9860	793	178	899	.7525	522	940	678	.8131	39
22	089	383	158	.9623	822	175	929	.7348	551	936	707	.7994	38
23	118	380	187	.9387	851	171	958	.7171	580	931	737	.7856	37
24	1·47	377	217	.9152	880	167	988	.6996	608	927	767	.7720	36
25	.11176	.99374	.11246	8.8919	.12908	.99163	.13017	7.6821	.14637	.98923	.14796	6.7584	35
26	205	370	276	.8686	937	160	047	.6647	666	919	826	.7448	34
27	234	367	305	.8455	966	156	076	.6473	695	914	856	.7313	33
28	263	364	335	.8225	995	152	106	.6301	723	910	886	.7179	32
29	291	360	364	.7996	.13024	148	136	.6129	752	906	915	.7045	31
30	.11320	.99357	.11394	8.7769	.13053	.99144	.13165	7.5958	.14781	.98902	.14945	6.6912	30
31	349	354	423	.7542	081	141	195	.5787	810	897	975	.6779	29
32	378	351	452	.7317	110	137	224	.5618	838	893	.15005	.6646	28
33	407	347	462	.7093	139	133	254	.5449	867	889	034	.6514	27
34	436	344	511	.6870	168	129	284	.5281	896	884	064	.6383	26
35	.11465	.99341	.11541	8.6648	.13197	.99125	.13313	7.5113	.14925	.98880	.15094	6.6252	25
36	494	337	570	.6427	226	122	343	.4947	954	876	124	.6122	24
37	523	334	600	.6208	254	118	372	.4781	982	871	153	.5992	23
38	552	331	629	.5989	283	114	402	.4615	.15011	867	183	.5863	22
39	580	327	659	.5772	312	110	432	.4451	040	863	213	.5734	21
40	.11609	.99324	.11688	8.5555	.13341	.99106	.13461	7.4287	.15069	.98858	.15243	6.5606	20
41	638	320	718	.5340	370	102	491	.4124	097	854	272	.5478	19
42	667	317	747	.5126	399	098	521	.3962	126	849	302	.5350	18
43	696	314	777	.4913	427	094	550	.3800	155	845	332	.5223	17
44	725	310	806	.4701	456	091	580	.3639	184	841	362	.5097	16
45	.11754	.99307	.11836	8.4490	.13485	.99087	.13609	7.3479	.15212	.98836	.15391	6.4971	15
46	783	303	865	.4280	514	083	639	.3319	241	832	421	.4846	14
47	812	300	895	.4071	543	079	669	.3160	270	827	451	.4721	13
48	840	297	924	.3863	572	075	698	.3002	299	823	481	.4596	12
49	869	293	954	.3656	600	071	728	.2844	327	818	511	.4472	11
50	.11898	.99290	.11983	8.3450	.13629	.99067	.13758	7.2687	.15356	.98814	.15540	6.4348	10
51	927	286	.12013	.3245	658	063	787	.2531	385	809	570	.4225	9
52	956	283	042	.3041	687	059	817	.2375	414	805	600	.4103	8
53	985	279	072	.2838	716	055	846	.2220	442	800	630	.3980	7
54	.12014	276	101	.2636	744	051	876	.2066	471	796	660	.3859	6
55	.12043	.99272	.12131	8.2434	.13773	.99047	.13906	7.1912	.15500	.98791	.15689	6.3737	5
56	071	269	160	.2234	802	043	935	.1759	529	787	719	.3617	4
57	100	265	190	.2035	831	039	965	.1607	557	782	749	.3496	3
58	129	262	219	.1837	860	035	995	.1455	586	778	779	.3376	2
59	158	258	249	.1640	889	031	.14024	.1304	615	773	809	.3257	1
60	.12187	.99255	.12278	8.1443	.13917	.99027	.14054	7.1154	.15643	.98769	.15838	6.3138	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	83°				82°				81°				

M.	9° Sin.	9° Cos.	9° Tan.	9° Cot.	10° Sin.	10° Cos.	10° Tan.	10° Cot.	11° Sin.	11° Cos.	11° Tan.	11° Cot.	
0	.15643	.98769	.15838	6.3138	.17365	.98481	.17633	5.6713	.19081	.98163	.19438	5.1446	60
1	672	764	868	.3019	393	476	663	.6617	109	157	468	.1366	59
2	701	760	898	.2901	422	471	693	.6521	138	152	498	.1286	58
3	730	755	928	.2783	451	466	723	.6425	167	146	529	.1207	57
4	758	751	958	.2666	479	461	753	.6329	195	140	559	.1128	56
5	.15787	.98746	.15988	6.2549	.17508	.98455	.17783	5.6234	.19224	.98135	.19589	5.1049	55
6	816	741	.16017	.2432	537	450	813	.6140	252	129	619	.0970	54
7	845	737	047	.2316	565	445	843	.6045	281	124	649	.0892	53
8	873	732	077	.2200	594	440	873	.5951	309	118	680	.0814	52
9	902	728	107	.2085	623	435	903	.5857	338	112	710	.0736	51
10	.15931	.98723	.16137	6.1970	.17651	.98430	.17933	5.5764	.19366	.98107	.19740	5.0658	50
11	959	718	167	.1856	680	425	963	.5671	395	101	770	.0581	49
12	988	714	196	.1742	708	420	993	.5578	423	096	801	.0504	48
13	.16017	709	226	.1628	737	414	.18023	.5485	452	090	831	.0427	47
14	046	704	256	.1515	766	409	053	.5393	481	084	861	.0350	46
15	.16074	.98700	.16286	6.1402	.17794	.98404	.18083	5.5301	.19509	.98079	.19891	5.0273	45
16	103	695	316	.1290	823	399	113	.5209	538	073	921	.0197	44
17	132	690	346	.1178	852	394	143	.5118	566	067	952	.0121	43
18	160	686	376	.1066	880	389	173	.5026	595	061	982	.0045	42
19	189	681	405	.0955	909	383	203	.4936	623	056	.20012	4.9969	41
20	.16218	.98676	.16435	6.0844	.17937	.98378	.18233	5.4845	.19652	.98050	.20042	4.9894	40
21	246	671	465	.0734	966	373	263	.4755	680	044	073	.9819	39
22	275	667	495	.0624	995	368	293	.4665	709	039	103	.9744	38
23	304	662	525	.0514	.18023	362	323	.4575	737	033	133	.9669	37
24	333	657	555	.0405	052	357	353	.4486	766	027	164	.9594	36
25	.16361	.98652	.16585	6.0296	.18081	.98352	.18384	5.4397	.19794	.98021	.20194	4.9520	35
26	390	648	615	.0188	109	347	414	.4308	823	016	224	.9446	34
27	419	643	645	.0080	138	341	444	.4219	851	010	254	.9372	33
28	447	638	674	5.9972	166	336	474	.4131	880	004	285	.9298	32
29	476	633	704	.9865	195	331	504	.4043	908	.97998	315	.9225	31
30	.16505	.98629	.16734	5.9758	.18224	.98325	.18534	5.3955	.19937	.97992	.20345	4.9152	30
31	533	624	764	.9651	252	320	564	.3868	965	987	376	.9078	29
32	562	619	794	.9545	281	315	594	.3781	994	981	406	.9006	28
33	591	614	824	.9439	309	310	624	.3694	.20022	975	436	.8933	27
34	620	609	854	.9333	338	304	654	.3607	051	969	466	.8860	26
35	.16648	.98604	.16884	5.9228	.18367	.98299	.18684	5.3521	.20079	.97963	.20497	4.8788	25
36	677	600	914	.9124	395	294	714	.3435	108	958	527	.8716	24
37	706	595	944	.9019	424	288	745	.3349	136	952	557	.8644	23
38	734	590	974	.8915	452	283	775	.3263	165	946	588	.8573	22
39	763	585	.17004	.8811	481	277	805	.3178	193	940	618	.8501	21
40	.16792	.98580	.17033	5.8708	.18509	.98272	.18835	5.3093	.20222	.97934	.20648	4.8430	20
41	820	575	063	.8605	538	267	865	.3008	250	928	679	.8359	19
42	849	570	093	.8502	567	261	895	.2924	279	922	709	.8288	18
43	878	565	123	.8400	595	256	925	.2839	307	916	739	.8218	17
44	906	561	153	.8298	624	250	955	.2755	336	910	770	.8147	16
45	.16935	.98556	.17183	5.8197	.18652	.98245	.18986	5.2672	.20364	.97905	.20800	4.8077	15
46	964	551	213	.8095	681	240	.19016	.2588	393	899	830	.8007	14
47	992	546	243	.7994	710	234	046	.2505	421	893	861	.7937	13
48	.17021	541	273	.7894	738	229	076	.2422	450	887	891	.7867	12
49	050	536	303	.7794	767	223	106	.2339	478	881	921	.7798	11
50	.17078	.98531	.17333	5.7694	.18795	.98218	.19136	5.2257	.20507	.97875	.20952	4.7729	10
51	107	526	363	.7594	824	212	166	.2174	535	869	982	.7659	9
52	136	521	393	.7495	852	207	197	.2092	563	863	.21013	.7591	8
53	164	516	423	.7396	881	201	227	.2011	592	857	043	.7522	7
54	193	511	453	.7297	910	196	257	.1929	620	851	073	.7453	6
55	.17222	.98506	.17483	5.7199	.18938	.98190	.19287	5.1848	.20649	.97845	.21104	4.7385	5
56	250	501	513	.7101	967	185	317	.1767	677	839	134	.7317	4
57	279	496	543	.7004	995	179	347	.1686	706	833	164	.7249	3
58	308	491	573	.6906	.19024	174	378	.1606	734	827	195	.7181	2
59	336	486	603	.6809	052	168	408	.1526	763	821	225	.7114	1
60	.17365	.98481	.17633	5.6713	.19081	.98163	.19438	5.1446	.20791	.97815	.21256	4.7046	0
	80° Cos.	80° Sin.	80° Cot.	80° Tan.	79° Cos.	79° Sin.	79° Cot.	79° Tan.	78° Cos.	78° Sin.	78° Cot.	78° Tan.	M.

	12°				13°				14°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.20791	.97815	.21256	4.7046	.22495	.97437	.23087	4.3315	.24192	.97030	.24933	4.0108	60
1	820	809	286	.6979	523	430	117	.3257	220	023	964	.0058	59
2	848	803	316	.6912	552	424	148	.3200	249	015	995	.0009	58
3	877	797	347	.6845	580	417	179	.3143	277	008	.25026	3.9959	57
4	905	791	377	.6779	608	411	209	.3086	305	001	056	.9910	56
5	.20933	.97784	.21408	4.6712	.22637	.97404	.23240	4.3029	.24333	.96994	.25087	3.9861	55
6	962	778	438	.6646	665	398	271	.2972	362	987	118	.9812	54
7	990	772	469	.6580	693	391	301	.2916	390	980	149	.9763	53
8	.21019	766	499	.6514	722	384	332	.2859	418	973	180	.9714	52
9	047	760	529	.6448	750	378	363	.2803	446	966	211	.9665	51
10	.21076	.97754	.21560	4.6382	.22778	.97371	.23393	4.2747	.24474	.96959	.25242	3.9617	50
11	104	748	590	.6317	807	365	424	.2691	503	952	273	.9568	49
12	132	742	621	.6252	835	358	455	.2635	531	945	304	.9520	48
13	161	735	651	.6187	863	351	485	.2580	559	937	335	.9471	47
14	189	729	682	.6122	892	345	516	.2524	587	930	366	.9423	46
15	.21218	.97723	.21712	4.6057	.22920	.97338	.23547	4.2468	.24615	.96923	.25397	3.9375	45
16	246	717	743	.5993	948	331	578	.2413	644	916	428	.9327	44
17	275	711	773	.5928	977	325	608	.2358	672	909	459	.9279	43
18	303	705	804	.5864	.23005	318	639	.2303	700	902	490	.9232	42
19	331	698	834	.5800	033	311	670	.2248	728	894	521	.9184	41
20	.21360	.97692	.21864	4.5736	.23062	.97304	.23700	4.2193	.24756	.96887	.25552	3.9136	40
21	388	686	895	.5673	090	298	731	.2139	784	880	583	.9089	39
22	417	680	925	.5609	118	291	762	.2084	813	873	614	.9042	38
23	445	673	956	.5546	146	284	793	.2030	841	866	645	.8995	37
24	474	667	986	.5483	175	278	823	.1976	869	858	676	.8947	36
25	.21502	.97661	.22017	4.5420	.23203	.97271	.23854	4.1922	.24897	.96851	.25707	3.8900	35
26	530	655	047	.5357	231	264	885	.1868	925	844	738	.8854	34
27	559	648	078	.5294	260	257	916	.1814	954	837	769	.8807	33
28	587	642	108	.5232	288	251	946	.1760	982	829	800	.8760	32
29	616	636	139	.5169	316	244	977	.1706	.25010	822	831	.8714	31
30	.21644	.97630	.22169	4.5107	.23345	.97237	.24008	4.1653	.25038	.96815	.25862	3.8667	30
31	672	623	200	.5045	373	230	039	.1600	066	807	893	.8621	29
32	701	617	231	.4983	401	223	069	.1547	094	800	924	.8575	28
33	729	611	261	.4922	429	217	100	.1493	122	793	955	.8528	27
34	758	604	292	.4860	458	210	131	.1441	151	786	986	.8482	26
35	.21786	.97598	.22322	4.4799	.23486	.97203	.24162	4.1388	.25179	.96778	.26017	3.8436	25
36	814	592	353	.4737	514	196	193	.1335	207	771	048	.8391	24
37	843	585	383	.4676	542	189	223	.1282	235	764	079	.8345	23
38	871	579	414	.4615	571	182	254	.1230	263	756	110	.8299	22
39	899	573	444	.4555	599	176	285	.1178	291	749	141	.8254	21
40	.21928	.97566	.22475	4.4494	.23627	.97169	.24316	4.1126	.25320	.96742	.26172	3.8208	20
41	956	560	505	.4434	656	162	347	.1074	348	734	203	.8163	19
42	985	553	536	.4373	684	155	377	.1022	376	727	235	.8118	18
43	.22013	547	567	.4313	712	148	408	.0970	404	719	266	.8073	17
44	041	541	597	.4253	740	141	439	.0918	432	712	297	.8028	16
45	.22070	.97534	.22628	4.4194	.23769	.97134	.24470	4.0867	.25460	.96705	.26328	3.7983	15
46	098	528	658	.4134	797	127	501	.0815	488	697	359	.7938	14
47	126	521	689	.4075	825	120	532	.0764	516	690	390	.7893	13
48	155	515	719	.4015	853	113	562	.0713	545	682	421	.7848	12
49	183	508	750	.3956	882	106	593	.0662	573	675	452	.7804	11
50	.22212	.97502	.22781	4.3897	.23910	.97100	.24624	4.0611	.25601	.96667	.26483	3.7760	10
51	240	496	811	.3838	938	093	655	.0560	629	660	515	.7715	9
52	268	489	842	.3779	966	086	686	.0509	657	653	546	.7671	8
53	297	483	872	.3721	995	079	717	.0459	685	645	577	.7627	7
54	325	476	903	.3662	.24023	072	747	.0408	713	638	608	.7583	6
55	.22353	.97470	.22934	4.3604	.24051	.97065	.24778	4.0358	.25741	.96630	.26639	3.7539	5
56	382	463	964	.3546	079	058	809	.0308	769	623	670	.7495	4
57	410	457	995	.3488	108	051	840	.0257	798	615	701	.7451	3
58	438	450	.23026	.3430	136	044	871	.0207	826	608	733	.7408	2
59	467	444	056	.3372	164	037	902	.0158	854	600	764	.7364	1
60	.22495	.97437	.23087	4.3315	.24192	.97030	.24933	4.0108	.25882	.96593	.26795	3.7321	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	77°				76°				75°				

	15°				16°				17°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.25882	.96593	.26795	3.7321	.27564	.96126	.28675	3.4874	.29237	.95630	.30573	3.2709	60
1	910	585	826	.7277	592	118	706	.4836	265	622	605	.2675	59
2	938	578	857	.7234	620	110	738	.4798	293	613	637	.2641	58
3	966	570	888	.7191	648	102	769	.4760	321	605	669	.2607	57
4	994	562	920	.7148	676	094	801	.4722	348	596	700	.2573	56
5	.26022	.96555	.26951	3.7105	.27704	.96086	.28832	3.4684	.29376	.95588	.30732	3.2539	55
6	050	547	982	.7062	731	078	864	.4646	404	579	764	.2506	54
7	079	540	.27013	.7019	759	070	895	.4608	432	571	796	.2472	53
8	107	532	044	.6976	787	062	927	.4570	460	562	828	.2438	52
9	135	524	076	.6933	815	054	958	.4533	487	554	860	.2405	51
10	.26163	.96517	.27107	3.6891	.27843	.96046	.28990	3.4495	.29515	.95545	.30891	3.2371	50
11	191	509	138	.6848	871	037	.29021	.4458	543	536	923	.2338	49
12	219	502	169	.6806	899	029	053	.4420	571	528	955	.2305	48
13	247	494	201	.6764	927	021	084	.4383	599	519	987	.2272	47
14	275	486	232	.6722	955	013	116	.4346	626	511	.31019	.2238	46
15	.26303	.96479	.27263	3.6680	.27983	.96005	.29147	3.4308	.29654	.95502	.31051	3.2205	45
16	331	471	294	.6638	.28011	.95997	179	.4271	682	493	083	.2172	44
17	359	463	326	.6596	039	989	210	.4234	710	485	115	.2139	43
18	387	456	357	.6554	067	981	242	.4197	737	476	147	.2106	42
19	415	448	388	.6512	095	972	274	.4160	765	467	178	.2073	41
20	.26443	.96440	.27419	3.6470	.28123	.95964	.29305	3.4124	.29793	.95459	.31210	3.2041	40
21	471	433	451	.6429	150	956	337	.4087	821	450	242	.2008	39
22	500	425	482	.6387	178	948	368	.4050	849	441	274	.1975	38
23	528	417	513	.6346	206	940	400	.4014	876	433	306	.1943	37
24	556	410	545	.6305	234	931	432	.3977	904	424	338	.1910	36
25	.26584	.96402	.27576	3.6264	.28262	.95923	.29463	3.3941	.29932	.95415	.31370	3.1878	35
26	612	394	607	.6222	290	915	495	.3904	960	407	402	.1845	34
27	640	386	638	.6181	318	907	526	.3868	987	398	434	.1813	33
28	668	379	670	.6140	346	898	558	.3832	.30015	389	466	.1780	32
29	696	371	701	.6100	374	890	590	.3796	043	380	498	.1748	31
30	.26724	.96363	.27732	3.6059	.28402	.95882	.29621	3.3759	.30071	.95372	.31530	3.1716	30
31	752	355	764	.6018	429	874	653	.3723	098	363	562	.1684	29
32	780	347	795	.5978	457	865	685	.3687	126	354	594	.1652	28
33	808	340	826	.5937	485	857	716	.3652	154	345	626	.1620	27
34	836	332	858	.5897	513	849	748	.3616	182	337	658	.1588	26
35	.26864	.96324	.27889	3.5856	.28541	.95841	.29780	3.3580	.30209	.95328	.31690	3.1556	25
36	892	316	921	.5816	569	832	811	.3544	237	319	722	.1524	24
37	920	308	952	.5776	597	824	843	.3509	265	310	754	.1492	23
38	948	301	983	.5736	625	816	875	.3473	292	301	786	.1460	22
39	976	293	.28015	.5696	652	807	906	.3438	320	293	818	.1429	21
40	.27004	.96285	.28046	3.5656	.28680	.95799	.29938	3.3402	.30348	.95284	.31850	3.1397	20
41	032	277	077	.5616	708	791	970	.3367	376	275	882	.1366	19
42	060	269	109	.5576	736	782	.30001	.3332	403	266	914	.1334	18
43	088	261	140	.5536	764	774	033	.3297	431	257	946	.1303	17
44	116	253	172	.5497	792	766	065	.3261	459	248	978	.1271	16
45	.27144	.96246	.28203	3.5457	.28820	.95757	.30097	3.3226	.30486	.95240	.32010	3.1240	15
46	172	238	234	.5418	847	749	128	.3191	514	231	042	.1209	14
47	200	230	266	.5379	875	740	160	.3156	542	222	074	.1178	13
48	228	222	297	.5339	903	732	192	.3122	570	213	106	.1146	12
49	256	214	329	.5300	931	724	224	.3087	597	204	139	.1115	11
50	.27284	.96206	.28360	3.5261	.28959	.95715	.30255	3.3052	.30625	.95195	.32171	3.1084	10
51	312	198	391	.5222	987	707	287	.3017	653	186	203	.1053	9
52	340	190	423	.5183	.29015	698	319	.2983	680	177	235	.1022	8
53	368	182	454	.5144	042	690	351	.2948	708	168	267	.0991	7
54	396	174	486	.5105	070	681	382	.2914	736	159	299	.0961	6
55	.27424	.96166	.28517	3.5067	.29098	.95673	.30414	3.2879	.30763	.95150	.32331	3.0930	5
56	452	158	549	.5028	126	664	446	.2845	791	142	363	.0899	4
57	480	150	580	.4989	154	656	478	.2811	819	133	396	.0868	3
58	508	142	612	.4951	182	647	509	.2777	846	124	428	.0838	2
59	536	134	643	.4912	209	639	541	.2743	874	115	460	.0807	1
60	.27564	.96126	.28675	3.4874	.29237	.95630	.30573	3.2709	.30902	.95106	.32492	3.0777	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	74°				73°				72°				

	18°				19°				20°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.30902	.95106	.32492	3.0777	.32557	.94552	.34433	2.9042	.34202	.93969	.36397	2.7475	60
1	929	097	524	.0746	584	542	465	.9015	229	959	430	.7450	59
2	957	088	556	.0716	612	533	498	.8987	257	949	463	.7425	58
3	985	079	588	.0686	639	523	530	.8960	284	939	496	.7400	57
4	.31012	070	621	.0655	667	514	563	.8933	311	929	529	.7376	56
5	.31040	.95061	.32653	3.0625	.32694	.94504	.34596	2.8905	.34339	.93919	.36562	2.7351	55
6	068	052	685	.0595	722	495	628	.8878	366	909	595	.7326	54
7	095	043	717	.0565	749	485	661	.8851	393	899	628	.7302	53
8	123	033	749	.0535	777	476	693	.8824	421	889	661	.7277	52
9	151	024	782	.0505	804	466	726	.8797	448	879	694	.7253	51
10	.31178	.95015	.32814	3.0475	.32832	.94457	.34758	2.8770	.34475	.93869	.36727	2.7228	50
11	206	006	846	.0445	859	447	791	.8743	503	859	760	.7204	49
12	233	.94997	878	.0415	887	438	824	.8716	530	849	793	.7179	48
13	261	988	911	.0385	914	428	856	.8689	557	839	826	.7155	47
14	289	979	943	.0356	942	418	889	.8662	584	829	859	.7130	46
15	.31316	.94970	.32975	3.0326	.32969	.94409	.34922	2.8636	.34612	.93819	.36892	2.7106	45
16	344	961	.33007	.0296	997	399	954	.8609	639	809	925	.7082	44
17	372	952	040	.0267	.33024	390	987	.8582	666	799	958	.7058	43
18	399	943	072	.0237	051	380	.35020	.8556	694	789	991	.7034	42
19	427	933	104	.0208	079	370	052	.8529	721	779	.37024	.7009	41
20	.31454	.94924	.33136	3.0178	.33106	.94361	.35085	2.8502	.34748	.93769	.37057	2.6985	40
21	482	915	169	.0149	134	351	118	.8476	775	759	090	.6961	39
22	510	906	201	.0120	161	342	150	.8449	803	748	123	.6937	38
23	537	897	233	.0090	189	332	183	.8423	830	738	157	.6913	37
24	565	888	266	.0061	216	322	216	.8397	857	728	190	.6889	36
25	.31593	.94878	.33298	3.0032	.33244	.94313	.35248	2.8370	.34884	.93718	.37223	2.6865	35
26	620	869	330	.0003	271	303	281	.8344	912	708	256	.6841	34
27	648	860	363	2.9974	298	293	314	.8318	939	698	289	.6818	33
28	675	851	395	.9945	326	284	346	.8291	966	688	322	.6794	32
29	703	842	427	.9916	353	274	379	.8265	993	677	355	.6770	31
30	.31730	.94832	.33460	2.9887	.33381	.94264	.35412	2.8239	.35021	.93667	.37388	2.6746	30
31	758	823	492	.9858	408	254	445	.8213	048	657	422	.6723	29
32	786	814	524	.9829	436	245	477	.8187	075	647	455	.6699	28
33	813	805	557	.9800	463	235	510	.8161	102	637	488	.6675	27
34	841	795	589	.9772	490	225	543	.8135	130	626	521	.6652	26
35	.31868	.94786	.33621	2.9743	.33518	.94215	.35576	2.8109	.35157	.93616	.37554	2.6628	25
36	896	777	654	.9714	545	206	608	.8083	184	606	588	.6605	24
37	923	768	686	.9686	573	196	641	.8057	211	596	621	.6581	23
38	951	758	718	.9657	600	186	674	.8032	239	585	654	.6558	22
39	979	749	751	.9629	627	176	707	.8006	266	575	687	.6534	21
40	.32006	.94740	.33783	2.9600	.33655	.94167	.35740	2.7980	.35293	.93565	.37720	2.6511	20
41	034	730	816	.9572	682	157	772	.7955	320	555	754	.6488	19
42	061	721	848	.9544	710	147	805	.7929	347	544	787	.6464	18
43	089	712	881	.9515	737	137	838	.7903	375	534	820	.6441	17
44	116	702	913	.9487	764	127	871	.7878	402	524	853	.6418	16
45	.32144	.94693	.33945	2.9459	.33792	.94118	.35904	2.7852	.35429	.93514	.37887	2.6395	15
46	171	684	978	.9431	819	108	937	.7827	456	503	920	.6371	14
47	199	674	.34010	.9403	846	098	969	.7801	484	493	953	.6348	13
48	227	665	043	.9375	874	088	.36002	.7776	511	483	986	.6325	12
49	254	656	075	.9347	901	078	035	.7751	538	472	.38020	.6302	11
50	.32282	.94646	.34108	2.9319	.33929	.94068	.36068	2.7725	.35565	.93462	.38053	2.6279	10
51	309	637	140	.9291	956	058	101	.7700	592	452	086	.6256	9
52	337	627	173	.9263	983	049	134	.7675	619	441	120	.6233	8
53	364	618	205	.9235	.34011	039	167	.7650	647	431	153	.6210	7
54	392	609	238	.9208	038	029	199	.7625	674	420	186	.6187	6
55	.32419	.94599	.34270	2.9180	.34065	.94019	.36232	2.7600	.35701	.93410	.38220	2.6165	5
56	447	590	303	.9152	093	009	265	.7575	728	400	253	.6142	4
57	474	580	335	.9125	120	.93999	298	.7550	755	389	286	.6119	3
58	502	571	368	.9097	147	989	331	.7525	782	379	320	.6096	2
59	529	561	400	.9070	175	979	364	.7500	810	368	353	.6074	1
60	.32557	.94552	.34433	2.9042	.34202	.93969	.36397	2.7475	.35837	.93358	.38386	2.6051	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	71°				70°				69°				

	21°				22°				23°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.35837	.93358	.38386	2.6051	.37461	.92718	.40403	2.4751	.39073	.92050	.42447	2.3559	60
1	864	348	420	.6028	488	707	436	.4730	100	039	482	.3539	59
2	891	337	453	.6006	515	697	470	.4709	127	028	516	.3520	58
3	918	327	487	.5983	542	686	504	.4689	153	016	551	.3501	57
4	945	316	520	.5961	569	675	538	.4668	180	005	585	.3483	56
5	.35973	.93306	.38553	2.5938	.37595	.92664	.40572	2.4648	.39207	.91994	.42619	2.3464	55
6	.36000	295	587	.5916	622	653	606	.4627	234	982	654	.3445	54
7	027	285	620	.5893	649	642	640	.4606	260	971	688	.3426	53
8	054	274	654	.5871	676	631	674	.4586	287	959	722	.3407	52
9	081	264	687	.5848	703	620	707	.4566	314	948	757	.3388	51
10	.36108	.93253	.38721	2.5826	.37730	.92609	.40741	2.4545	.39341	.91936	.42791	2.3369	50
11	135	243	754	.5804	757	598	775	.4525	367	925	826	.3351	49
12	162	232	787	.5782	784	587	809	.4504	394	914	860	.3332	48
13	190	222	821	.5759	811	576	843	.4484	421	902	894	.3313	47
14	217	211	854	.5737	838	565	877	.4464	448	891	929	.3294	46
15	.36244	.93201	.38888	2.5715	.37865	.92554	.40911	2.4443	.39474	.91879	.42963	2.3276	45
16	271	190	921	.5693	892	543	945	.4423	501	868	998	.3257	44
17	298	180	955	.5671	919	532	979	.4403	528	856	.43032	.3238	43
18	325	169	988	.5649	946	521	.41013	.4383	555	845	067	.3220	42
19	352	159	.39022	.5627	973	510	047	.4362	581	833	101	.3201	41
20	.36379	.93148	.39055	2.5605	.37999	.92499	.41081	2.4342	.39608	.91822	.43136	2.3183	40
21	406	137	089	.5583	.38026	488	115	.4322	635	810	170	.3164	39
22	434	127	122	.5561	053	477	149	.4302	661	799	205	.3146	38
23	461	116	156	.5539	080	466	183	.4282	688	787	239	.3127	37
24	488	106	190	.5517	107	455	217	.4262	715	775	274	.3109	36
25	.36515	.93095	.39223	2.5495	.38134	.92444	.41251	2.4242	.39741	.91764	.43308	2.3090	35
26	542	084	257	.5473	161	432	285	.4222	768	752	343	.3072	34
27	569	074	290	.5452	188	421	319	.4202	795	741	378	.3053	33
28	596	063	324	.5430	215	410	353	.4182	822	729	412	.3035	32
29	623	052	357	.5408	241	399	387	.4162	848	718	447	.3017	31
30	.36650	.93042	.39391	2.5386	.38268	.92388	.41421	2.4142	.39875	.91706	.43481	2.2998	30
31	677	031	425	.5365	295	377	455	.4122	902	694	516	.2980	29
32	704	020	458	.5343	322	366	490	.4102	928	683	550	.2962	28
33	731	010	492	.5322	349	355	524	.4083	955	671	585	.2944	27
34	758	.92999	526	.5300	376	343	558	.4063	982	660	620	.2925	26
35	.36785	.92988	.39559	2.5279	.38403	.92332	.41592	2.4043	.40008	.91648	.43654	2.2907	25
36	812	978	593	.5257	430	321	626	.4023	035	636	689	.2889	24
37	839	967	626	.5236	456	310	660	.4004	062	625	724	.2871	23
38	867	956	660	.5214	483	299	694	.3984	088	613	758	.2853	22
39	894	945	694	.5193	510	287	728	.3964	115	601	793	.2835	21
40	.36921	.92935	.39727	2.5172	.38537	.92276	.41763	2.3945	.40141	.91590	.43828	2.2817	20
41	948	924	761	.5150	564	265	797	.3925	168	578	862	.2799	19
42	975	913	795	.5129	591	254	831	.3906	195	566	897	.2781	18
43	.37002	902	829	.5108	617	243	865	.3886	221	555	932	.2763	17
44	029	892	862	.5086	644	231	899	.3867	248	543	966	.2745	16
45	.37056	.92881	.39896	2.5065	.38671	.92220	.41933	2.3847	.40275	.91531	.44001	2.2727	15
46	083	870	930	.5044	698	209	968	.3828	301	519	036	.2709	14
47	110	859	963	.5023	725	198	.42002	.3808	328	508	071	.2691	13
48	137	849	997	.5002	752	186	036	.3789	355	496	105	.2673	12
49	164	838	.40031	.4981	778	175	070	.3770	381	484	140	.2655	11
50	.37191	.92827	.40065	2.4960	.38805	.92164	.42105	2.3750	.40408	.91472	.44175	2.2637	10
51	218	816	098	.4939	832	152	139	.3731	434	461	210	.2620	9
52	245	805	132	.4918	859	141	173	.3712	461	449	244	.2602	8
53	272	794	166	.4897	886	130	207	.3693	488	437	279	.2584	7
54	299	784	200	.4876	912	119	242	.3673	514	425	314	.2566	6
55	.37326	.92773	.40234	2.4855	.38939	.92107	.42276	2.3654	.40541	.91414	.44349	2.2549	5
56	353	762	267	.4834	966	096	310	.3635	567	402	384	.2531	4
57	380	751	301	.4813	993	085	345	.3616	594	390	418	.2513	3
58	407	740	335	.4792	.39020	073	379	.3597	621	378	453	.2496	2
59	434	729	369	.4772	046	062	413	.3578	647	366	488	.2478	1
60	.37461	.92718	.40403	2.4751	.39073	.92050	.42447	2.3559	.40674	.91355	.44523	2.2460	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	68°				67°				66°				

	24°				25°				26°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.40674	.91355	.44523	2.2460	.42262	.90631	.46631	2.1445	.43837	.89879	.48773	2.0503	60
1	700	343	558	.2443	288	618	666	.1429	863	867	809	.0488	59
2	727	331	593	.2425	315	606	702	.1413	889	854	845	.0473	58
3	753	319	627	.2408	341	594	737	.1396	916	841	881	.0458	57
4	780	307	662	.2390	367	582	772	.1380	942	828	917	.0443	56
5	.40806	.91295	.44697	2.2373	.42394	.90569	.46808	2.1364	.43968	.89816	.48953	2.0428	55
6	833	283	732	.2355	420	557	843	.1348	994	803	989	.0413	54
7	860	272	767	.2338	446	545	879	.1332	.44020	790	.49026	.0398	53
8	886	260	802	.2320	473	532	914	.1315	046	777	062	.0383	52
9	913	248	837	.2303	499	520	950	.1299	072	764	098	.0368	51
10	.40939	.91236	.44872	2.2286	.42525	.90507	.46985	2.1283	.44098	.89752	.49134	2.0353	50
11	966	224	907	.2268	552	495	.47021	.1267	124	739	170	.0338	49
12	992	212	942	.2251	578	483	056	.1251	151	726	206	.0323	48
13	.41019	200	977	.2234	604	470	092	.1235	177	713	242	.0308	47
14	045	188	.45012	.2216	631	458	128	.1219	203	700	278	.0293	46
15	.41072	.91176	.45047	2.2199	.42657	.90446	.47163	2.1203	.44229	.89687	.49315	2.0278	45
16	098	164	082	.2182	683	433	199	.1187	255	674	351	.0263	44
17	125	152	117	.2165	709	421	234	.1171	281	662	387	.0248	43
18	151	140	152	.2148	736	408	270	.1155	307	649	423	.0233	42
19	178	128	187	.2130	762	396	305	.1139	333	636	459	.0219	41
20	.41204	.91116	.45222	2.2113	.42788	.90383	.47341	2.1123	.44359	.89623	.49495	2.0204	40
21	231	104	257	.2096	815	371	377	.1107	385	610	532	.0189	39
22	257	092	292	.2079	841	358	412	.1092	411	597	568	.0174	38
23	284	080	327	.2062	867	346	448	.1076	437	584	604	.0160	37
24	310	068	362	.2045	894	334	483	.1060	464	571	640	.0145	36
25	.41337	.91056	.45397	2.2028	.42920	.90321	.47519	2.1044	.44490	.89558	.49677	2.0130	35
26	363	044	432	.2011	946	309	555	.1028	516	545	713	.0115	34
27	390	032	467	.1994	972	296	590	.1013	542	532	749	.0101	33
28	416	020	502	.1977	999	284	626	.0997	568	519	786	.0086	32
29	443	008	538	.1960	.43025	271	662	.0981	594	506	822	.0072	31
30	.41469	.90996	.45573	2.1943	.43051	.90259	.47698	2.0965	.44620	.89493	.49858	2.0057	30
31	496	984	608	.1926	077	246	733	.0950	646	480	894	.0042	29
32	522	972	643	.1909	104	233	769	.0934	672	467	931	.0028	28
33	549	960	678	.1892	130	221	805	.0918	698	454	967	.0013	27
34	575	948	713	.1876	156	208	840	.0903	724	441	.50004	1.9999	26
35	.41602	.90936	.45748	2.1859	.43182	.90196	.47876	2.0887	.44750	.89428	.50040	1.9984	25
36	628	924	784	.1842	209	183	912	.0872	776	415	076	.9970	24
37	655	911	819	.1825	235	171	948	.0856	802	402	113	.9955	23
38	681	899	854	.1808	261	158	984	.0840	828	389	149	.9941	22
39	707	887	889	.1792	287	146	.48019	.0825	854	376	185	.9926	21
40	.41734	.90875	.45924	2.1775	.43313	.90133	.48055	2.0809	.44880	.89363	.50222	1.9912	20
41	760	863	960	.1758	340	120	091	.0794	906	350	258	.9897	19
42	787	851	995	.1742	366	108	127	.0778	932	337	295	.9883	18
43	813	839	.46030	.1725	392	095	163	.0763	958	324	331	.9868	17
44	840	826	065	.1708	418	082	198	.0748	984	311	368	.9854	16
45	.41866	.90814	.46101	2.1692	.43445	.90070	.48234	2.0732	.45010	.89298	.50404	1.9840	15
46	892	802	136	.1675	471	057	270	.0717	036	285	441	.9825	14
47	919	790	171	.1659	497	045	306	.0701	062	272	477	.9811	13
48	945	778	206	.1642	523	032	342	.0686	088	259	514	.9797	12
49	972	766	242	.1625	549	019	378	.0671	114	245	550	.9782	11
50	.41998	.90753	.46277	2.1609	.43575	.90007	.48414	2.0655	.45140	.89232	.50587	1.9768	10
51	.42024	741	312	.1592	602	.89994	450	.0640	166	219	623	.9754	9
52	051	729	348	.1576	628	981	486	.0625	192	206	660	.9740	8
53	077	717	383	.1560	654	968	521	.0609	218	193	696	.9725	7
54	104	704	418	.1543	680	956	557	.0594	243	180	733	.9711	6
55	.42130	.90692	.46454	2.1527	.43706	.89943	.48593	2.0579	.45269	.89167	.50769	1.9697	5
56	156	680	489	.1510	733	930	629	.0564	295	153	806	.9683	4
57	183	668	525	.1494	759	918	665	.0549	321	140	843	.9669	3
58	209	655	560	.1478	785	905	701	.0533	347	127	879	.9654	2
59	235	643	595	.1461	811	892	737	.0518	373	114	916	.9640	1
60	.42262	.90631	.46631	2.1445	.43837	.89879	.48773	2.0503	.45399	.89101	.50953	1.9626	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	65°				64°				63°				

M.	27° Sin.	27° Cos.	27° Tan.	27° Cot.	28° Sin.	28° Cos.	28° Tan.	28° Cot.	29° Sin.	29° Cos.	29° Tan.	29° Cot.	M.
0	.45399	.89101	.50953	1.9626	.46947	.88295	.53171	1.8807	.48481	.87462	.55431	1.8040	60
1	425	087	989	.9612	973	281	208	.8794	506	448	469	.8028	59
2	451	074	.51026	.9598	999	267	246	.8781	532	434	507	.8016	58
3	477	061	063	.9584	.47024	254	283	.8768	557	420	545	.8003	57
4	503	048	099	.9570	050	240	320	.8755	583	406	583	.7991	56
5	.45529	.89035	.51136	1.9556	.47076	.88226	.53358	1.8741	.48608	.87391	.55621	1.7979	55
6	554	021	173	.9542	101	213	395	.8728	634	377	659	.7966	54
7	580	008	209	.9528	127	199	432	.8715	659	363	697	.7954	53
8	606	.88995	246	.9514	153	185	470	.8702	684	349	736	.7942	52
9	632	981	283	.9500	178	172	507	.8689	710	335	774	.7930	51
10	.45658	.88968	.51319	1.9486	.47204	.88158	.53545	1.8676	.48735	.87321	.55812	1.7917	50
11	684	955	356	.9472	229	144	582	.8663	761	306	850	.7905	49
12	710	942	393	.9458	255	130	620	.8650	786	292	888	.7893	48
13	736	928	430	.9444	281	117	657	.8637	811	278	926	.7881	47
14	762	915	467	.9430	306	103	694	.8624	837	264	964	.7868	46
15	.45787	.88902	.51503	1.9416	.47332	.88089	.53732	1.8611	.48862	.87250	.56003	1.7856	45
16	813	888	540	.9402	358	075	769	.8598	888	235	041	.7844	44
17	839	875	577	.9388	383	062	807	.8585	913	221	079	.7832	43
18	865	862	614	.9375	409	048	844	.8572	938	207	117	.7820	42
19	891	848	651	.9361	434	034	882	.8559	964	193	156	.7808	41
20	.45917	.88835	.51688	1.9347	.47460	.88020	.53920	1.8546	.48989	.87178	.56194	1.7796	40
21	942	822	724	.9333	486	006	957	.8533	.49014	164	232	.7783	39
22	968	808	761	.9319	511	.87993	995	.8520	040	150	270	.7771	38
23	994	795	798	.9306	537	979	.54032	.8507	065	136	309	.7759	37
24	.46020	782	835	.9292	562	965	070	.8495	090	121	347	.7747	36
25	.46046	.88768	.51872	1.9278	.47588	.87951	.54107	1.8482	.49116	.87107	.56385	1.7735	35
26	072	755	909	.9265	614	937	145	.8469	141	093	424	.7723	34
27	097	741	946	.9251	639	923	183	.8456	166	079	462	.7711	33
28	123	728	983	.9237	665	909	220	.8443	192	064	501	.7699	32
29	149	715	.52020	.9223	690	896	258	.8430	217	050	539	.7687	31
30	.46175	.88701	.52057	1.9210	.47716	.87882	.54296	1.8418	.49242	.87036	.56577	1.7675	30
31	201	688	094	.9196	741	868	333	.8405	268	021	616	.7663	29
32	226	674	131	.9183	767	854	371	.8392	293	007	654	.7651	28
33	252	661	168	.9169	793	840	409	.8379	318	.86993	693	.7639	27
34	278	647	205	.9155	818	826	446	.8367	344	978	731	.7627	26
35	.46304	.88634	.52242	1.9142	.47844	.87812	.54484	1.8354	.49369	.86964	.56769	1.7615	25
36	330	620	279	.9128	869	798	522	.8341	394	949	808	.7603	24
37	355	607	316	.9115	895	784	560	.8329	419	935	846	.7591	23
38	381	593	353	.9101	920	770	597	.8316	445	921	885	.7579	22
39	407	580	390	.9088	946	756	635	.8303	470	906	923	.7567	21
40	.46433	.88566	.52427	1.9074	.47971	.87743	.54673	1.8291	.49495	.86892	.56962	1.7556	20
41	458	553	464	.9061	997	729	711	.8278	521	878	.57000	.7544	19
42	484	539	501	.9047	.48022	715	748	.8265	546	863	039	.7532	18
43	510	526	538	.9034	048	701	786	.8253	571	849	078	.7520	17
44	536	512	575	.9020	073	687	824	.8240	596	834	116	.7508	16
45	.46561	.88499	.52613	1.9007	.48099	.87673	.54862	1.8228	.49622	.86820	.57155	1.7496	15
46	587	485	650	.8993	124	659	900	.8215	647	805	193	.7485	14
47	613	472	687	.8980	150	645	938	.8202	672	791	232	.7473	13
48	639	458	724	.8967	175	631	975	.8190	697	777	271	.7461	12
49	664	445	761	.8953	201	617	.55013	.8177	723	762	309	.7449	11
50	.46690	.88431	.52798	1.8940	.48226	.87603	.55051	1.8165	.49748	.86748	.57348	1.7437	10
51	716	417	836	.8927	252	589	089	.8152	773	733	386	.7426	9
52	742	404	873	.8913	277	575	127	.8140	798	719	425	.7414	8
53	767	390	910	.8900	303	561	165	.8127	824	704	464	.7402	7
54	793	377	947	.8887	328	546	203	.8115	849	690	503	.7391	6
55	.46819	.88363	.52985	1.8873	.48354	.87532	.55241	1.8103	.49874	.86675	.57541	1.7379	5
56	844	349	.53022	.8860	379	518	279	.8090	899	661	580	.7367	4
57	870	336	059	.8847	405	504	317	.8078	924	646	619	.7355	3
58	896	322	096	.8834	430	490	355	.8065	950	632	657	.7344	2
59	921	308	134	.8820	456	476	393	.8053	975	617	696	.7332	1
60	.46947	.88295	.53171	1.8807	.48481	.87462	.55431	1.8040	.50000	.86603	.57735	1.7321	0
	62° Cos.	62° Sin.	62° Cot.	62° Tan.	61° Cos.	61° Sin.	61° Cot.	61° Tan.	60° Cos.	60° Sin.	60° Cot.	60° Tan.	M.

	30°				31°				32°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.50000	.86603	.57735	1.7321	.51504	.85717	.60086	1.6643	.52992	.84805	.62487	1.6003	60
1	025	588	774	.7309	529	702	126	.6632	.53017	789	527	.5993	59
2	050	573	813	.7297	554	687	165	.6621	041	774	568	.5983	58
3	076	559	851	.7286	579	672	205	.6610	066	759	608	.5972	57
4	101	544	890	.7274	604	657	245	.6599	091	743	649	.5962	56
5	.50126	.86530	.57929	1.7262	.51628	.85642	.60284	1.6588	.53115	.84728	.62689	1.5952	55
6	151	515	968	.7251	653	627	324	.6577	140	712	730	.5941	54
7	176	501	.58007	.7239	678	612	364	.6566	164	697	770	.5931	53
8	201	486	046	.7228	703	597	403	.6555	189	681	811	.5921	52
9	227	471	085	.7216	728	582	443	.6545	214	666	852	.5911	51
10	.50252	.86457	.58124	1.7205	.51753	.85567	.60483	1.6534	.53238	.84650	.62892	1.5900	50
11	277	442	162	.7193	778	551	522	.6523	263	635	933	.5890	49
12	302	427	201	.7182	803	536	562	.6512	288	619	973	.5880	48
13	327	413	240	.7170	828	521	602	.6501	312	604	.63014	.5869	47
14	352	398	279	.7159	852	506	642	.6490	337	588	055	.5859	46
15	.50377	.86384	.58318	1.7147	.51877	.85491	.60681	1.6479	.53361	.84573	.63095	1.5849	45
16	403	369	357	.7136	902	476	721	.6469	386	557	136	.5839	44
17	428	354	396	.7124	927	461	761	.6458	411	542	177	.5829	43
18	453	340	435	.7113	952	446	801	.6447	435	526	217	.5818	42
19	478	325	474	.7102	977	431	841	.6436	460	511	258	.5808	41
20	.50503	.86310	.58513	1.7090	.52002	.85416	.60881	1.6426	.53484	.84495	.63299	1.5798	40
21	528	295	552	.7079	026	401	921	.6415	509	480	340	.5788	39
22	553	281	591	.7067	051	385	960	.6404	534	464	380	.5778	38
23	578	266	631	.7056	076	370	.61000	.6393	558	448	421	.5768	37
24	603	251	670	.7045	101	355	040	.6383	583	433	462	.5757	36
25	.50628	.86237	.58709	1.7033	.52126	.85340	.61080	1.6372	.53607	.84417	.63503	1.5747	35
26	654	222	748	.7022	151	325	120	.6361	632	402	544	.5737	34
27	679	207	787	.7011	175	310	160	.6351	656	386	584	.5727	33
28	704	192	826	.6999	200	294	200	.6340	681	370	625	.5717	32
29	729	178	865	.6988	225	279	240	.6329	705	355	666	.5707	31
30	.50754	.86163	.58905	1.6977	.52250	.85264	.61280	1.6319	.53730	.84339	.63707	1.5697	30
31	779	148	944	.6965	275	249	320	.6308	754	324	748	.5687	29
32	804	133	983	.6954	299	234	360	.6297	779	308	789	.5677	28
33	829	119	.59022	.6943	324	218	400	.6287	804	292	830	.5667	27
34	854	104	061	.6932	349	203	440	.6276	828	277	871	.5657	26
35	.50879	.86089	.59101	1.6920	.52374	.85188	.61480	1.6265	.53853	.84261	.63912	1.5647	25
36	904	074	140	.6909	399	173	520	.6255	877	245	953	.5637	24
37	929	059	179	.6898	423	157	561	.6244	902	230	994	.5627	23
38	954	045	218	.6887	448	142	601	.6234	926	214	.64035	.5617	22
39	979	030	258	.6875	473	127	641	.6223	951	198	076	.5607	21
40	.51004	.86015	.59297	1.6864	.52498	.85112	.61681	1.6212	.53975	.84182	.64117	1.5597	20
41	029	000	336	.6853	522	096	721	.6202	.54000	167	158	.5587	19
42	054	.85985	376	.6842	547	081	761	.6191	024	151	199	.5577	18
43	079	970	415	.6831	572	066	801	.6181	049	135	240	.5567	17
44	104	956	454	.6820	597	051	842	.6170	073	120	281	.5557	16
45	.51129	.85941	.59494	1.6808	.52621	.85035	.61882	1.6160	.54097	.84104	.64322	1.5547	15
46	154	926	533	.6797	646	020	922	.6149	122	088	363	.5537	14
47	179	911	573	.6786	671	005	962	.6139	146	072	404	.5527	13
48	204	896	612	.6775	696	.84989	.62003	.6128	171	057	446	.5517	12
49	229	881	651	.6764	720	974	043	.6118	195	041	487	.5507	11
50	.51254	.85866	.59691	1.6753	.52745	.84959	.62083	1.6107	.54220	.84025	.64528	1.5497	10
51	279	851	730	.6742	770	943	124	.6097	244	009	569	.5487	9
52	304	836	770	.6731	794	928	164	.6087	269	.83994	610	.5477	8
53	329	821	809	.6720	819	913	204	.6076	293	978	652	.5468	7
54	354	806	849	.6709	844	897	245	.6066	317	962	693	.5458	6
55	.51379	.85792	.59888	1.6698	.52869	.84882	.62285	1.6055	.54342	.83946	.64734	1.5448	5
56	404	777	928	.6687	893	866	325	.6045	366	930	775	.5438	4
57	429	762	967	.6676	918	851	366	.6034	391	915	817	.5428	3
58	454	747	.60007	.6665	943	836	406	.6024	415	899	858	.5418	2
59	479	732	046	.6654	967	820	446	.6014	440	883	899	.5408	1
60	.51504	.85717	.60086	1.6643	.52992	.84805	.62487	1.6003	.54464	.83867	.64941	1.5399	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	59°				58°				57°				

	33°				34°				35°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.54464	.83867	.64941	1.5399	.55919	.82904	.67451	1.4826	.57358	.81915	.70021	1.4281	60
1	488	851	982	.5389	943	887	493	.4816	381	899	064	.4273	59
2	513	835	.65024	.5379	968	871	536	.4807	405	882	107	.4264	58
3	537	819	065	.5369	992	855	578	.4798	429	865	151	.4255	57
4	561	804	106	.5359	.56016	839	620	.4788	453	848	194	.4246	56
5	.54586	.83788	.65148	1.5350	.56040	.82822	.67663	1.4779	.57477	.81832	.70238	1.4237	55
6	610	772	189	.5340	064	806	705	.4770	501	815	281	.4229	54
7	635	756	231	.5330	088	790	748	.4761	524	798	325	.4220	53
8	659	740	272	.5320	112	773	790	.4751	548	782	368	.4211	52
9	683	724	314	.5311	136	757	832	.4742	572	765	412	.4202	51
10	.54708	.83708	.65355	1.5301	.56160	.82741	.67875	1.4733	.57596	.81748	.70455	1.4193	50
11	732	692	397	.5291	184	724	917	.4724	619	731	499	.4185	49
12	756	676	438	.5282	208	708	960	.4715	643	714	542	.4176	48
13	781	660	480	.5272	232	692	.68002	.4705	667	698	586	.4167	47
14	805	645	521	.5262	256	675	045	.4696	691	681	629	.4158	46
15	.54829	.83629	.65563	1.5253	.56280	.82659	.68088	1.4687	.57715	.81664	.70673	1.4150	45
16	854	613	604	.5243	305	643	130	.4678	738	647	717	.4141	44
17	878	597	646	.5233	329	626	173	.4669	762	631	760	.4132	43
18	902	581	688	.5224	353	610	215	.4659	786	614	804	.4124	42
19	927	565	729	.5214	377	593	258	.4650	810	597	848	.4115	41
20	.54951	.83549	.65771	1.5204	.56401	.82577	.68301	1.4641	.57833	.81580	.70891	1.4106	40
21	975	533	813	.5195	425	561	343	.4632	857	563	935	.4097	39
22	999	517	854	.5185	449	544	386	.4623	881	546	979	.4089	38
23	.55024	501	896	.5175	473	528	429	.4614	904	530	.71023	.4080	37
24	048	485	938	.5166	497	511	471	.4605	928	513	066	.4071	36
25	.55072	.83469	.65980	1.5156	.56521	.82495	.68514	1.4596	.57952	.81496	.71110	1.4063	35
26	097	453	.66021	.5147	545	478	557	.4586	976	479	154	.4054	34
27	121	437	063	.5137	569	462	600	.4577	999	462	198	.4045	33
28	145	421	105	.5127	593	446	642	.4568	.58023	445	242	.4037	32
29	169	405	147	.5118	617	429	685	.4559	047	428	285	.4028	31
30	.55194	.83389	.66189	1.5108	.56641	.82413	.68728	1.4550	.58070	.81412	.71329	1.4019	30
31	218	373	230	.5099	665	396	771	.4541	094	395	373	.4011	29
32	242	356	272	.5089	689	380	814	.4532	118	378	417	.4002	28
33	266	340	314	.5080	713	363	857	.4523	141	361	461	.3994	27
34	291	324	356	.5070	736	347	900	.4514	165	344	505	.3985	26
35	.55315	.83308	.66398	1.5061	.56760	.82330	.68942	1.4505	.58189	.81327	.71549	1.3976	25
36	339	292	440	.5051	784	314	985	.4496	212	310	593	.3968	24
37	363	276	482	.5042	808	297	.69028	.4487	236	293	637	.3959	23
38	388	260	524	.5032	832	281	071	.4478	260	276	681	.3951	22
39	412	244	566	.5023	856	264	114	.4469	283	259	725	.3942	21
40	.55436	.83228	.66608	1.5013	.56880	.82248	.69157	1.4460	.58307	.81242	.71769	1.3934	20
41	460	212	650	.5004	904	231	200	.4451	330	225	813	.3925	19
42	484	195	692	.4994	928	214	243	.4442	354	208	857	.3916	18
43	509	179	734	.4985	952	198	286	.4433	378	191	901	.3908	17
44	533	163	776	.4975	976	181	329	.4424	401	174	946	.3899	16
45	.55557	.83147	.66818	1.4966	.57000	.82165	.69372	1.4415	.58425	.81157	.71990	1.3891	15
46	581	131	860	.4957	024	148	416	.4406	449	140	.72034	.3882	14
47	605	115	902	.4947	047	132	459	.4397	472	123	078	.3874	13
48	630	098	944	.4938	071	115	502	.4388	496	106	122	.3865	12
49	654	082	986	.4928	095	098	545	.4379	519	089	167	.3857	11
50	.55678	.83066	.67028	1.4919	.57119	.82082	.69588	1.4370	.58543	.81072	.72211	1.3848	10
51	702	050	071	.4910	143	065	631	.4361	567	055	255	.3840	9
52	726	034	113	.4900	167	048	675	.4352	590	038	299	.3831	8
53	750	017	155	.4891	191	032	718	.4344	614	021	344	.3823	7
54	775	001	197	.4882	215	015	761	.4335	637	004	388	.3814	6
55	.55799	.82985	.67239	1.4872	.57238	.81999	.69804	1.4326	.58661	.80987	.72432	1.3806	5
56	823	969	282	.4863	262	982	847	.4317	684	970	477	.3798	4
57	847	953	324	.4854	286	965	891	.4308	708	953	521	.3789	3
58	871	936	366	.4844	310	949	934	.4299	731	936	565	.3781	2
59	895	920	409	.4835	334	932	977	.4290	755	919	610	.3772	1
60	.55919	.82904	.67451	1.4826	.57358	.81915	.70021	1.4281	.58779	.80902	.72654	1.3764	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	56°				55°				54°				

	36°				37°				38°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.58779	.80902	.72654	1.3764	.60182	.79864	.75355	1.3270	.61566	.78801	.78129	1.2799	60
1	802	885	699	.3755	205	846	401	.3262	589	783	175	.2792	59
2	826	867	743	.3747	228	829	447	.3254	612	765	222	.2784	58
3	849	850	788	.3739	251	811	492	.3246	635	747	269	.2776	57
4	873	833	832	.3730	274	793	538	.3238	658	729	316	.2769	56
5	.58896	.80816	.72877	1.3722	.60298	.79776	.75584	1.3230	.61681	.78711	.78363	1.2761	55
6	920	799	921	.3713	321	758	629	.3222	704	694	410	.2753	54
7	943	782	966	.3705	344	741	675	.3214	726	676	457	.2746	53
8	967	765	.73010	.3697	367	723	721	.3206	749	658	504	.2738	52
9	990	748	055	.3688	390	706	767	.3198	772	640	551	.2731	51
10	.59014	.80730	.73100	1.3680	.60414	.79688	.75812	1.3190	.61795	.78622	.78598	1.2723	50
11	037	713	144	.3672	437	671	858	.3182	818	604	645	.2715	49
12	061	696	189	.3663	460	653	904	.3175	841	586	692	.2708	48
13	084	679	234	.3655	483	635	950	.3167	864	568	739	.2700	47
14	108	662	278	.3647	506	618	996	.3159	887	550	786	.2693	46
15	.59131	.80644	.73323	1.3638	.60529	.79600	.76042	1.3151	.61909	.78532	.78834	1.2685	45
16	154	627	368	.3630	553	583	088	.3143	932	514	881	.2677	44
17	178	610	413	.3622	576	565	134	.3135	955	496	928	.2670	43
18	201	593	457	.3613	599	547	180	.3127	978	478	975	.2662	42
19	225	576	502	.3605	622	530	226	.3119	.62001	460	.79022	.2655	41
20	.59248	.80558	.73547	1.3597	.60645	.79512	.76272	1.3111	.62024	.78442	.79070	1.2647	40
21	272	541	592	.3588	668	494	318	.3103	046	424	117	.2640	39
22	295	524	637	.3580	691	477	364	.3095	069	405	164	.2632	38
23	318	507	681	.3572	714	459	410	.3087	092	387	212	.2624	37
24	342	489	726	.3564	738	441	456	.3079	115	369	259	.2617	36
25	.59365	.80472	.73771	1.3555	.60761	.79424	.76502	1.3072	.62138	.78351	.79306	1.2609	35
26	389	455	816	.3547	784	406	548	.3064	160	333	354	.2602	34
27	412	438	861	.3539	807	388	594	.3056	183	315	401	.2594	33
28	436	420	906	.3531	830	371	640	.3048	206	297	449	.2587	32
29	459	403	951	.3522	853	353	686	.3040	229	279	496	.2579	31
30	.59482	.80386	.73996	1.3514	.60876	.79335	.76733	1.3032	.62251	.78261	.79544	1.2572	30
31	506	368	.74041	.3506	899	318	779	.3024	274	243	591	.2564	29
32	529	351	086	.3498	922	300	825	.3017	297	225	639	.2557	28
33	552	334	131	.3490	945	282	871	.3009	320	206	686	.2549	27
34	576	316	176	.3481	968	264	918	.3001	342	188	734	.2542	26
35	.59599	.80299	.74221	1.3473	.60991	.79247	.76964	1.2993	.62365	.78170	.79781	1.2534	25
36	622	282	267	.3465	.61015	229	.77010	.2985	388	152	829	.2527	24
37	646	264	312	.3457	038	211	057	.2977	411	134	877	.2519	23
38	669	247	357	.3449	061	193	103	.2970	433	116	924	.2512	22
39	693	230	402	.3440	084	176	149	.2962	456	098	972	.2504	21
40	.59716	.80212	.74447	1.3432	.61107	.79158	.77196	1.2954	.62479	.78079	.80020	1.2497	20
41	739	195	492	.3424	130	140	242	.2946	502	061	067	.2489	19
42	763	178	538	.3416	153	122	289	.2938	524	043	115	.2482	18
43	786	160	583	.3408	176	105	335	.2931	547	025	163	.2475	17
44	809	143	628	.3400	199	087	382	.2923	570	007	211	.2467	16
45	.59832	.80125	.74674	1.3392	.61222	.79069	.77428	1.2915	.62592	.77988	.80258	1.2460	15
46	856	108	719	.3384	245	051	475	.2907	615	970	306	.2452	14
47	879	091	764	.3375	268	033	521	.2900	638	952	354	.2445	13
48	902	073	810	.3367	291	016	568	.2892	660	934	402	.2437	12
49	926	056	855	.3359	314	.78998	615	.2884	683	916	450	.2430	11
50	.59949	.80038	.74900	1.3351	.61337	.78980	.77661	1.2876	.62706	.77897	.80498	1.2423	10
51	972	021	946	.3343	360	962	708	.2869	728	879	546	.2415	9
52	995	003	991	.3335	383	944	754	.2861	751	861	594	.2408	8
53	.60019	.79986	.75037	.3327	406	926	801	.2853	774	843	642	.2401	7
54	042	968	082	.3319	429	908	848	.2846	796	824	690	.2393	6
55	.60065	.79951	.75128	1.3311	.61451	.78891	.77895	1.2838	.62819	.77806	.80738	1.2386	5
56	089	934	173	.3303	474	873	941	.2830	842	788	786	.2378	4
57	112	916	219	.3295	497	855	988	.2822	864	769	834	.2371	3
58	135	899	264	.3287	520	837	.78035	.2815	887	751	882	.2364	2
59	158	881	310	.3278	543	819	082	.2807	909	733	930	.2356	1
60	.60182	.79864	.75355	1.3270	.61566	.78801	.78129	1.2799	.62932	.77715	.80978	1.2349	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	53°				52°				51°				

	39°				40°				41°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.62932	.77715	.80978	1.2349	.64279	.76604	.83910	1.1918	.65606	.75471	.86929	1.1504	60
1	955	696	.81027	.2342	301	586	960	.1910	628	452	980	.1497	59
2	977	678	075	.2334	323	567	.84009	.1903	650	433	.87031	.1490	58
3	.63000	660	123	.2327	346	548	059	.1896	672	414	082	.1483	57
4	022	641	171	.2320	368	530	108	.1889	694	395	133	.1477	56
5	.63045	.77623	.81220	1.2312	.64390	.76511	.84158	1.1882	.65716	.75375	.87184	1.1470	55
6	068	605	268	.2305	412	492	208	.1875	738	356	236	.1463	54
7	090	586	316	.2298	435	473	258	.1868	759	337	287	.1456	53
8	113	568	364	.2290	457	455	307	.1861	781	318	338	.1450	52
9	135	550	413	.2283	479	436	357	.1854	803	299	389	.1443	51
10	.63158	.77531	.81461	1.2276	.64501	.76417	.84407	1.1847	.65825	.75280	.87441	1.1436	50
11	180	513	510	.2268	524	398	457	.1840	847	261	492	.1430	49
12	203	494	558	.2261	546	380	507	.1833	869	241	543	.1423	48
13	225	476	606	.2254	568	361	556	.1826	891	222	595	.1416	47
14	248	458	655	.2247	590	342	606	.1819	913	203	646	.1410	46
15	.63271	.77439	.81703	1.2239	.64612	.76323	.84656	1.1812	.65935	.75184	.87698	1.1403	45
16	293	421	752	.2232	635	304	706	.1806	956	165	749	.1396	44
17	316	402	800	.2225	657	286	756	.1799	978	146	801	.1389	43
18	338	384	849	.2218	679	267	806	.1792	.66000	126	852	.1383	42
19	361	366	898	.2210	701	248	856	.1785	022	107	904	.1376	41
20	.63383	.77347	.81946	1.2203	.64723	.76229	.84906	1.1778	.66044	.75088	.87955	1.1369	40
21	406	329	995	.2196	746	210	956	.1771	066	069	.88007	.1363	39
22	428	310	.82044	.2189	768	192	.85006	.1764	088	050	059	.1356	38
23	451	292	092	.2181	790	173	057	.1757	109	030	110	.1349	37
24	473	273	141	.2174	812	154	107	.1750	131	011	162	.1343	36
25	.63496	.77255	.82190	1.2167	.64834	.76135	.85157	1.1743	.66153	.74992	.88214	1.1336	35
26	518	236	238	.2160	856	116	207	.1736	175	973	265	.1329	34
27	540	218	287	.2153	878	097	257	.1729	197	953	317	.1323	33
28	563	199	336	.2145	901	078	308	.1722	218	934	369	.1316	32
29	585	181	385	.2138	923	059	358	.1715	240	915	421	.1310	31
30	.63608	.77162	.82434	1.2131	.64945	.76041	.85408	1.1708	.66262	.74896	.88473	1.1303	30
31	630	144	483	.2124	967	022	458	.1702	284	876	524	.1296	29
32	653	125	531	.2117	989	003	509	.1695	306	857	576	.1290	28
33	675	107	580	.2109	.65011	.75984	559	.1688	327	838	628	.1283	27
34	698	088	629	.2102	033	965	609	.1681	349	818	680	.1276	26
35	.63720	.77070	.82678	1.2095	.65055	.75946	.85660	1.1674	.66371	.74799	.88732	1.1270	25
36	742	051	727	.2088	077	927	710	.1667	393	780	784	.1263	24
37	765	033	776	.2081	100	908	761	.1660	414	760	836	.1257	23
38	787	014	825	.2074	122	889	811	.1653	436	741	888	.1250	22
39	810	.76996	874	.2066	144	870	862	.1647	458	722	940	.1243	21
40	.63832	.76977	.82923	1.2059	.65166	.75851	.85912	1.1640	.66480	.74703	.88992	1.1237	20
41	854	959	972	.2052	188	832	963	.1633	501	683	.89045	.1230	19
42	877	940	.83022	.2045	210	813	.86014	.1626	523	664	097	.1224	18
43	899	921	071	.2038	232	794	064	.1619	545	644	149	.1217	17
44	922	903	120	.2031	254	775	115	.1612	566	625	201	.1211	16
45	.63944	.76884	.83169	1.2024	.65276	.75756	.86166	1.1606	.66588	.74606	.89253	1.1204	15
46	966	866	218	.2017	298	738	216	.1599	610	586	306	.1197	14
47	989	847	268	.2009	320	719	267	.1592	632	567	358	.1191	13
48	.64011	828	317	.2002	342	700	318	.1585	653	548	410	.1184	12
49	033	810	366	.1995	364	680	368	.1578	675	528	463	.1178	11
50	.64056	.76791	.83415	1.1988	.65386	75661	.86419	1.1571	.66697	.74509	.89515	1.1171	10
51	078	772	465	.1981	408	642	470	.1565	718	489	567	.1165	9
52	100	754	514	.1974	430	623	521	.1558	740	470	620	.1158	8
53	123	735	564	.1967	452	604	572	.1551	762	451	672	.1152	7
54	145	717	613	.1960	474	585	623	.1544	783	431	725	.1145	6
55	.64167	.76698	.83662	1.1953	.65496	.75566	.86674	1.1538	.66805	.74412	.89777	1.1139	5
56	190	679	712	.1946	518	547	725	.1531	827	392	830	.1132	4
57	212	661	761	.1939	540	528	776	.1524	848	373	883	.1126	3
58	234	642	811	.1932	562	509	827	.1517	870	353	935	.1119	2
59	256	623	860	.1925	584	490	878	.1510	891	334	988	.1113	1
60	.64279	.76604	.83910	1.1918	.65606	.75471	.86929	1.1504	.66913	.74314	.90040	1.1106	0
	Cos.	Sin.	Cot.	Tan	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	50°				49°				48°				

	42°				43°				44°				
M.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	
0	.66913	.74314	.90040	1.1106	.68200	.73135	.93252	1.0724	.69466	.71934	.96569	1.0355	60
1	935	295	093	.1100	221	116	306	.0717	487	914	625	.0349	59
2	956	276	146	.1093	242	096	360	.0711	508	894	681	.0343	58
3	978	256	199	.1087	264	076	415	.0705	529	873	738	.0337	57
4	999	237	251	.1080	285	056	469	.0699	549	853	794	.0331	56
5	.67021	.74217	.90304	1.1074	.68306	.73036	.93524	1.0692	.69570	.71833	.96850	1.0325	55
6	043	198	357	.1067	327	016	578	.0686	591	813	907	.0319	54
7	064	178	410	.1061	349	.72996	633	.0680	612	792	963	.0313	53
8	086	159	463	.1054	370	976	688	.0674	633	772	.97020	.0307	52
9	107	139	516	.1048	391	957	742	.0668	654	752	076	.0301	51
10	.67129	.74120	.90569	1.1041	.68412	.72937	.93797	1.0661	.69675	.71732	.97133	1.0295	50
11	151	100	621	.1035	434	917	852	.0655	696	711	189	.0289	49
12	172	080	674	.1028	455	897	906	.0649	717	691	246	.0283	48
13	194	061	727	.1022	476	877	961	.0643	737	671	302	.0277	47
14	215	041	781	.1016	497	857	.94016	.0637	758	650	359	.0271	46
15	.67237	.74022	.90834	1.1009	.68518	.72837	.94071	1.0630	.69779	.71630	.97416	1.0265	45
16	258	002	887	.1003	539	817	125	.0624	800	610	472	.0259	44
17	280	.73983	940	.0996	561	797	180	.0618	821	590	529	.0253	43
18	301	963	993	.0990	582	777	235	.0612	842	569	586	.0247	42
19	323	944	.91046	.0983	603	757	290	.0606	862	549	643	.0241	41
20	.67344	.73924	.91099	1.0977	.68624	.72737	.94345	1.0599	.69883	.71529	.97700	1.0235	40
21	366	904	153	.0971	645	717	400	.0593	904	508	756	.0230	39
22	387	885	206	.0964	666	697	455	.0587	925	488	813	.0224	38
23	409	865	259	.0958	688	677	510	.0581	946	468	870	.0218	37
24	430	846	313	.0951	709	657	565	.0575	966	447	927	.0212	36
25	.67452	.73826	.91366	1.0945	.68730	.72637	.94620	1.0569	.69987	.71427	.97984	1.0206	35
26	473	806	419	.0939	751	617	676	.0562	.70008	407	.98041	.0200	34
27	495	787	473	.0932	772	597	731	.0556	029	386	098	.0194	33
28	516	767	526	.0926	793	577	786	.0550	049	366	155	.0188	32
29	538	747	580	.0919	814	557	841	.0544	070	345	213	.0182	31
30	.67559	.73728	.91633	1.0913	.68835	.72537	.94896	1.0538	.70091	.71325	.98270	1.0176	30
31	580	708	687	.0907	857	517	952	.0532	112	305	327	.0170	29
32	602	688	740	.0900	878	497	.95007	.0526	132	284	384	.0164	28
33	623	669	794	.0894	899	477	062	.0519	153	264	441	.0158	27
34	645	649	847	.0888	920	457	118	.0513	174	243	499	.0152	26
35	.67666	.73629	.91901	1.0881	.68941	.72437	.95173	1.0507	.70195	.71223	.98556	1.0147	25
36	688	610	955	.0875	962	417	229	.0501	215	203	613	.0141	24
37	709	590	.92008	.0869	983	397	284	.0495	236	182	671	.0135	23
38	730	570	062	.0862	.69004	377	340	.0489	257	162	728	.0129	22
39	752	551	116	.0856	025	357	395	.0483	277	141	786	.0123	21
40	.67773	.73531	.92170	1.0850	.69046	.72337	.95451	1.0477	.70298	.71121	.98843	1.0117	20
41	795	511	224	.0843	067	317	506	.0470	319	100	901	.0111	19
42	816	491	277	.0837	088	297	562	.0464	339	080	958	.0105	18
43	837	472	331	.0831	109	277	618	.0458	360	059	.99016	.0099	17
44	859	452	385	.0824	130	257	673	.0452	381	039	073	.0094	16
45	.67880	.73432	.92439	1.0818	.69151	.72236	.95729	1.0446	.70401	.71019	.99131	1.0088	15
46	901	413	493	.0812	172	216	785	.0440	422	.70998	189	.0082	14
47	923	393	547	.0805	193	196	841	.0434	443	978	247	.0076	13
48	944	373	601	.0799	214	176	897	.0428	463	957	304	.0070	12
49	965	353	655	.0793	235	156	952	.0422	484	937	362	.0064	11
50	.67987	.73333	.92709	1.0786	.69256	.72136	.96008	1.0416	.70505	.70916	.99420	1.0058	10
51	.68008	314	763	.0780	277	116	064	.0410	525	896	478	.0052	9
52	029	294	817	.0774	298	095	120	.0404	546	875	536	.0047	8
53	051	274	872	.0768	319	075	176	.0398	567	855	594	.0041	7
54	072	254	926	.0761	340	055	232	.0392	587	834	652	.0035	6
55	.68093	.73234	.92980	1.0755	.69361	.72035	.96288	1.0385	.70608	.70813	.99710	1.0029	5
56	115	215	.93034	.0749	382	015	344	.0379	628	793	768	.0023	4
57	136	195	088	.0742	403	.71995	400	.0373	649	772	826	.0017	3
58	157	175	143	.0736	424	974	457	.0367	670	752	884	.0012	2
59	179	155	197	.0730	445	954	513	.0361	690	731	942	.0006	1
60	.68200	.73135	.93252	1.0724	.69466	.71934	.96569	1.0355	.70711	.70711	1.0000	1.0000	0
	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	M.
	47°				46°				45°				

	0°		1°		2°		3°		4°		
M.	Sin.	Tan.	Sin.	Tan.	Sin.	Tan.	Sin.	Tan.	Sin.	Tan.	M.
	4.68		4.68		4.68		4.68		4.68		
0	5575	5575	5553	5619	5487	5751	5376	5972	5222	6281	0
1	5575	5575	5552	5620	5485	5754	5374	5976	5219	6287	1
2	5575	5575	5551	5622	5484	5757	5372	5981	5216	6293	2
3	5575	5575	5551	5623	5482	5760	5370	5985	5213	6299	3
4	5575	5575	5550	5625	5481	5763	5367	5990	5210	6305	4
5	5575	5575	5549	5627	5479	5766	5365	5994	5207	6311	5
6	5575	5575	5548	5628	5478	5769	5363	5999	5204	6317	6
7	5575	5575	5547	5630	5476	5773	5361	6004	5201	6323	7
8	5574	5576	5547	5632	5475	5776	5358	6008	5198	6329	8
9	5574	5576	5546	5633	5473	5779	5356	6013	5195	6335	9
10	5574	5576	5545	5635	5471	5782	5354	6017	5192	6341	10
11	5574	5576	5544	5637	5470	5785	5351	6022	5189	6348	11
12	5574	5577	5543	5638	5468	5788	5349	6027	5186	6354	12
13	5574	5577	5542	5640	5467	5792	5347	6031	5183	6360	13
14	5574	5577	5541	5642	5465	5795	5344	6036	5180	6366	14
15	5573	5578	5540	5644	5463	5798	5342	6041	5177	6372	15
16	5573	5578	5539	5646	5462	5802	5340	6046	5173	6379	16
17	5573	5578	5539	5648	5460	5805	5337	6051	5170	6385	17
18	5573	5579	5538	5649	5458	5808	5335	6055	5167	6391	18
19	5573	5579	5537	5651	5457	5812	5332	6060	5164	6398	19
20	5572	5580	5536	5653	5455	5815	5330	6065	5161	6404	20
21	5572	5580	5535	5655	5453	5818	5327	6070	5158	6410	21
22	5572	5581	5534	5657	5451	5822	5325	6075	5154	6417	22
23	5572	5581	5533	5659	5450	5825	5322	6080	5151	6423	23
24	5571	5582	5532	5661	5448	5829	5320	6085	5148	6430	24
25	5571	5583	5531	5663	5446	5833	5317	6090	5145	6436	25
26	5571	5583	5530	5665	5444	5836	5315	6095	5141	6443	26
27	5570	5584	5529	5668	5443	5840	5312	6100	5138	6449	27
28	5570	5584	5527	5670	5441	5843	5310	6105	5135	6456	28
29	5570	5585	5526	5672	5439	5847	5307	6110	5132	6462	29
30	5569	5586	5525	5674	5437	5851	5305	6116	5128	6469	30
31	5569	5587	5524	5676	5435	5854	5302	6121	5125	6476	31
32	5569	5587	5523	5679	5433	5858	5300	6126	5122	6482	32
33	5568	5588	5522	5681	5431	5862	5297	6131	5118	6489	33
34	5568	5589	5521	5683	5430	5866	5294	6136	5115	6496	34
35	5567	5590	5520	5685	5428	5869	5292	6142	5112	6503	35
36	5567	5591	5518	5688	5426	5873	5289	6147	5108	6509	36
37	5566	5592	5517	5690	5424	5877	5286	6152	5105	6516	37
38	5566	5593	5516	5693	5422	5881	5284	6158	5101	6523	38
39	5566	5593	5515	5695	5420	5885	5281	6163	5098	6530	39
40	5565	5594	5514	5697	5418	5889	5278	6168	5095	6537	40
41	5565	5595	5512	5700	5416	5893	5276	6174	5091	6544	41
42	5564	5596	5511	5702	5414	5897	5273	6179	5088	6551	42
43	5564	5598	5510	5705	5412	5900	5270	6185	5084	6557	43
44	5563	5599	5509	5707	5410	5905	5268	6190	5081	6564	44
45	5562	5600	5507	5710	5408	5909	5265	6196	5077	6571	45
46	5562	5601	5506	5713	5406	5913	5262	6201	5074	6578	46
47	5561	5602	5505	5715	5404	5917	5259	6207	5070	6585	47
48	5561	5603	5503	5718	5402	5921	5256	6212	5067	6593	48
49	5560	5604	5502	5720	5400	5925	5254	6218	5963	6600	49
50	5560	5605	5501	5723	5398	5929	5251	6224	5060	6607	50
51	5559	5607	5499	5726	5396	5933	5248	6229	5056	6614	51
52	5558	5608	5498	5729	5394	5937	5245	6235	5053	6621	52
53	5558	5609	5497	5731	5392	5942	5242	6241	5049	6628	53
54	5557	5611	5495	5734	5389	5946	5239	6246	5045	6635	54
55	5556	5612	5494	5737	5387	5950	5237	6252	5042	6643	55
56	5556	5613	5492	5740	5385	5955	5234	6258	5038	6650	56
57	5555	5615	5491	5743	5383	5959	5231	6264	5034	6657	57
58	5554	5616	5490	5745	5381	5963	5228	6269	5031	6665	58
59	5554	5618	5488	5748	5379	5968	5225	6275	5027	6672	59
M.	Sin.	Tan.	Sin.	Tan.	Sin.	Tan.	Sin.	Tan.	Sin.	Tan.	M.

AN ELEMENTARY TREATISE

ON

SURVEYING AND NAVIGATION

BY

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PREFACE.

THIS brief work on Surveying and Navigation is intended for those students who desire to supplement the study of Trigonometry with a brief course on its applications to those subjects.

No attempt has been made to treat the subjects fully. Special effort has, however, been made to have the work correct and accurate as far as it goes, and it is believed that the student who afterwards becomes a surveyor or navigator will have nothing to unlearn that he has learned from this work.

The author wishes to acknowledge his obligations to Messrs. W. & L. E. Gurley, Troy, N.Y., for the use of plates from which were made the cuts of the instruments found throughout the work.

ARTHUR G. ROBBINS.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
September, 1896.

CONTENTS.

LAND SURVEYING.

SURVEYING is the art of measuring and locating lines and angles on the earth's surface.

Representation of these measurements on paper is called *plotting.*

By applying the rules of geometry and trigonometry to these measurements, distances and quantities may be computed.

In plane surveying all distances measured are horizontal distances.

The instruments for measuring distances are as follows:

Gunter's or surveyor's chain, made of iron or steel wire, contains

$$100 \text{ links} = 4 \text{ rods} = 66 \text{ feet.}$$
$$1 \text{ link} = 7\tfrac{92}{100} \text{ inches.}$$
$$10 \text{ sq. chains} = 1 \text{ Acre.}$$

Engineer's chain, 100 ft. long, contains 100 links, each one foot long.

The steel tape, which is made of a continuous ribbon of steel, generally in lengths of 50 or 100 ft., is graduated to feet, tenths and hundredths of a foot.

The metallic tape, generally 50 ft. in length, is made of cloth, into which is woven a number of fine wires to prevent stretching. It is graduated to feet, tenths and half-tenths.

Wooden rods, for measuring short distances, are graduated to read by a vernier to one-thousandth of a foot.

The inch is not used in surveying field work.

To measure a Straight Line with the Chain.

The work is done by two persons using a chain and a set of eleven marking-pins. Sometimes two lining-poles are used to mark the ends of the line.

The fore chainman takes ten of the eleven pins, and sets a pin at the end of each chain length, after the rear chainman has lined it in with the point to which the distance is to be measured. The rear chainman lines

in the pin to be set by the fore chainman, holds the rear end of the chain against the last pin set by the fore chainman until the next one is set, and takes up each pin after the next forward one is set.

As soon as the fore chainman sets his last pin, he calls up and receives from the rear chainman his ten pins, taking care to count them to see that none are lost, records one tally, *i.e.* ten chains, and the work is continued till the end of the line is reached. The total distance is found by counting ten chains for each tally, one chain for each pin in the hand of the rear chainman, and the fractional part of a chain between the last pin set and the end of the line. The last pin should remain in the ground and not be counted as in the hand of the rear chainman.

When a line is being measured down hill, the forward end of the chain should be held at the same level as the rear end, and the point transferred to the ground by a plumb line, or, less accurately, by a lining-pole suspended between the thumb and finger. In all cases the chain must be held horizontal, whether the ground is level or not. The tendency, when learning to chain on a steep hill, is to hold the chain inclined at a less angle with the ground than the one between the horizontal and the surface of the ground.

Because of the large number of wearing surfaces, usually 600 in the surveyor's and engineer's chain, the length of every chain should be frequently compared with a standard.

A new steel tape may be used as a standard when there is none other convenient. The comparison should be made on a smooth and level surface, such as a level sidewalk or long hallway. Provision is made for shortening the chain by turning a nut at the end of either one of the end links.

If a line has been measured with a chain afterwards found to be too long or too short, the measured length should be corrected. In making this correction it should be remembered that the *true* length of a line, measured with a chain that is too long, is *greater* than the measured length, and *vice versa.*

The degree of precision obtained ought to depend upon the character of the work, and not upon the character of the ground. With ordinary skill in chaining, the error in measuring a line along the highways of a town or village should not exceed one part in four or five thousand. If no more care be used in measuring a line through the tangled undergrowth of some forest lands, an error of one in three or four hundred might be expected.

By applying the rules of geometry and trigonometry, it is practicable to measure angles, considerable areas, and distances to inaccessible objects by means of the chain.

Given the line **ab**, *to locate on the ground the line* **be** *perpendicular to* **ab** *at* **b**.

Select any convenient point as *c*, one chain length from *b*. With one end of the chain held at *c*, swing the other end to *d*, in the line *ab*, using

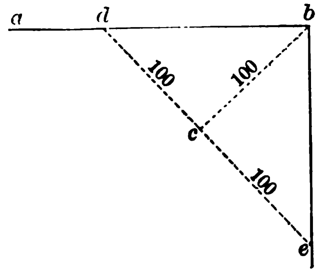

FIG. 1.

a lining-pole at *d* to aid in lining in from *a* if necessary. With one end of the chain still held at *c*, swing the other end to locate *e* in the line *dc* produced. *be* is the perpendicular required.

Another Method.

Measure $cb = 40$ ft. Hold one end of the chain at *c*, and the 80-ft. mark at *b*. Let a third person take the chain at the 50-ft. mark and pull both parts *cd* and *bd* taut. *bd* is the line required.

Other methods equally feasible will suggest themselves to the student.

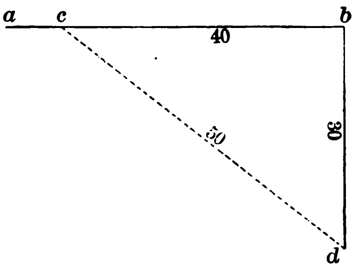

FIG. 2.

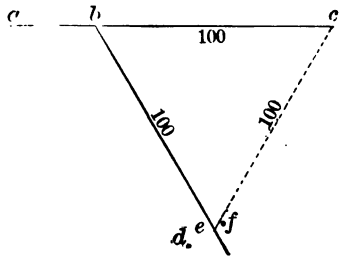

FIG. 3.

Given a line **ab** *on the ground, to locate a line starting at* **b** *and making an angle of* 60° *with this line.*

Prolong *ab* (Fig. 3) any convenient distance, as 100 ft., to *c*. Hold one end of the chain at *b*, and with the same length *bc* locate a number of pins at *d*, *f*, etc., a foot or two apart and at a distance from *c* as nearly equal to *bc* as can be estimated. Measure from *c* the distance $ce = cb$, lining in the point *e* between the two adjacent pins (*d* and *f* in this case). *be* is the line required.

To lay out Any Given Angle, as 22° 30′, with the Chain.

By trigonometry, twice the sine of one-half of an angle is equal to its chord, the radius being unity.

Given the line **ab** *on the ground to locate a line* **cd**, *making the angle* **bcd** = 22° 30′.

From *c* measure *ce* any convenient distance, as 100 ft. Twice the sine of one-half of 22° 30′ = .3902. Multiply this by distance *ce* (100 ft.

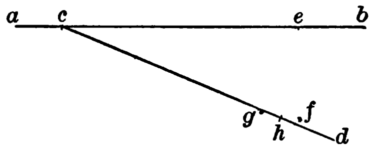

FIG. 4.

in this case). From *e*, with radius 39.02 ft., locate pins on arc *fg* one or two feet apart. From *c*, with radius 100 ft., locate the point *h* on the arc *fg*. *cd* passing through *h* is the line required.

By the reverse of the above process, the angle between any two lines located on the ground may be measured.

Figures 5, 6, 7, and 8 illustrate how lines may be prolonged through obstacles, and distances to inaccessible objects measured by application of these problems.

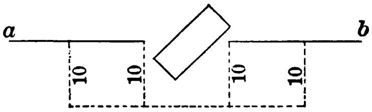

FIG. 5.

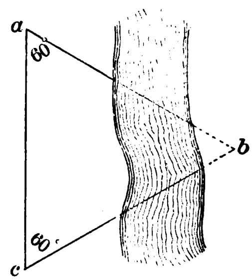

FIG. 6.

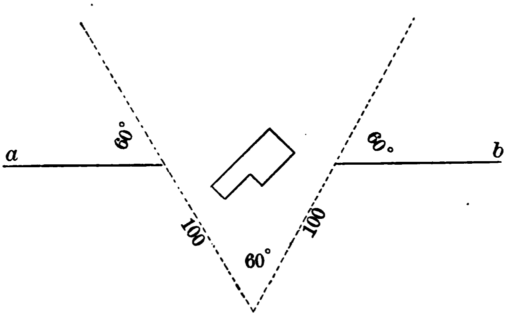

FIG. 7.

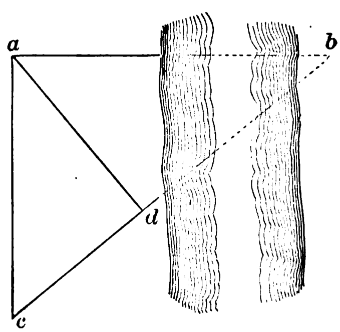

FIG. 8.
ab : *ac* : : *ad* : *cd*

Areas of fields may be determined by measuring the length of lines dividing the field into triangles and computing the area of each triangle

separately. In dividing the field into triangles, much more accurate results will be obtained if the lines to be measured are so chosen as to divide the field into as few and as nearly equilateral triangles as possible.

The area of a narrow, irregular area such as that shown at *abc* (Fig. 9) can best be determined by measuring at regular intervals the lengths of

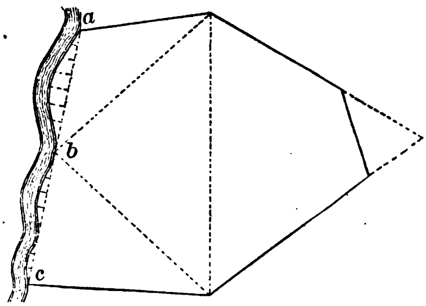

FIG. 9.

right-angle offsets to the lines *ab* and *bc* and then computing the area by the following rule, known as the trapezoidal rule,

$$A = \frac{d}{2}(h + 2\,\Sigma h_i + h'),$$

in which A is the area, d the common distance between the offsets, h and h' the two end offsets, and $2\,\Sigma h_i$ twice the sum of all the offsets between the two end offsets.

This rule assumes that the bounding line between any two adjacent offsets is a straight line.

THE COMPASS.

The compass, shown in Fig. 10, is an instrument for determining the angle that any line makes with the magnetic meridian.

It consists essentially of a circle, graduated to half-degrees, and numbered from 0° to 90° each way from the north and south points.

The sights are in the prolongation of the line N. S. Balanced on a steel pivot at the centre of the circle, is a magnetic needle, which points in the direction of the magnetic meridian. The compass is connected with the tripod by a ball and socket joint, which allows the compass to be turned in any direction for the purpose of levelling.

By setting the compass anywhere in a line and levelling it, by aid of the spirit bubbles shown on the compass box, then turning the sights till they point in the direction of the line, the angle which this line makes

with the magnetic meridian may be read, by observing the number of degrees between the N. or S. point of the compass box and the N. or S. end of the needle. This angle is called the *bearing* of the line. Bearings

Fig. 10.

are read from the N. or S. point of the compass box, 90° in each direction. The letters which mark the E. and W. points on the compass box are reversed to aid in reading bearings, as is shown in Figs. 11, 12, 13, and 14.

The bearing of the line *ab*, Fig. 11, is N. 20° W.
The bearing of the line *ab*, Fig. 12, is N. 20° E.
The bearing of the line *ab*, Fig. 13, is S. 20° E.
The bearing of the line *ab*, Fig. 14, is S. 20° W.

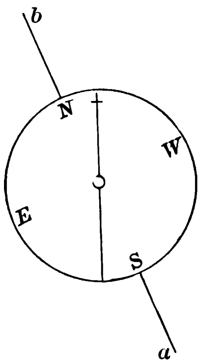

Fig. 11.

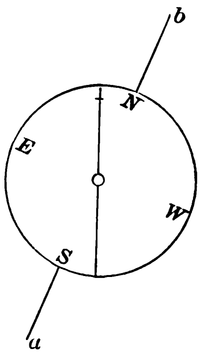

Fig. 12.

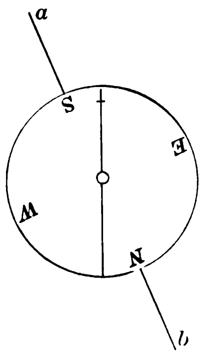

Fig. 13.

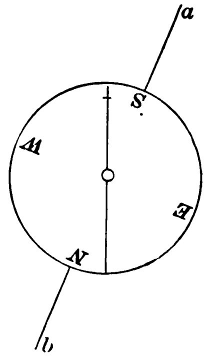

Fig. 14.

If the observer directs the N. point of the compass box in the direction of the line whose bearing is to be taken, and reads the N. end of the needle, the letters on the compass box between which that end of the needle lies are the ones to be used in recording the bearing. A careful observance of this rule will prevent the student from reading bearings incorrectly when learning to use the compass.

The magnetic needle does not point true north. Moreover, it does not always point in the same direction.

The angle which the magnetic needle makes with the true meridian is called the *declination* of the needle. Changes in the declination are called *variations* of the needle. The principal variations are, secular variation, daily variation, irregular variations.

The secular variation is a slow and continuous change in the pointing of the needle, which may be compared to the movement of a pendulum requiring centuries to make a single oscillation.

The daily variation is small, seldom exceeding six or eight minutes, and in ordinary surveying is neglected.

Irregular variations may occur at any time. They follow no known law.

In order to learn what the declination of the needle is at any time, the compass should be frequently set in a true meridian line[1] and the declination measured.

The United States Coast and Geodetic Survey publishes from time to time, charts of the United States, on which are drawn lines connecting places at which the declination at any given time is the same. These lines are called *isogonic lines.* The line connecting places where there is no declination is called the *agonic line.* In 1890 this line passed a little to the east of Charleston, S.C., through North Carolina, Virginia, Ohio, and Michigan. At all places to the east of this line, the declination is *west,* and at all places to the west of this line the declination is *east.*

At present the *west* declinations are increasing about two to four minutes a year, while the *east* declinations are decreasing; or, in other words, the agonic line is moving west.

From observations extending over a number of years it is possible to devise formulas by which the declination of the needle may be computed, with a considerable degree of precision, for some years in the future.

The following are a number of such formulas taken from tables in the United States Coast and Geodetic Survey Report for 1886:

$D =$ the declination, $+$ when east, and $-$ when west. $m =$ time $- 1850$, expressed in years and fraction of a year.

[1] A method of establishing a true meridian line is explained in the chapter on Practical Astronomy.

Name of Station and State.	Latitude.	West Longitude.	The Magnetic Declination expressed as a Function of Time.
	° ′	° ′	° ° °
Montreal, Can.	45 30.5	73 34.6	$D=+11.88+4.17\sin(1.50m-18.5)$
Eastport, Me.	44 54.4	66 59.2	$D=+15.14+3.90\sin(1.20m+31.7)$
Portland, Me.	43 38.8	70 16.6	$D=+11.26+3.16\sin(1.33m+5.8)$
Burlington, Vt.	44 28.5	73 12.0	$D=+10.81+3.65\sin(1.30m-20.5)$ $+0.18\sin(7.00m+132.0)$
Hanover, N.H.	43 42.3	72 17.1	$D=+9.80+4.02\sin(1.40m-14.1)$
Rutland, Vt.	43 36.5	72 55.5	$D=+10.03+3.82\sin(1.50m-24.3)$
Portsmouth, N.H. . . .	43 04.3	70 42.5	$D=+10.71+3.36\sin(1.44m-7.4)$
Boston, Mass.	42 21.5	71 03.9	$D=+9.48+2.94\sin(1.30m+3.7)$
Hartford, Conn.. . . .	41 45.9	72 40.4	$D=+8.06+2.90\sin(1.25m-26.4)$
Albany, N.Y.	42 39.2	73 45.8	$D=+8.17+3.02\sin(1.44m-8.3)$
Harrisburg, Pa.	40 15.9	76 52.9	$D=+2.93+2.98\sin(1.50m+0.2)$
Baltimore, Md.	39 17.8	76 37.0	$D=+3.20+2.57\sin(1.45m-21.2)$
Charleston, S.C.. . . .	32 46.6	79 55.8	$D=-2.14+2.77\sin(1.40m-3.1)$
Key West, Fla.	24 33.5	81 48.5	$D=-3.70+3.16\sin(1.35m-35.1)$
New Orleans, La. . . .	29 57.2	90 03.9	$D=-5.61+2.57\sin(1.40m-61.9)$
Cincinnati, O.	39 08.6	84 25.3	$D=-2.40+2.62\sin(1.42m-39.8)$
Pittsburgh, Pa.	40 27.6	80 00.8	$D=+1.85+2.45\sin(1.45m-28.4)$
Cleveland, O..	41 30.3	81 42.0	$D=+0.10+2.07\sin(1.40m-6.2)$
Detroit, Mich.	42 20.0	83 03.0	$D=-0.97+2.21\sin(1.50m-15.3)$
Buffalo, N.Y..	42 52.8	78 53.5	$D=+3.66+3.47\sin(1.40m-27.8)$
Sitka, Alaska	57 02.9	135 19.7	$D=-25.79+3.30\sin(1.30m-104.2)$
Port Townsend, Wash. T.,	48 07.0	122 44.9	$D=-18.84+3.00\sin(1.45m-122.1)$
San Francisco, Cal. . .	37 47.5	122 27.3	$D=-13.94+2.65\sin(1.05m-135.5)$
Monterey, Cal.	36 36.1	121 53.6	$D=-13.79+2.65\sin(1.10m-156.4)$
San Diego, Cal.	32 42.1	117 14.3	$D=-11.78+1.90\sin(1.15m-151.6)$

It is in relocating the lines of an old survey when the location of some of the corners has been lost that a knowledge of the changes in the declination of the needle is essential to the surveyor. For example, the declination of the needle at New Haven, Conn., was

5° 45′ W. in 1761,
5° 15′ W. in 1780,
4° 30′ W. in 1819,
5° 30′ W. in 1828,
6° 00′ W. in 1838.

Therefore, a line that had a bearing of N. 10° W. in 1761, had a bearing of

N. 10° 30′ W. in 1780,
N. 11° 15′ W. in 1819,
N. 10° 15′ W. in 1828,
N. 9° 45′ W. in 1838.

At a place in Illinois the declination of the needle in 1821 was 8° 00′ E., and in 1843 it was 7° 15′ E.; therefore a line that in 1821 had a bearing of S. 20° W., had, in 1843, a bearing of S. 20° 45′ W.

When the bearing of a line is being read, see that there is no iron or steel near, to turn the needle from its normal position. Some of the things most liable to cause this are, the chain or pins, hatchet, covered steel buttons on coat, steel-bowed spectacles, keys or knife in vest pocket. In fact, any bit of iron held near the needle may attract it appreciably.

To measure the bearing of a line when local attraction cannot be avoided, as, for example, when the line is an iron fence or a railroad track, measure the bearing in the usual way, and also, from the same place, measure the bearing to a point where no local attraction exists. Next set the compass over this latter point and measure the bearing to the first point. This bearing being correct, the bearing of the fence or track may be computed.

Example. — Compass at a (Fig. 15), bearing ab = N. 20° E.; bearing ac = N. 85° E.; therefore angle cab = 65°.

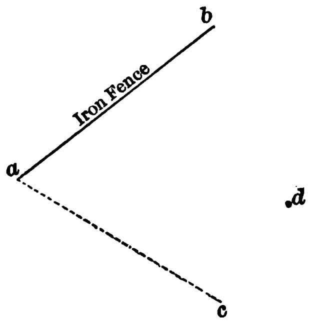

FIG. 15.

Compass at c, bearing ca = N. 83° W., which is the true bearing of ca, provided there is no local attraction at c; therefore the true bearing of ac is S. 83° E.; and since angle bac = 65°, the bearing of ab is N. 32° E. instead of N. 20° E.

In order to prove that there is no local attraction at c, select a fourth point d and measure the bearing cd; then at d measure the bearing dc. If one is exactly the reverse of the other, there is no local attraction at either c or d.

To make a compass survey of a field, the bearing and reverse bearing of all the sides should be taken, and the length of all the sides measured.

The following are samples of field notes of compass surveys:

	Station.	Bearing.	Reverse Bearing.	Distance.	Station.	Bearing.	Reverse Bearing.	Distance.	
		°	°	chains		°	°	chains	
(1)	1	N. 37 E.	S. 37 W.	15.32	1	N. 75¼ W.	—	5.22	(3)
	2	N. 46½ W.	S. 46¾ E.	4.53	2	S. 77 W.	—	10.60	
	3	S. 43½ W.	N. 43¼ E.	13.75	3	S. 74¾ W.	—	4.57	
	4	S. 26 E.	N. 26 W.	5.00	4	N. 86 E.	—	3.84	
	5	S. 57 E.	N. 57 W.	1.60	5	S. 50 E.	—	4.00	
					6	S. 27½ E.	—	3.93	
(2)	1	N. 67 E.	S. 67 W.	3.66	7	N. 65½ E.	—	7.90	
	2	S. 24½ E.	N. 24½ W.	0.95	8	N. 23 E.	—	2.17	
	3	S. 36¼ E.	N. 36¼ W.	1.34	9	N. 33 E.	—	1.00	
	4	S. 53½ E.	N. 53½ W.	2.00	10	N. 46½ E.	—	1.84	
	5	S. 42¼ E.	N. 42 W.	1.14	11	N. 60½ E.	—	1.40	
	6	S. 35¼ E.	N. 35½ W.	2.52					
	7	S. 74¼ W.	N. 74¼ E.	3.20	1	N. 89¼ W.	S. 89 E.	4.74	(4)
	8	N. 33 W.	S. 32¾ E.	3.30	2	N. 17½ W.	S. 17½ E.	12.50	
	9	N. 50¾ W.	S. 50¾ E.	1.77	3	S. 73¾ E.	N. 73½ W.	15.36	
	10	N. 61¾ W.	S. 61½ E.	1.14	4	S. 38½ W.	N. 38½ E.	9.87	
	11	N. 47½ W.	S. 47¼ E.	1.53					

(2) Area 2 A. 29 Rds. (3) Area 7 A. 155 Rds. (4) Area 9 A. 127 Rds. *Ans.*

If the readings of the two ends of the needle are not alike, the trouble is probably due to a bent needle or a bent pivot. If the needle is bent, the difference in the readings of the two ends will be the same, whatever the direction of the compass sights. If the difference in the readings of the two ends of the needle changes with the direction of the compass sights, the pivot is bent, and the needle may or may not be straight.

To straighten the pivot, turn the compass box till the difference in the readings of the two ends of the needle is greatest, then bend the pivot at right angles to the direction of the needle. After the pivot is straightened, to straighten the needle, bend till the readings of the two ends are the same.

To remagnetize the Needle.

Remove the glass cover from the compass box and take the needle from the pivot. Place the needle on a flat surface and draw an ordinary bar magnet from the centre of the needle to the end, repeating the operation a number of times. In returning the magnet from the end of the needle to the centre, it should be carried several inches away from the needle

and not moved back close to its surface. Use the *north* pole of the magnet on the *south* end of the needle, and *vice versa.*

Sometimes when the surface of the glass cover to the compass has been rubbed with a dry cloth, electricity will be developed and cause the needle to adhere to the glass when lowered to the pivot. A touch on the glass with the moistened finger will remove the difficulty and cause the needle to swing free.

Always keep the *needle raised from the pivot except when taking a bearing,* otherwise the sharp point of the pivot will become blunt, and the pointing of the needle much less precise.

CALCULATION OF THE AREA.

The *latitude* of a line, or its northing or southing, is the distance that one end of the line is north or south of the other end.

It is equal to the length of the line, multiplied by the natural cosine of its bearing.

The *departure* of a line, or its easting or westing, is the distance that one end of the line is east or west of the other end.

It equals the length of the line multiplied by the natural sine of its bearing.

Obviously in a closed field the sum of the northings should equal the sum of the southings, and the sum of the eastings equal the sum of the westings.

It is customary to consider north latitudes and east departures *positive,* and south latitudes and west departures *negative.*

Compute all the latitudes and departures. The difference between the sum of the northings and the sum of the southings is the error in latitude. The difference between the sum of the eastings and the sum of the westings is the error in departure. The square root of the sum of the squares of these errors is the *error of closure.* This should not exceed a certain percentage of the perimeter, the amount depending upon the precision required for the work in hand.

After determining the error in latitude and in departure, this error must be distributed among the several courses, in order that the latitudes and departures shall balance exactly. This may be done by the following rule:

The sum of all the latitudes, or departures, is to any latitude, or departure, as the total error in latitude, or departure, is to the correction to be applied to that latitude, or departure.

This simply distributes the error throughout the whole length of the survey. It would be more precise, if not scientific, to make the correc-

tion to the lines or bearings that were incorrectly measured. The surveyor, from his knowledge of the survey, should distribute the error among those lines and bearings in which the natural difficulties to doing correct work were the greatest.

The surveyor should be cautious about increasing the length of any of the lines, in order to balance the survey, since any chainman, however careful, is much more likely to make the measured length of a line too *long* rather than too *short*, because of the difficulty in drawing the chain perfectly straight and horizontal.

After balancing the latitudes and departures, compute the *double meridian distances.*

The double meridian distance of any line is equal to twice the distance of its centre from any chosen meridian.

It will be found convenient to choose the meridian passing through the extreme east or west point of a survey, from which to compute the double meridian distances, in order that they may all have the same algebraic sign. This point may generally be easily determined by a simple inspection of the field notes. Having selected this point, call the line starting from it the *first course.*

RULE. — *The double meridian distance of the first course is equal to its departure.*

The double meridian distance of the second course is equal to the double meridian distance of the first course, plus its departure, plus the departure of the second course.

The double meridian distance of any course is equal to the double meridian distance of the preceding course, plus its departure, plus the departure of the course itself.

The double meridian distance of the last course should equal its departure, which checks the computation.

Multiply the double meridian distance of each course by its corrected northing or southing, observing that the product of a northing multiplied by a positive double meridian distance gives a positive result, and that the product of a southing multiplied by a positive double meridian distance gives a negative result.

The algebraic sum of all these products is twice the area of the field.

The following is the computation of the area from the field notes given in Example 1, page 10:

Station.	Bearing.	Distance.	Latitude.		Departure.		Balanced.		D.M.D.	+Areas.	−Areas.
			N+	S−	E+	W−	Latitude.	Departure.			
1	N. 37° E.	15.32	12.24	—	9.23	—	+12.23	+9.22	16.28	199.104	—
2	N. 46½ W.	4.53	3.11	—	—	3.28	+ 3.11	−3.28	22.22	69.104	—
3	S. 43½ W.	13.75	—	9.97	—	9.46	− 9.98	−9.47	9.47	—	94.511
4	S. 26 E.	5.00	—	4.49	2.19	—	− 4.49	+2.19	2.19	—	9.833
5	S. 57 E.	1.60	—	0.87	1.34	—	− 0.87	+1.34	5.72	—	4.976
			15.35	15.33	12.76	12.74				268.208	109.320
			15.33		12.74					109.320	
			error in southing, 0.02.		0.02, error in westing.					2)158.888	
										79.444 sq. chains	
										= 7 acres 151.1 rods.	

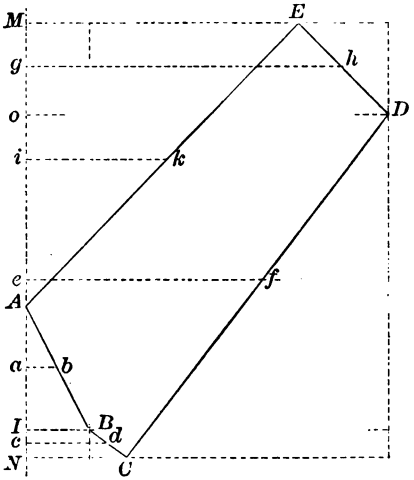

FIG. 16.

ABCDE (Fig. 16) shows a plot of the field, the area of which is computed above. *MN* is a meridian through *A*, the most westerly point. (Sta. 4 in field notes.)

Course.	Latitude.	D.M.D.	+Areas.	− Areas.
AB	*AI*	2 *ab*	——	2 *ABI*
BC	*IN*	2 *cd*	——	2 *IBCN*
CD	*No*	2 *ef*	2 *NCDo*	——
DE	*oM*	2 *gh*	2 *oDEM*	——
EA	*MA*	2 *ik*	——	2 *AME*

It is clear from Fig. 16 that each area given in the last table is equal to the product of the latitude of the course forming one of its limits, into the double meridian distance of that course. It is also evident that the difference between the + areas and the − areas is equal to twice the area *ABCDE*.

Plotting.

A survey may be plotted by laying off the angles with a protractor, and the distances, to the desired scale, with any convenient form of scale; or the corners of the field may be plotted by means of total latitudes and departures.

The protractor, one form of which is shown in Fig. 17, is generally made of a circular or semi-circular piece of brass or German silver, graduated to degrees on its circumference.

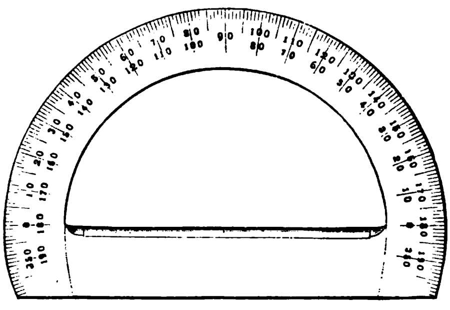

FIG. 17.

To lay off an angle with the protractor, place the centre mark over the vertex of the angle, with the diameter through the 0° mark along the given line. Mark off the number of degrees contained in the angle, and connect this point with the vertex.

In every case the line from which the angle is laid off should extend beyond the circumference of the protractor in both directions, to insure an accurate result.

To plot by Total Latitudes and Departures.

The total latitude, or departure, of any station is equal to the algebraic sum of all the latitudes, or departures, of all the preceding courses.

The following are the total latitudes and departures of the survey computed on page 13:

Station.	Total Latitude.	Total Departure.
A	0	0
B	− 4.49	+ 2.19
C	− 5.36	+ 3.53
D	+ 6.88	+ 12.75
E	+ 9.98	+ 9.47
A	0	0

Draw a meridian line, and locate at any convenient point along this line the point having 0 for its total latitude and departure. Lay off above this point the greatest positive total latitude, and below this point the greatest negative total latitude. At each of these latter points erect perpendiculars, and give each a length equal to the greatest total departure. Draw a line connecting the ends of these perpendiculars, and a rectangle will be formed, inside of which the plot is to be located. From these lines locate each station of the survey, using the total latitudes as ordinates and the total departures as abscissas. Draw lines connecting each adjacent station, and the survey is plotted. To check the work, scale each line and see if it agrees with the measured length. (*ABCDE*, Fig. 16, is plotted by total latitudes and departures.)

Every plot should show, in addition to the outline of the field, the length and bearing of each line, the date of the survey, the declination of the magnetic needle at the time the survey was made, the scale of the plot, the location of the field, and the name of the surveyor.

When it is not practicable to measure one line of a survey, the missing data may be supplied by the computation, as in that case the difference between the northings and the southings is the latitude of the missing line, and the difference between the eastings and westings is its departure. Divide the departure by the latitude, and the quotient is the natural tangent of the missing course. If practicable, all of the courses and distances should be measured, as whenever one is omitted the whole error of closure is, of necessity, thrown into the missing line, and the surveyor has no knowledge of the magnitude of the error.

PARTING OFF LAND.

Suppose that in the field plotted on page 13, it is required to locate from a point F (Fig. 18) on the line CD, eight chains from C, a line FX

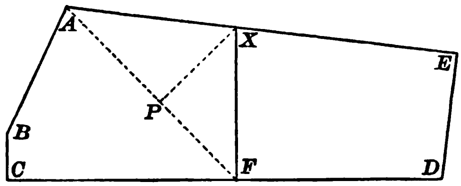

FIG. 18.

such that the area $ABCFX$ shall be four acres. It is evident that the point X will fall somewhere on the line AE.

Draw AF. Compute the area $ABCF$.

The latitude, departure, and bearing of AF may be determined as in the case of the missing bearing and distance of any line. Subtract the area $ABCF$ from four acres, and there remains the area of the triangle AFX. Compute the length of PX, perpendicular to AF ($\frac{1}{2} AF \times PX$ = area AFX). With the angle XAF and the length PX given, AX may be computed, and X located by measuring this distance from A.

RUNNING A RANDOM LINE.

If for any reason, in running from one station of a survey to another, the two points are not intervisible, and there is no dividing fence, a random line is first run in as nearly the right direction as can be judged, till a point is reached, at which an offset at right angles to the random line

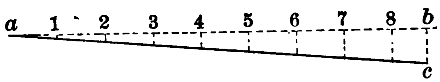

FIG. 19.

will pass through the other end of the true line. Stakes should be set at regular intervals, say one chain apart, along the random line, from which points on the true line may be located as shown in Fig. 19.

$ab = 8.75$ ch., $bc = .35$ ch.; then to locate a point on the line ac opposite the stake at 8 the offset is $\frac{8}{8.75} \times .35 = .32$ ch. The offset from the stake at 7 is .28 ch., and so on.

If the line divides two wood lots, one of which is to be cut, the line is marked by blazing adjacent trees on the side nearest the line, as shown in Fig. 20.

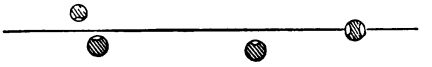

FIG. 20.

To replace a broken boundary line by a single straight line, without changing the area of the adjoining fields.

Let *ab* (Fig. 21) be a curved or broken line separating the two fields. It is required to locate a line, extending from *a* to the line *cd*, such that the area on either side of this line will be the same as that on either side of the line *ab*.

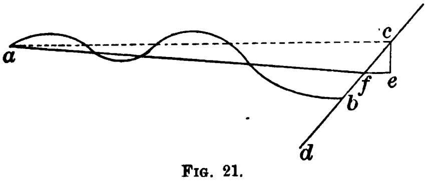

FIG. 21.

From *a* run a random line *ac*. From this random line measure offsets to the curved line *ab* and compute the area on either side of *ac* between it and *ab*. Divide the difference of these areas by half of *ac* and measure this distance, *ce* in the figure, from *c*, perpendicular to *ac*; through *e*, parallel to *ac*, lay off *ef* to *cd*. *af* is the line required.

THE TRANSIT.

The transit, two forms of which are shown in Fig. 22, is an instrument for measuring angles independent of the magnetic needle and with much greater accuracy.

The telescope differs from an ordinary telescope in that there is placed between the object-glass and the eye-piece, a ring carrying two fine spider threads called "cross-hairs." These are used to define the line of sight through the telescope, in the same way as do the compass sights on the compass.

The telescope carries with itself an index, which moves about a graduated circle and marks the number of degrees and minutes contained between two successive sightings of the telescope.

The circle is graduated to half-degrees, or sometimes into twenty-minute spaces. The index carries a vernier, which is a device for reading accurately fractional parts of a degree. The single-minute vernier is generally made as follows:

A space on the vernier, equal to twenty-nine half-degrees on the circle, is divided into thirty equal parts. This makes each division on the vernier one-thirtieth of half a degree shorter than a half-degree, or one minute shorter.

FIG. 22.—PLAIN TRANSIT.

If the zero of the vernier is set opposite the zero of the circle, the first division of the vernier falls one minute short of the first division of the circle, the fifth division five minutes short, and so on, till the thirtieth

division on the vernier is reached. This division falls thirty minutes short of the thirtieth division on the circle, and coincides with the twenty-ninth half-degree division.

FIG. 22. — ENGINEER'S TRANSIT.

By moving the vernier till the first division from the zero coincides with the first division of the circle, the telescope is turned through an angle of one minute. By turning the vernier till its fifth division coin-

cides with the fifth division of the circle, the telescope is moved through an angle of five minutes, and so on.

To read the angle for any setting of the vernier, count, on the circle, the number of degrees and half-degrees between the zero of the circle and the zero of the vernier; then look along the *vernier*, till a division is found which coincides exactly with one of the divisions on the circle; the number of this division *on the vernier* is the number of minutes to be added to the degrees and half-degree, if any, to give the true reading.

The following is a general rule for determining the smallest reading of any vernier:

Divide the value of the smallest division on the circle by the number of divisions on the vernier.

For example, thirty divisions on vernier and circle divided to thirty-minute spaces,

$$\frac{30'}{30} = 1'. \tag{1}$$

Forty divisions on vernier; circle divided to twenty-minute spaces,

$$\frac{20'}{40} = 30''. \tag{2}$$

Sixty divisions on vernier; circle divided to twenty-minute spaces,

$$\frac{20'}{60} = 20''. \tag{3}$$

Sixty divisions on vernier; circle divided to ten-minute spaces,

$$\frac{10'}{60} = 10''. \tag{4}$$

Ten divisions on vernier; rod divided to hundredths of a foot,

$$\frac{.01 \text{ ft.}}{10} = .001 \text{ ft.} \tag{5}$$

Ten divisions on vernier; scale divided to tenths of an inch,

$$\frac{.1 \text{ in.}}{10} = .01 \text{ in.} \tag{6}$$

Twenty-five divisions on vernier; scale divided to .05 in.,

$$\frac{.05 \text{ in.}}{25} = .002 \text{ in.} \tag{7}$$

(1), (2), and (3) are different forms of transit verniers, (4) is a sextant vernier, (5) is the form of vernier used on Boston and New York levelling-rods, and (6) and (7) are two forms of vernier used on mercurial barometers.

To measure an Angle with the Transit.

Bring the plumb-bob, hung from the transit, exactly over the point at which the angle is to be measured. Place the plate bubbles parallel to opposite levelling-screws and level, by grasping opposite levelling-screws between the thumb and forefinger of each hand and turning both thumbs in or out, as is necessary. (The bubble will move in the same direction as the left thumb in turning the levelling-screws.)

Set the zero of the vernier opposite the zero of the circle; focus the eye-piece on the cross-hairs, and the object-glass on the object to be sighted; tighten the lower clamp and bring the image of the object, defining one line of angle, in exact coincidence with the vertical cross-hair, by means of the lower tangent screws. Loosen the upper clamp; sight the telescope to the object defining the second line of the angle; tighten the upper clamp and bring the image of the object in exact coincidence with the vertical cross-hair, by means of the upper tangent screw. Read on the circle the number of degrees passed over by the index and on the vernier opposite the line in coincidence with a line on the circle the number of minutes to be added to the circle reading, to give the correct reading.

Generally there are four verniers on a transit, one on each side of the zero of each of the two opposite indices. The student, when learning to use the transit, should read that vernier which lies in the same direction from the zero set, as that in which the zero of the vernier was turned in sighting the second object.

In making a survey of a closed field with the transit, either the interior angles or the deflection angles may be measured. If the interior angles are measured, their sum should equal twice as many right angles as the field has sides, less four right angles. If the deflection angles are measured, the sum of all the right deflections should differ from the sum of all the left deflections by 360°.

In either case the bearing of each line should be measured with the needle. The bearing of each line should also be computed by assuming the computed bearing of one line (generally the first) equal to its observed bearing, and then computing the bearing of each succeeding line from the deflection angle. If the computed bearing does not agree approximately with the observed bearing, both should be remeasured to discover if any mistake has been made.

In Fig. 23, *a*, *b*, *d*, *e*, and *f* are right deflection angles; *c* and *g* are left deflection angles.

Areas may be determined from the computed bearings and the distances in the same way as with compass surveys.

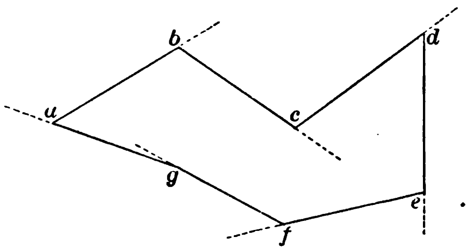

Fig. 23.

Adjustments of the Transit.

First. To make the plate bubbles parallel to the horizontal circle. Level the bubbles in the ordinary way; then revolve the instrument halfway round. If either bubble runs toward one end of its tube, turn the adjusting screw at the end of the bubble tube till the bubble moves halfway back to the centre. Repeat the operation till the bubble will remain in the centre during a whole revolution of the instrument.

Second. To adjust the line of collimation. This consists in placing the vertical cross-hair in the optical axis of the telescope, so that a straight line may be prolonged by revolving the telescope on its horizontal axis.

Sight the telescope on some point *A* 200 to 300 ft. away, and clamp the spindle; then revolve the telescope on its horizontal axis and locate a point *B* in the line of sight. Loosen the clamp and revolve the instrument on its vertical axis till *A* is sighted. Clamp the spindle and again revolve the telescope on its horizontal axis, and note whether the line of sight strikes the point *B*. If it strikes to one side, call this point *C*. To adjust, move the cross-hair ring till the line of sight strikes a point one-fourth of the distance from *C* toward *B*. Test the correctness of the adjustment by repeating. To move the cross-hair ring, loosen the capstan-headed screw on one side of the telescope tube and tighten the opposite one. Unless the telescope is "inverting," the cross-hair ring must be moved in the direction opposite to what appears to be correct.

Third. To make the axis of the telescope horizontal, so that the line of sight will move in a vertical line. Set up the instrument near some high object, as a steeple; sight the high point and clamp the spindle. Depress the telescope and locate a point, in the line of sight, nearly on a

level with the telescope. Loosen the clamp; revolve the instrument on its vertical axis and the telescope on its horizontal axis; sight the low point and clamp the spindle. Raise the telescope and note if the line of sight strikes the high point. If the line of sight is to one side, the standard on the opposite side is too high, and the axis of the telescope must be moved till the line of sight moves half-way back toward the high point first sighted. To move the axis of the telescope, turn the screw under the bearing at one end of the axis. Repeat the operation in order to see if the adjustment has been correctly made.

STADIA SURVEYING.

In addition to the centre horizontal cross-hair in the telescope, most modern engineers' transits have two other horizontal hairs, situated at equal distances above and below the centre one. These are called *stadia hairs.*

They are placed at such a distance apart that the distance intercepted by them, on a rod held vertical, is one-hundredth of the distance to the rod, from a point in front of the object-glass equal to the focal length of the telescope; or, expressed in formula,

$$d = 100\,s + f + c.$$

d = distance from the centre of the instrument.

s = space on rod intercepted by stadia hairs.

f = focal length of telescope, *i.e.* distance from the object-glass to the cross-hairs = from three-fourths to one foot in most transits.

c = distance from the object-glass to the centre of the instrument (= one-half foot, about).

If the rod is not at the same level as the transit, and is still held vertical, the horizontal distance and the difference in elevation may be found by the following formulas:

$$\text{Horizontal distance} = s\cos^2 v + (c + f)\cos v.$$

$$\text{Difference in elevation} = s\,\tfrac{1}{2}\sin 2\,v + (f + c)\sin v.$$

v = vertical angle (either elevation or depression).

The difference in elevation given, is the vertical distance between the centre of the telescope and the point on the rod intercepted by the centre horizontal cross-hair. By sighting at a point on the rod equal to the height of the telescope, the difference in elevation between the surface of the ground at the instrument and at the rod is given.

The tables computed by Mr. Arthur Winslow of the State Geological Survey of Pennsylvania, give values of $s \cos^2 v$ and $\frac{1}{2} s \sin 2 v$ for angles from 0° to 30° with $s = 1$. By use of these tables the reduction of stadia notes is made quite easy.

Example in Use of Tables.

Vertical angle $= + 4° 28'$ $(c + f) = 1.25 = c$ in table.

Rod reading $= 4.42$ ft.

Difference in elevation $= 4.42 \times 7.76 + .10 = 34.40$ ft.

Horizontal distance $= 4.42 \times 99.39 + 1.25 = 440.55$ ft.

The stadia furnishes a very rapid method of measuring distances when an error of one part in four or five hundred is admissible.

PUBLIC LAND SURVEYS.

The larger part of the land north of the Ohio River and west of the Mississippi is laid out according to law, by lines running north and south, and east and west.

The method is as follows:

A *principal meridian* is run due north. At intervals of twenty-four miles north of latitude 35° and at intervals of thirty miles south of this latitude, *standard parallels* are run due east and west. At intervals of forty-two miles, lines are run north and south, connecting standard parallels.

This divides the surface into figures shaped as shown in Fig. 24.

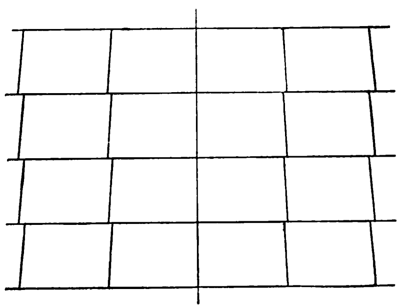

FIG. 24.

Each of these approximately rectangular plots, or *checks*, as they are called, is divided into townships which are six miles square, "as nearly as may be." The bounding lines of the townships are run as follows:

Starting at a, six miles from o (Fig. 25), run a line due north six miles to b, establishing section and quarter-section corners every one-half mile. From b run a random line toward c. c has been already established on the principal meridian. In all probability the random line will not strike c exactly, but somewhere north or south of it. In this case the true line must be run from c to b by correcting each one-half-mile point established on the random line. In the same way run from b to d due north; then a random line from d toward e; then correcting this from e to d.

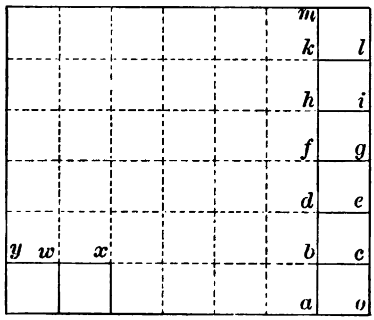

FIG. 25.

When the line between the sixth and seventh township lines is run, a random line is run both east and west from w toward x and y, and so on till the whole check is divided into townships.

Each township is subdivided into sections numbered as shown in Fig. 26.

Each section is divided into quarter-sections called respectively the N.E., N.W., S.E., and S.W. quarter-section.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 26.

The lines of townships running east and west from a principal meridian are called *townships*. The lines of townships running north and south of a standard parallel are called *ranges;* thus

N.E. $\frac{1}{4}$ sec. 18 T. 3 N. R. 5 W.

is the northeast quarter of section eighteen, in the township in the third line of townships north of a standard parallel, and the fifth range west of a principal meridian.

Burt's solar compass, shown in Fig. 27, is the instrument commonly used in running lines on government surveys.

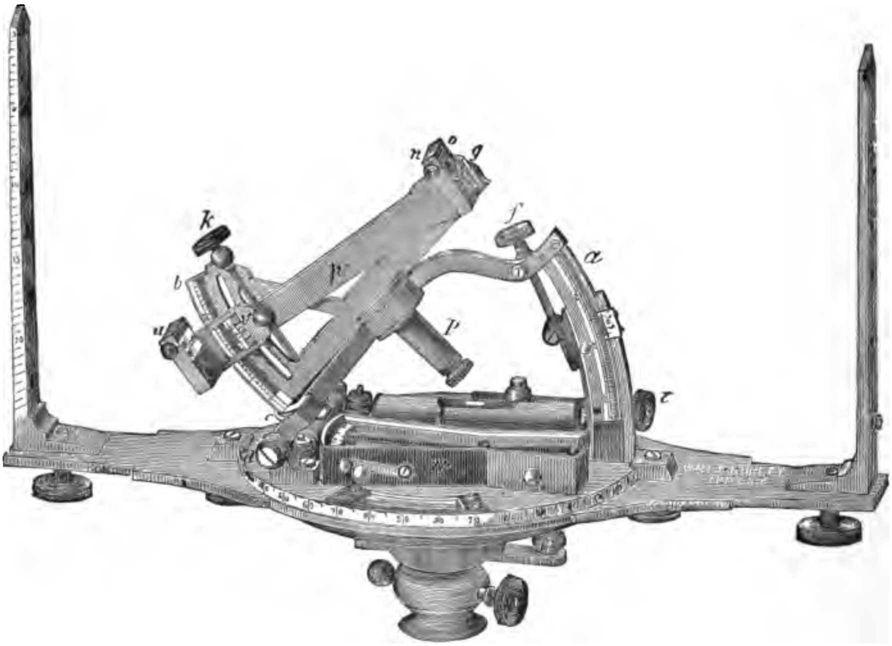

FIG. 27.

This instrument is devised to determine a true north and south line by means of an observation on the sun.

At *a* is the latitude arc, on which is laid off the latitude of the place; *b* is the declination arc, on which is laid off the declination of the sun (corrected for refraction) at the time when the observation is made.

The compass is then turned about on its vertical axis, and at the same time the arm *h* about the polar axis *p*, till an image of the sun, formed by a lens at *g*, falls between the four lines on the silvered plate at *u*. The polar axis *p* is then parallel to the axis of the earth, and the compass

sights are in a true north and south line. Do not confuse the false image, sometimes reflected from the arm *h*, with the true image formed by the sunlight passing directly from the lens to the silvered plate. The false image is much less bright, and its outline is less clearly defined than that of the real image.

To determine the Declination of the Sun at Any Time and Place.

The Nautical Almanac gives in advance the declination of the sun for apparent and for mean noon at Greenwich for each day in the year. It also gives the change in declination for one hour. By multiplying this change by the number of hours after noon and adding the result to or subtracting it from the declination at noon, according as the declination is increasing or decreasing, the declination for any time may be determined.

Having the declination at Greenwich, the time when the declination is the same at any other place is known, if the difference between local and Greenwich time is known; or, in other words, if the longitude is known. For example, the declination in New England at 7 A.M. is the same as it is at mean noon, of the same day, at Greenwich, since the longitude of New England is five hours west of Greenwich. The declination in the Middle States at 10 A.M. is the same as that at Greenwich at 4 P.M. of the same day, since their longitude is six hours west of Greenwich.

When the declination is *north*, the correction for refraction is to be *added* to the declination obtained from the Nautical Almanac, and when it is *south*, the refraction correction is to be *subtracted*, to give the angle to be laid off on the declination arc of the solar compass.[1]

Tables giving the correction for refraction for different hours of the day, for different latitudes, and for different declinations, have been computed. Those published by Messrs. W. & L. E. Gurley in their manual were prepared especially for use in connection with the solar compass.

THE LEVEL.

This instrument, shown in Fig. 28, is used to determine the difference in elevation of different stations.

It consists essentially of a telescope, attached to the under side of which is a delicate spirit level. The whole, mounted on a tripod, may be freely revolved about a vertical axis.

The instrument is levelled by bringing the bubble into the centre of the tube when successively parallel to one set of opposite levelling-screws

[1] This is true for places north of the equator; for places south of the equator the words *north* and *south* should be interchanged.

and then parallel to the other set. The levelling-rod, two forms of which are shown in Fig. 29, is used to measure the distance down from the level line of sight to any point the elevation of which is desired.

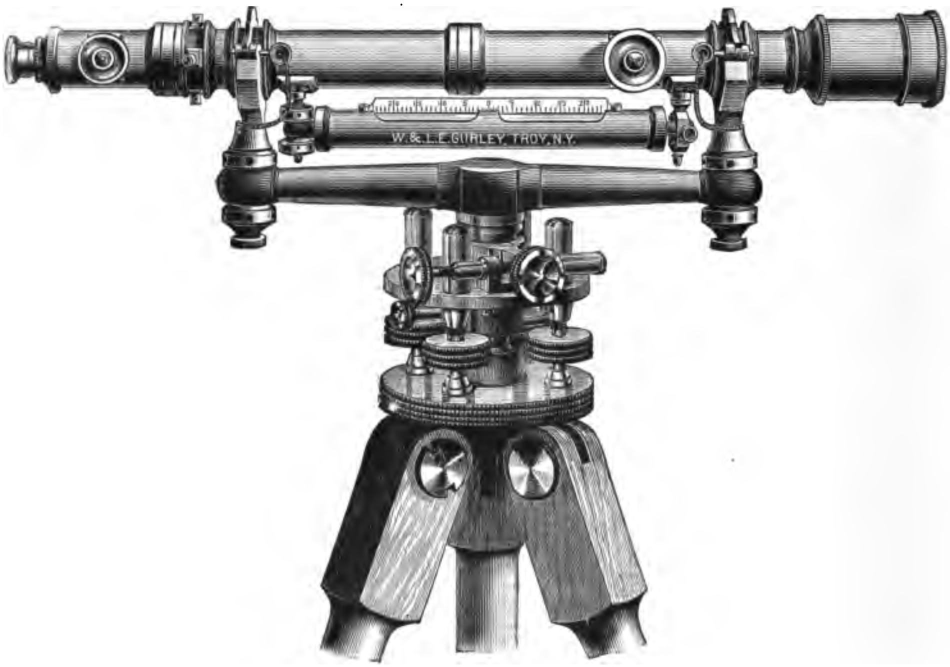

FIG. 28.

To determine the difference in elevation between two points, the instrument is set between them and levelled, and the distance of each point below the line of sight determined. The difference between these distances is the difference in elevation.

To determine each of these distances, the rodman holds the rod on one of the points, and moves the target up, or down, as directed by the leveller, till the horizontal line on the target exactly coincides with the line defined by the horizontal cross-hair in the telescope. He then records the rod reading. The rod is then taken to the second point, the telescope is revolved on its vertical axis till the rod is sighted, the target is again set, and the rod reading taken. The leveller should see to it that the bubble is in the centre of the tube at the instant of sighting, as the fact that the instrument has been once levelled will not insure its remaining so. If, for any reason, the difference in elevation of the two points cannot be determined by a single setting of the instrument, determine successively

the difference in elevation of a number of intermediate points, as shown in Fig. 30.

The algebraic sum of these differences is the difference in elevation.

Profile Levelling.

In order to make a profile of the surface of the ground along any given line, elevations are taken at equal (generally 100 ft.) intervals. The elevations are measured above an assumed horizontal plane called the *datum plane.* This plane should be below the lowest point on the surface, to avoid negative readings.

A *bench-mark* (B. M.) is some permanent mark, the elevation of which has been determined and recorded together with a description, by which it may be found at any time.

A *height of instrument* (H. of I.) is the elevation of the line of sight through the level above the datum plane. It is found by adding a back-sight to the elevation of a bench-mark or turning-point. A *back-sight* or + *sight* (B. S.) is a rod reading taken on a bench-mark or turning-point, and is used to determine the height of instrument. A *fore-sight,* or —*sight* (F. S.) is a rod reading taken on a point, the elevation of which is to be determined. This elevation is determined by subtracting the fore-sight from the height of instrument.

A *turning-point* (T. P.) is a point on which both a back-sight and a fore-sight are taken. It is used to determine a new height of instrument. A turning-point should be solid, and not one whose elevation can be changed by pressure of the rod while the sights are being taken.

To determine the elevations for a profile, stakes are first set along the line at intervals of 100 ft., and numbered 0, 1, 2, 3, etc., to the end of the line.

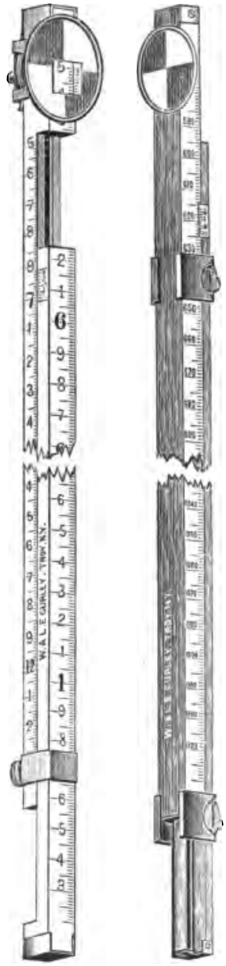

FIG. 29.

The level is set near the 0 station, and a back-sight is taken on the nearest bench-mark. Fore-sights are then taken on as many stations as can be reached from that setting of the level. A turning-point is then chosen and a fore-sight taken on it. The level is then carried forward along the line and set up again and levelled. A back-sight is then taken

on the turning-point, a new height of instrument determined, and then fore-sights are taken on as many more stations as may be sighted from this new position of the level. In this way the work is carried on till the end of the line is reached.

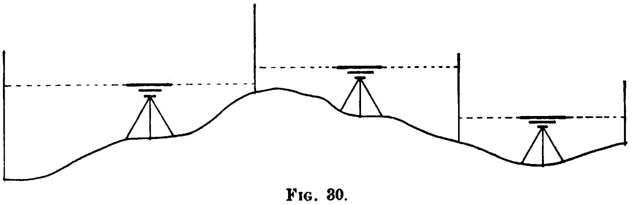

FIG. 30.

The following is a set of profile level notes:

Station.	B. S.	H. of I.	F. S.	Elevation.
B. M.	7.206	114.900		107.694
0			4.2	110.7
1			5.1	109.8
2			6.3	108.6
3			4.9	110.0
T. P.	5.182	116.210	3.872	111.028
4			4.2	112.0
+ 40			5.7	110.5
5			2.7	113.5
6			1.8	114.4
B. M.			0.987	115.223

Figure 31 shows a profile plotted from the above notes.

In this way profiles for railroad lines, water-pipe lines, highways, etc., are determined.

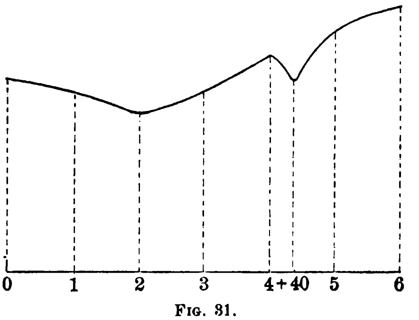

FIG. 31.

Whenever excavation or filling is to be made over considerable areas, as in excavating for reservoirs, filling marshes, etc., the amount of material

to be moved may be determined by what is known as cross-sectioning the area. This is done by running transit lines at right angles to each other to divide the field into squares, and then determining the elevation at each intersection. From these elevations, and the area of the total number of squares, the amount of material above any plane may be computed by multiplying the area of each square by the mean of the heights of the four corners above the given plane.

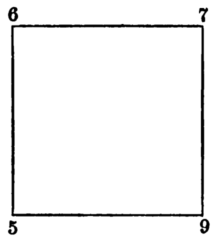

FIG. 32.

Let Fig. 32 represent a piece of land 100 ft. square, and the figures at each corner the elevations in feet; then the cubic contents above a two-foot horizontal plane are $\frac{4+5+3+7}{4 \times 27} \times 100 \times 100$ cu. yds.

When there are a large number of squares, take the sum of the heights at all of the corners common to one square, plus twice the sum of the heights at all the corners common to two squares, plus three times the sum of the heights at all the corners common to three squares, plus four times the sum of the heights at all of the corners common to four squares;

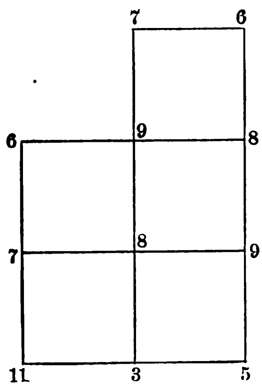

FIG. 33.

divide the result by 4, multiply by the area of one square, and the result is the contents in cubic feet. Divide by 27 to reduce to cubic yards.

Figure 33 represents a plot of land, the area of each square being 100 ft. The figures at the corners are the elevations in feet. The volume in cubic yards above a zero plane is:

$$100 \times 100 \times \frac{11+6+7+6+5+2(8+9+3+7)+3 \times 9+4 \times 8}{4 \times 27} = 13703.7 \text{ cu. yds.}$$

Adjustments of the Wye Level.

First. The adjustment of the line of collimation. This consists in placing the intersection of the cross-hairs in the centre of the wye-rings. Loosen the clips that hold the telescope in the wyes. Sight the vertical cross-hair on some well-defined line 200 to 300 ft. away, and clamp the spindle. Turn the telescope half-way round in the wyes and note if the vertical cross-hair still coincides with the line first sighted. If it does not, bring the line of sight half-way back to this position by moving the cross-hair ring as is done in adjusting the line of collimation in the transit. Adjust the horizontal cross-hair in the same way. When this adjustment is made, the intersection of the cross-hairs will remain on a point while the telescope is turned through a whole revolution in the wyes.

Second. To make the bubble parallel to the line of sight. Bring the telescope parallel to two opposite levelling-screws and clamp the spindle. Bring the bubble into the centre of the tube by turning the levelling-screws, and then turn the telescope a few degrees in the wyes. Should the bubble move toward one end of the tube, it would show that a vertical plane through the axis of the telescope is not parallel to a vertical plane through the bubble tube. To correct this, move the bubble to the centre by turning the two capstan-headed screws, giving a horizontal motion to one end of the bubble tube.

Repeat if necessary, till the bubble will remain in the centre when the telescope is revolved in the wyes five or ten degrees either side of its normal position. Next reverse the telescope in the wyes, and note if the bubble moves towards one end of the tube. If it does, that end is too high and must be lowered, or the opposite one raised, till the bubble takes a position half-way back toward the centre. This is done by turning the capstan-headed screws, giving a vertical motion to one end of the bubble tube.

Third. To make the line of sight and the bubble at right angles to the vertical axis of the instrument. Fasten the clips holding the telescope in the wyes; bring the bubble parallel to two opposite levelling-screws, and level; revolve the instrument 180° on the vertical axis, and if the bubble moves toward one end of the tube, turn the capstan-screws at the end of

the wyes till the bubble moves half-way back to the centre. Relevel, and repeat the operation to test the adjustment.

Contour lines are lines connecting points of equal elevation.

A shore line of a pond represents a contour line. If the water in the pond should rise a given amount, as 10 ft., then that shore line would represent another contour line 10 ft. higher than the first.

Contour lines furnish an easy and accurate way of representing on a map the relief of a region.

Such maps are valuable for many engineering purposes, among which are, location of routes for roads, railroads, water-pipes, aqueducts, military movements, etc.

Contour maps on a small scale are often made by interpolating the contour lines on a plot that has been cross-sectioned as shown in Fig. 34.

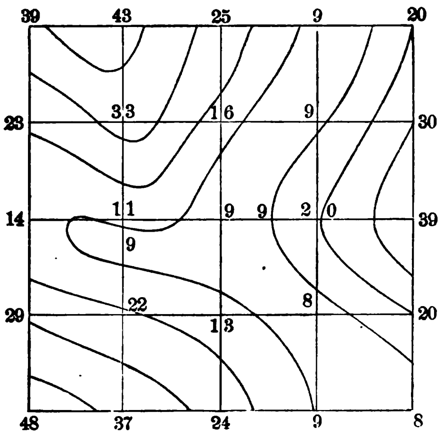

FIG. 34.

PRACTICAL ASTRONOMY AND NAVIGATION.

THE navigator, and sometimes the surveyor, needs a knowledge of astronomy sufficient to enable him to determine his position (latitude and longitude) on the surface of the earth. In the case of the navigator the error should not exceed the distance from the ship's deck to the horizon line, a distance seldom exceeding five or six miles.

The *zenith* is the point vertically above the observer, and is 180° from the direction taken by a plumb-bob.

The *nadir* is the point vertically underneath the observer, and lies in the direction taken by a plumb-bob at rest. The zenith and nadir change with every change in the observer's position.

The *plane of the horizon* is a plane passing through the observer's position, and everywhere at right angles to the direction of the zenith. Planes at right angles to the plane of the horizon, and containing the line connecting the zenith and nadir, are called *planes of altitude.*

The two *celestial poles* are the points where the axis about which the earth revolves, would, if produced, pierce the heavens. The one which is above the horizon in the northern hemisphere is called the *north celestial pole ;* the other is called the *south celestial pole.*

The two poles are independent of the observer's position.

The circle in which a plane passing through the earth's centre, and at right angles to the line connecting north and south poles, cuts the surface of the earth, is called the *terrestrial equator.* If this plane be extended, the circle in which it cuts the heavens is called the *celestial equator.*

Hour circles are circles containing the line connecting the poles and at right angles to the plane of the equator.

The plane containing the hour circle passing through the zenith intersects the horizon in its north and south points.

The *altitude* of a point is its angular distance above the horizon, measured in a vertical circle passing through the point. The zenith distance is the complement of the altitude.

The *azimuth* of a point is the angle at the zenith between the meridian and a vertical circle passing through the point. Azimuth is generally

measured from the south through the west, north, and east, from 0° to 360°.

The declination of a point is its angular distance from the equator measured on an hour circle passing through the point. Declination is considered positive when the point is north of the equator, and negative when south of it. Polar distance is the complement of the declination.

The hour angle of a point is the angle at the pole between the meridian and the hour circle passing through the point, or it may be defined as the arc of the equator intercepted by these two circles. Hour angles are usually measured from the south point of the equator from 0° to 360°, or from zero hours to twenty-four hours in the direction of the motion of the hands of a watch.

The *latitude* of a point on the earth's surface is equal to the altitude of the pole. The observer's latitude is considered positive when he is north of the equator, and negative when south of it.

Longitude is the distance east or west of any assumed meridian. The meridian passing through the Greenwich, England, observatory is the one commonly used. Longitude is measured 180° or twelve hours east and west of the assumed meridian.

The Nautical Almanac is a book, published by the government, which contains, among other data, the right ascensions and declinations of the sun, moon, planets, and certain fixed stars, the semi-diameters of the sun and moon, and the equation of time for Greenwich apparent and mean noon of each day in the year — all computed several years in advance.

The following is taken from the tables giving data in regard to the sun:

JULY, 1896, AT GREENWICH APPARENT NOON.

Day of the Week.	Day of the Month.	The Sun's Apparent Right Ascension.	The Sun's Difference for One Hour	The Sun's Apparent Declination.	The Sun's Difference for One Hour	The Sun's Semi-diameter.	Equation of Time to be added to Apparent Time.	Difference for One Hour.
		h. m. s.	s.	° ′ ″	″	′ ″	m. s.	s.
Wednesday,	1	6 43 49.95	10.332	N 23 4 19.8	−10.81	15 46.16	3 40.69	0.474
Thursday .	2	6 47 57.80	10.321	22 59 48.2	11.82	15 46.15	3 51.95	0.463
Friday . .	3	6 52 5.38	10.310	22 54 52.5	12.82	15 46.15	4 2.94	0.452
Saturday .	4	6 56 12.68	10.297	22 49 32.7	−13.82	15 46.15	4 13.64	0.439
SUNDAY.	5	7 0 19.65	10.284	22 43 49.1	14.81	15 46.16	4 24.03	0.426
Monday. .	6	7 4 26.29	10.269	22 37 41.7	15.80	15 46.17	4 34.09	0.412

The **sextant** shown in Fig. 35 is a hand instrument for measuring angles. Since it does not require a rigid support, it is particularly adapted to measuring angles from a ship at sea.

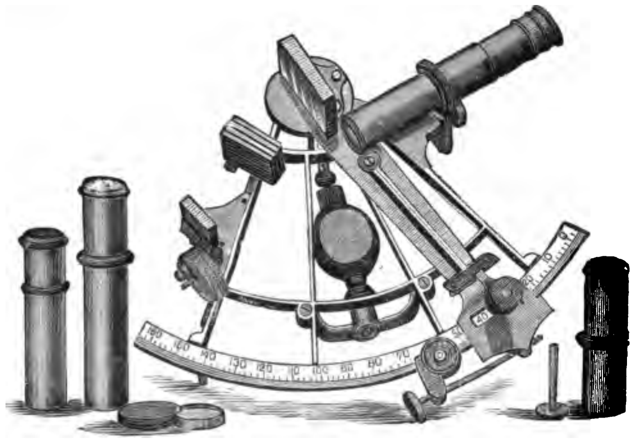

FIG. 35.

The construction of the sextant depends upon the principle of optics, that if a ray of light be twice reflected from two plane mirrors, its angular change in direction is equal to twice the angle of the mirrors.

At I (Fig. 36) is the index glass which rotates with the arm IV. This arm carries at V a vernier which moves along the graduated arc AB. At H is the horizon glass, one half of which is silvered and the other half clear.

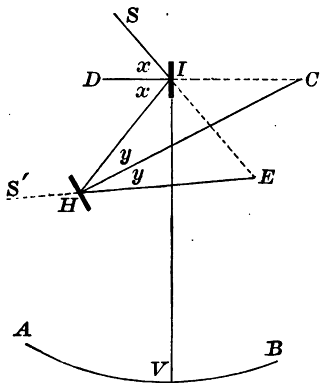

FIG. 36.

A ray of light coming from the direction S is reflected from I and H, and enters the eye at E. As the arm IV is moved along the arc, the image of S

travels across the mirror at H. When the image is seen exactly in the line $S'E$, which is the path of a ray of light from S' through the clear part of the glass at H, the angle of the mirrors is one-half the angle SES'. The arc AB is so graduated that any angle, as SES', may be read directly by the vernier at V.

To prove that $SES' =$ twice the angle between the mirrors, draw DC perpendicular to the mirror at I, and HC perpendicular to the mirror at H. Then $DCH =$ angle between the mirrors.

$$\angle SES' = 2x - 2y;$$

$$\angle DCH = x - y;$$

$$\therefore\ SES' = 2\,DCH.$$

If the two mirrors are not parallel when the vernier reads zero there is an *index error*. This error is a constant correction which must be made to all angles measured with the instrument.

To determine the index error, measure the angular diameter of the sun by bringing the discs of the two images in contact.

Call the diameter of the sun d, the sextant reading r, and the index error e.

Then $d = r + e$.

Now move the vernier till the discs are again in contact, the image that was first above being now underneath.

The zero of the vernier will now probably be back of the zero of the circle.

Call this reading $-r'$.

Then $$-d = -r' + e.$$

Hence $$e = \frac{r' - r}{2}.$$

In order to obtain the true altitude of a heavenly body, the observed altitude measured with the sextant must be corrected for dip, parallax, refraction, and, in the case of the sun or moon, for semi-diameter.

Dip.

Altitudes at sea are measured from the visible horizon, which is below the true horizon an amount depending upon the height of the observer above the surface of the water.

Let OH (Fig. 37) be the true horizon, and OH' the visible horizon. $OA = a =$ the observer's height above the water. $AC = BC = R =$ mean radius of the earth $= 20{,}900{,}000$ ft. approximately. $HOH' = ACB = D =$

the dip of the horizon. $\operatorname{Tan} D = \frac{OB}{BC} = \frac{\sqrt{2\,Ra + a^2}}{R} = \sqrt{\frac{2\,a}{R} + \frac{a^2}{R^2}}$. $\frac{a^2}{R^2}$ is very small and may be neglected. The angle D is also small, and may be taken as $D \tan 1'$. Substituting, we have

$$D \text{ in minutes} = \frac{1}{\tan 1'} \times \frac{1}{3233}\sqrt{a}$$
$$= .94\sqrt{a},$$

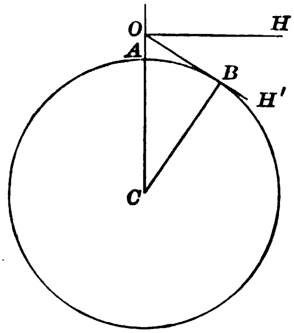

FIG. 37.

or the dip in minutes is approximately the square root of the observer's height in feet above the water.

The correction for dip must be subtracted from the observed altitude to give the true altitude.

Parallax.

The difference in direction of a heavenly body as seen by an observer on the earth's surface and as it would be seen from the earth's centre is called *parallax.*

The magnitude of the parallax depends upon the altitude of the heavenly body and the ratio of the earth's radius to the distance of the heavenly body. The horizontal parallax is the parallax when the heavenly body is in the horizon. It may be found as follows:

FIG. 38.

Let P (Fig. 38) = horizontal parallax, AC = the earth's radius, BC = the distance of the heavenly body from the earth's centre.

$$\operatorname{Sin} P = \frac{AC}{BC}.$$

The parallax of a heavenly body in any other position may be found as follows: Let ZAB (Fig. 39) be the observed zenith distance. $ABC = p$ = parallax;

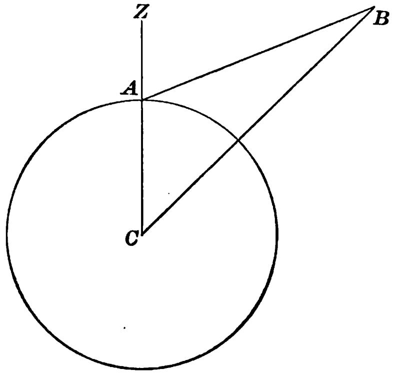

FIG. 39.

then
$$BC : AC :: \sin BAZ : \sin ABC,$$

$$\sin ABC = \frac{\sin BAZ \times AC}{BC}.$$

$\frac{AC}{BC}$ = horizontal parallax. Substituting this, we have

$$\sin p = \sin P \sin BAZ.$$

Both p and P are small; therefore we may without excessive error assume

$$p = P \sin BAZ,$$

or the parallax in any position is equal to the horizontal parallax into the sine of the observed zenith distance.

The correction for parallax must be added to the observed altitude to give the true altitude. The parallax of the sun is small, never exceeding 9″, and in this work is neglected.

Refraction.

A ray of light coming obliquely through the atmosphere is bent out of a straight line so that the observer sees a heavenly body *above* its true position.

Let SO (Fig. 40) be the direction of the ray before refraction, and OS'' that of the refracted ray; then, from the laws of refraction, OS, OZ, OS'' lie in the same plane, and $\frac{\sin ZOS}{\sin Z'OS''}$ is constant for the same media, whatever the angle SOZ. Call this ratio i; then $\sin(x+y) = i \sin x$,

$$\sin x \cos y + \cos x \sin y = i \sin x.$$

The angle of refraction y is small, and $\sin y = y$ approximately, and $\cos y = 1$ approximately. Substituting these values,

$$\sin x + y\cos x = i\sin x, \quad y = (i-1)\tan x.$$

Let Y equal the refraction when $ZOS = 45°$; then $\tan x = 1$ and $Y = i - 1$. Substituting these values we have,

$$y = Y\tan x;$$

or the refraction for any zenith distance is equal to the product of refraction at 45° into the tangent of the observed zenith distance.

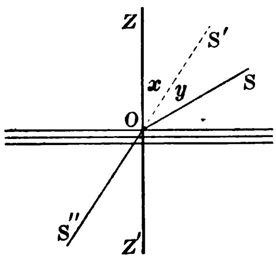

FIG. 40.

With the barometer at 30 in. and the thermometer at 50° F., the refraction at 45° is 58.2″.

For all altitudes of 20° or over measured with a sextant or engineer's transit it is permissible to take the *refraction correction in minutes equal to the natural tangent of the observed zenith distance.* For altitudes less than 20° the correction obtained in this way will be too large.

If the altitude of a heavenly body is measured with a sextant, the true altitude = observed altitude − dip + parallax − refraction.

If the transit is used, there is no correction for dip.

When the body observed is the sun or moon, the altitude of one limb is measured, and the angular semi-diameter must be added or subtracted according as the lower or the upper limb is observed.

To find Latitude.

First. By observation on a circumpolar star at upper or lower culmination.

Let P (Fig. 41) represent the pole, S' a circumpolar star at upper and S a circumpolar star at lower culmination.

Let $h' = S'H' =$ the true altitude of S', and h the true altitude of S. Let $d = QS' =$ the declination; then latitude $= h + (90° - d)$ for lower culmination, and latitude $= h' - (90° - d)$ for upper culmination.

The latitude may also be found by observing, when possible, the altitude of a circumpolar star at both upper and lower culmination, and taking one-half the sum.

Double latitude $= h + (90° - d) + h' - (90° - d)$, or latitude $= \frac{1}{2}(h + h')$. In this case it is not necessary to know the declination.

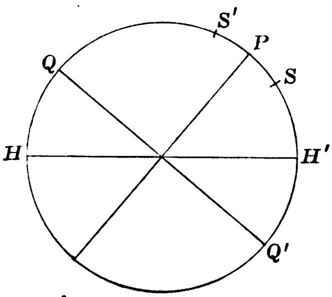

FIG. 41.

For the greater part of the year this observation cannot be taken, because one of the culminations occurs during daylight.

Example. — The altitude of Polaris at upper culmination, as observed with an engineer's transit, was 45° 28'. Declination = 88° 45' 15"; what was the latitude?

$$\begin{array}{ll}
45° \, 28' & = \text{obs. alt.} \\
\underline{\quad\; 01'} & = \text{refr. cor.} \\
45° \, 27' & = \text{true altitude.} \\
\underline{88° \, 45' \, 15''} & \\
134° \, 12' \, 15'' & \\
\underline{90°} & \\
44° \, 12' \, 15'' \text{ N.} & = \text{latitude.}
\end{array}$$

Second. By observing the meridian altitude of any star.

In the northern hemisphere the meridian altitude of any star that culminates south of the zenith, minus its declination, is equal to the altitude of the equator. This is the complement of the latitude. If the star culminates north of the zenith, $(180° - h)$ must be substituted for the altitude.

Example. — The observed meridian altitude of the sun's lower limb, as measured from the visible horizon at sea, was 33° 22' 30"; sun's semi-diameter, 16'; height of eye above surface of water, 16 ft.; sun's declination south, 13° 03'. Required latitude.

$$\begin{array}{rl}
33^\circ\, 22'\, 30'' & \text{obs. alt.} \\
\underline{1'\, 30''} & \text{refr. cor.} \\
33^\circ\, 21' & \\
\underline{04'} & \text{cor. for dip.} \\
33^\circ\, 17' & \\
\underline{16'} & \text{sun's diam.} \\
33^\circ\, 33' & \text{true altitude of sun's centre.} \\
-\,\underline{13^\circ\, 03'} & \text{sun's declination.} \\
46^\circ\, 36' & = \text{co. latitude.} \\
43^\circ\, 24'\,\text{N.} & = \text{latitude.}
\end{array}$$

Third. By the altitude of the sun or a star in any position, the time being known.

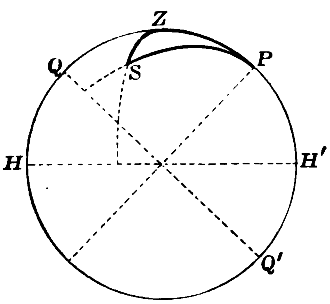

FIG. 42.

This requires the solution of a spherical triangle. Let S (Fig. 42) represent the position of the sun or a star whose altitude has been measured. In the triangle ZSP,

$$ZS = 90^\circ - \text{altitude},$$
$$SP = 90^\circ - \text{declination},$$
$$ZP = 90^\circ - \text{latitude},$$

and the angle $ZPS = 360^\circ -$ time expressed in degrees. Therefore in the triangle ZSP we have two sides, and the angle opposite one of them given, to find the third side, which is the complement of the latitude required.

Time.

Apparent time is the time of the true sun.

An apparent solar day is the interval of time between two successive transits of the sun over the same meridian.

Because the motion of the earth about the sun is not uniform, and also because it moves in the plane of the ecliptic instead of that of the equator, solar days are not of the same length, the greatest variation being about sixteen minutes.

To obviate this difficulty, an imaginary sun is supposed to revolve in the equator, about the earth, in the same time that the real sun appears to revolve in the ecliptic: this latter gives uniform motion.

A mean solar day is the interval of time between two successive transits of this imaginary sun over the same meridian. The equation of time is the time that has to be added algebraically to apparent time to give mean time.

The civil day begins at midnight, and is divided into two parts of twelve hours each. The astronomical day begins at noon of the civil day, and is divided into twenty-four hours.

To change from civil time to astronomical time: If it is A.M. civil time, subtract one day and add twelve hours; if it is P.M., omit the P.M.

To change apparent time to mean solar time: Add algebraically the equation of time to apparent time, and the result is mean solar time.

To determine local time by observing the altitude of the sun in the morning or afternoon, the latitude being known: In Fig. 42, let S represent the sun, the altitude of which has been measured, Z the zenith, and P the pole. In the triangle ZSP, $SZ = 90° -$ altitude, $PS = 90° -$ declination, and $PZ = 90° -$ latitude.

Let $h =$ altitude, $l =$ latitude, $d =$ declination, and $z =$ zenith distance;

$$\text{then } \sin \tfrac{1}{2}\, SPZ = \sqrt{\frac{\sin \frac{1}{2}[z + (l - d)] \sin \frac{1}{2}[z - (l - d)]}{\cos l \cos d}}.$$

The angle SPZ is the hour angle if the altitude is measured in the afternoon. If the altitude is measured in the morning, the hour angle is 360° minus the angle obtained by the solution of the triangle. The hour angle changed to hours, minutes, and seconds is local apparent time, which may be changed to mean local time by adding the equation of time.

Example. — Aug. 11, 1894, A.M. In latitude 42° 30′ N. the observed altitude of the sun's lower limb was 38° 19′; height of eye, 25 ft.; sun's declination, 15° 12′ N.; semi-diameter, 16′; equation of time, $+ 5^{m.} 01^{s.}$. Required local time.

	$42° 30' = l$	colog cos =	0.1324
	$15° 12' = d$	colog cos =	0.0155
$38° 19'$	$27° 18' = (l - d)$		
$5'$ dip.	$51° 32' = z$		
$1'$ refr. cor.	$2)\overline{78° 50'}$		
$38° 13'$	$39° 25' = \frac{1}{2}[z + (l - d)]$	log sin =	9.8027
$16'$ semi-diam.	$2)\overline{24° 14'}$		
$38° 28' = h.$	$12° 07' = \frac{1}{2}[z - (l - d)]$	log sin =	9.3220
			$2)\overline{9.2726}$
		log sin $\frac{1}{2}$ SPZ =	9.6363
		$\frac{1}{2}$ SPZ =	$25° 38'.9$
		SPZ =	$51° 17'.8$.

$360° - SPZ = 308° 42'.2 = 20$ h. 34 m. 49 s.

$= 8^{h.} 34^{m.} 49^{s.}$ A.M. apparent time
$\quad 5 \quad 01$ eq. of t.

$8^{h.} 39^{m.} 50^{s.}$ A.M. mean local time.
Ans.

Aug. 6, 1894, P.M. In latitude 42° 30′ N. the observed altitude of the sun's upper limb was 32° 55′; height of eye, 16 ft.; sun's declination, 16° 32′ N.; semi-diameter, 16′; equation of time, $+ 5^{m.} 39^{s.}$. Required mean local time. $4^{h.} 08^{m.} 27^{s.}$. *Ans.*

To establish a Meridian Line by an Observation on the North Star (Polaris) at Elongation.

Polaris appears to revolve about the pole in a small circle in a little less than twenty-four hours. When at its extreme east or west point, it is said to be at elongation. Fifteen or twenty minutes before the star gets to its elongation, set up the transit at one end of the line to be established and sight on the star. The star will move toward the east if the elongation is east, and toward the west if the elongation is west. When the star reaches its greatest elongation, it will move vertically along the cross-hair for some time, and then leave it in a direction opposite to its former motion. As long as the star moves toward the point of its elongation, the cross-hair must be kept sighted on it, by turning the tangent screw. When the point of elongation is reached, the line defined by the transit must be established on the ground, by driving a stake or spike in this line at some point 100 to 200 ft. from the transit.

To establish a meridian, lay off from this line the azimuth of Polaris, to the *right* if west elongation was observed, and to the *left* if east elongation was observed, the transit being set over the south end of the line.

To find Azimuth.

$$\sin \text{azimuth} = \frac{\sin \text{polar distance}}{\cos \text{latitude}}.$$

It is necessary to luminate the cross-hairs when observing at night. This is done by means of a lantern, and a reflector made to fit over the object-glass end of the telescope.

A temporary reflector may be made by fastening over the object-glass end of the telescope a piece of oiled paper, leaving a hole one-fourth to one-half inch in diameter over the centre of the object-glass.

The times at which the elongations of Polaris occur are given in Bulletin No. 14, published by the United States Coast and Geodetic Survey, at Washington.

NAVIGATION.

There are two ways by which the navigator determines the position of his vessel at sea: by *dead reckoning* and by *observation*.

In navigation by dead reckoning the courses and distances run are measured by the compass and log.

In navigation by observation the position is determined by observing the altitude of the sun or some other heavenly body.

The mariner's compass, shown in Fig. 43, consists of a circular card divided into thirty-two equal parts called points. Attached to the under

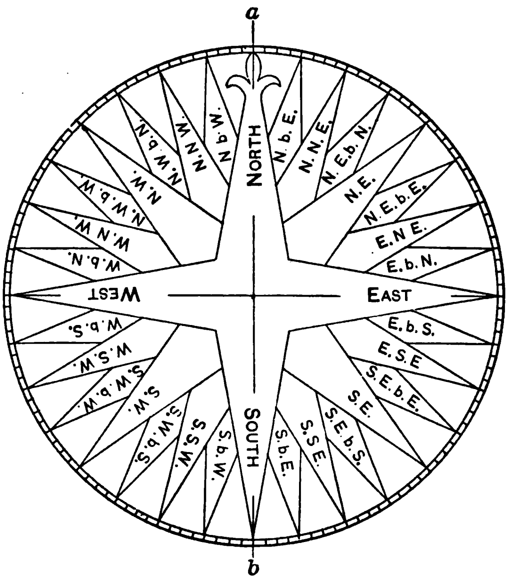

FIG. 43.

side of the card, in the direction of its north and south line, is a magnetic needle, supported at its centre on a pivot. Both swing in gimbals in a box, so that the card remains horizontal whatever the inclination of the ship. On the inside of the box are two points, *a* and *b*, the line connecting which is parallel to the ship's keel; so that if *a* is toward the bow, the point on the card opposite it will give the ship's course.

Each of the thirty-two points are subdivided into quarter-points. Naming the points in order from the north through the east, south, and west, is called *boxing the compass*.

The compass gives the magnetic course. In order to determine the true course, the magnetic course must be corrected for *variation* and *deviation*.

Variation of the mariner's compass is the angle that the needle makes with the true meridian. It is the same as *declination* of the surveyor's compass.

Deviation is the change in the direction of the pointing of the needle caused by the iron and steel used in constructing the ship. Deviation changes with the direction in which the ship is headed. It is generally counteracted as much as possible by placing permanent magnets near the compass.

Leeway is the angle which a vessel's course makes with her keel. It is caused by the wind sliding the vessel to leeward.

Although leeway is not a compass error, the effect is the same as if it were, and the compass course must be corrected for it, to give the true course of the ship.

When the variation is $\substack{\text{west}\\\text{east}}$, the true course is to the $\substack{\text{left}\\\text{right}}$ of the compass course. The same rule applies in correcting for deviation.

The correction for leeway on the $\substack{\text{starboard}\\\text{port}}$ tack is the same as for $\substack{\text{west}\\\text{east}}$ variation.

A ship is on the $\substack{\text{starboard}\\\text{port}}$ tack when the wind blows on the $\substack{\text{right}\\\text{left}}$ side of the ship.

Compass Course.	Variation.	Deviation.	Leeway.	True Course.
W.S.W.	¼ pt.W.	½ pt.W.	¼ pt. port.	S.W. by W. ½ W.
N. by E.	1 pt.E.	¼ pt.W.	½ pt.starb.	N. by E. ¼ E.
N.W. ¾ N.	2½ pts.W.	¼ pt.E.	½ pt. starb.	W.N.W.

The common log consists of a reel, line, and chip. The chip (Fig. 44) is a flat piece of wood shaped like a sector of a circle, weighted with lead on its curved edge to make it stand upright in the water. A jerk on the line

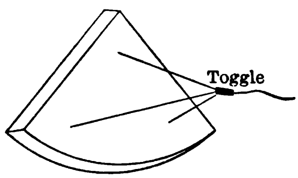

FIG. 44.

will free the two lower cords at the toggle, causing the chip to lie flat, so that it may be easily hauled aboard. With this log a sand-glass is used to measure the time. The line is graduated to the same part of a knot as

the part of an hour measured by the sand-glass. This makes the knot 47′ 4″ long when a twenty-eight-second glass is used. Vessels going at high speed use a fourteen-second glass.

The patent log is an instrument fitted with curved blades, shaped somewhat like a ship's propeller, which cause it to revolve when drawn through the water. The number of revolutions are registered on a dial, which is graduated so as to record the revolutions per minute in knots per hour.

The log gives the speed of the vessel in relation to the water. If there are currents, then the speed indicated by the log must be corrected; *i.e.* if sailing against a current, deduct its speed from that given by the log; if sailing with the current, add its speed.

In a head wind the log is apt to overrate, and *vice versa.*

A knot is equal to one minute (1′) of latitude, or to one minute of longitude at the equator. Thus if a vessel sails due N. 60 knots from latitude 40° S., she will arrive in latitude 39° S. Since all meridians converge, a knot is equal to a minute of longitude only at the equator.

The difference in longitude in minutes of angle may, however, be found when sailing due E. or W., by dividing the distance in knots by the cosine of the latitude.

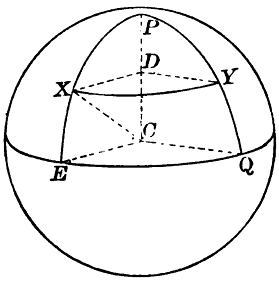

FIG. 45.

Let P (Fig. 45) be the pole, EQ an arc of the equator, XY a parallel in latitude EX; then $\angle ECX = \angle CXD =$ latitude $= L$. Cos $CXD = \frac{DX}{CX}$; or $\cos L = \frac{DX}{R}$. $R =$ radius.

Since similar arcs are to each other as their radii,

$$EQ : XY :: R : R \cos L.$$

Hence $EQ = \frac{XY}{\cos L}$. But EQ is the difference in longitude, and XY is the distance sailed E. or W.

A ship sails 55 knots due E. from latitude 40, longitude 65 W. Find her longitude.

$$\frac{55}{.766} = 72' = 1^\circ\ 12'. \quad 65^\circ - 1^\circ\ 12' = 63^\circ\ 48'\ \text{W.} \qquad \textit{Ans.}$$

Middle Latitude Sailing.

If a vessel sails on any course other than a meridian or parallel the new position may be found by computing the latitude and departure in the same way that they are computed in compass surveys. The latitude gives the distance sailed N. or S. of the starting-point, and the departure the distance E. or W. In like manner, if a vessel sails on several courses, the algebraic sum of all the latitudes gives the distance sailed N. or S., and the algebraic sum of all the departures gives the distance sailed E. or W. If the distances are in knots, the algebraic sum of the latitudes is the difference of latitude in minutes of angle. To determine the difference in longitude, the algebraic sum of the departures must be divided by the cosine of the latitude; but the course sailed was not on any one parallel of latitude. We may, however, determine the difference in longitude by dividing the departure by the cosine of the mean of the latitudes of the two ends of the course; or, in other words, by dividing the departure by the cosine of the middle latitude. This will give the difference in longitude somewhat less than the true difference. In low latitudes, where the difference in latitude of the two ends of the course is not great, the error is small.

A vessel sails from lat. 40° 28′ N., long. 74° 01′ W., as follows:

E. by N.	10 knots,
E.S.E.	20 "
S.E. by E.	26 "
S.S.W.	15 "

Require her lat. and long.

Course.	Angle.	Distance. Knots.	Latitude.		Departure.	
			N.	S.	E.	W.
E. by N.	78° 45′	10	1.95	——	9.81	——
E.S.E.	67° 30′	20	——	14.44	21.62	——
S.E. by E.	56° 15′	26	——	7.65	18.48	——
S.S.W.	22° 30′	15	——	13.86	——	5.74
			1.95	35.95	49.91	5.74
				1.95	5.74	
		Diff. in lat. in min. =		34.00	44.17	

$$40° 28' - 34' = 39° 54' \text{ N.} = \text{latitude.}$$

$$40° 11' = \text{middle latitude.} \quad \cos = .7640.$$

$$\frac{44.17}{.7640} = 58' = \text{diff. in long. in minutes.}$$

$$74° 01' - 58' = 73° 03' \text{ W.} = \text{longitude.}$$

A chronometer is a well-made and nicely adjusted timepiece swung in gimbals and set to Greenwich time.

The *rate* of a chronometer is the amount of time that it gains or loses in a day.

The difference between the chronometer time and Greenwich time and the rate of the chronometer are determined when in port. Knowing these, the Greenwich time may be determined, when desired, during the voyage.

On June 12 a chronometer compared with Greenwich time was $2^{\text{m.}}\ 14^{\text{s.}}$ fast; its rate was $1.1^{\text{s.}}$ losing. What was the Greenwich time corresponding to $4^{\text{h.}}\ 40^{\text{m.}}\ 17^{\text{s.}}$ June 20?

$$8 \times 1.1^{\text{s.}} = 9^{\text{s.}} -. \quad 2^{\text{m.}}\ 14^{\text{s.}} - 9^{\text{s.}} = 2^{\text{m.}}\ 5^{\text{s.}},$$

$$4^{\text{h.}}\ 40^{\text{m.}}\ 17^{\text{s.}} - 2^{\text{m.}}\ 5^{\text{s.}} = 4^{\text{h.}}\ 38^{\text{m.}}\ 12^{\text{s.}} \text{ Greenwich time.}$$

Since position determined by dead reckoning is always liable to error, it is customary to determine the position of a ship by observation, daily if possible.

In determining the position of a ship by observation, the latitude is found by measuring the meridian altitude of the sun, as explained in the chapter on astronomy.

The longitude is determined by measuring the altitude of the sun when it bears nearly E. or W., and computing local time. The ship's chronometer gives Greenwich time. The difference between these is the difference in longitude. The longitude is W. if the Greenwich time is later than the local time, and E. if it is earlier.

The change in latitude during the interval of time between the observation for latitude and that for time is found by dead reckoning. It is necessary to compute this in order to get the latitude to be used in solving for time.

On Feb. 28, 1896, at $21^{\text{h.}}\ 40^{\text{m.}}$ approximate Greenwich time, the observed meridian altitude of the sun's lower limb was $33° 36' 30''$, the sun bearing south. Height of eye above the sea 20 ft.; sun's declination $7° 41' 30''$ S.; semi-diameter $16'$. Required the latitude. $48° 32'$ N. *Ans.*

On May 20, 1896, at $22^{\text{h.}}\ 20^{\text{m}}$ approximate Greenwich time, the observed meridian altitude of the sun's lower limb was $68° 12' 30''$, the

sun bearing south. Height of eye 25 ft.; sun's declination 20° 20′ N.; semi-diameter 16′. Required the latitude. 41° 57′ N. *Ans.*

On Sept. 29, 1896, at $0^{h.}$ $20^{m.}$ approximate Greenwich time, the observed meridian altitude of the sun's upper limb was 63° 25′, the sun bearing north. Height of eye 20 ft.; sun's declination 2° 43′ S.; semi-diameter 16′. Required the latitude. 29° 39′ S. *Ans.*

On March 11, 1896, A.M., in latitude 47° 28′ N., the observed altitude of the sun's lower limb was 26° 25′. The corresponding Greenwich time was $23^{h.}$ $31^{m.}$ $14^{s.}$. Height of eye 25 ft.; sun's declination 3° 24′ S.; semi-diameter 16′; equation of time $+ 9^{m.}$ $59^{s.}$. Required the longitude. $2^{h.}$ $14^{m.}$ $30^{s.}$ W. *Ans.*

On Sept. 6, 1896, in latitude 36° 18′ N., the observed altitude of the sun's lower limb was 33° 14′. The corresponding Greenwich time was $17^{h.}$ $20^{m.}$ $41^{s.}$. Height of eye 20 ft.; sun's declination 6° 16′ N.; semi-diameter 16′; equation of time $- 1^{m.}$ $51^{s.}$. Required the longitude. $10^{h.}$ $08^{m.}$ $08^{s.}$ E. *Ans.*

STADIA TABLES.[1]

M.	0°		1°		2°		3°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
$c = 0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$c = 1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$c = 1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

[1] These tables were computed by Mr. Arthur Winslow of the State Geological Survey of Pennsylvania.

M.	4°		5°		6°		7°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$c = 0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$c = 1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$c = 1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

M.	8°		9°		10°		11°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0'	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$c = 0.75$	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
$c = 1.00$	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$c = 1.25$	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

M.	12°		13°		14°		15°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	25.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c = 0.75$	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$c = 1.00$	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$c = 1.25$	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

M.	16°		17°		18°		19°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.32	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
$c = 0.75$	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$c = 1.00$	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$c = 1.25$	1.20	0.36	1.19	0.38	1.19	0.40	1.18	0.42

M.	20°		21°		22°		23°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87 66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85 19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c = 0.75$	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$c = 1.00$	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$c = 1.25$	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

M.	24°		25°		26°		27°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$c = 0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$c = 1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$c = 1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

M.	28°		29°		30°	
	hor. dist.	diff. elev.	hor. dist.	diff. elev.	hor. dist.	diff. elev.
0′	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
$c = 0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$c = 1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$c = 1.25$	1.10	0.60	1.09	0.62	1.08	0.64

A TABLE OF MEAN REFRACTIONS IN DECLINATION.

To apply on the declination arc of Solar Attachment of either Compasses or Transits.

Computed by Edward W. Arms, C. E., for W. & L. E. Gurley, Troy, N. Y.

HOUR ANGLE.	DECLINATIONS.								
	For Latitude 15°.								
	+20°	+15°	+10°	+5°	0°	--5°	−10°	−15°	−20°
0 h.	−05″	0″	+05″	10″	15″	21″	27″	33″	40″
2	−03	+02	07	12	18	23	29	36	43
3	+01	05	11	16	22	28	34	41	49
4	08	12	19	24	30	37	44	53	1′04
5	29	34	41	49	59	1′10	1′24	1′43	2 08
	For Latitude 17° 30′.								
0 h.	−02″	+02″	08″	13″	18″	24″	30″	36″	44″
2	0	05	10	15	21	27	33	40	48
3	+02	10	15	21	27	33	40	48	57
4	13	18	28	29	35	43	51	1′01	1′13
5	34	41	49	58	1′10	1′23	1′41	2 06	2 42
	For Latitude 20°.								
0 h.	0″	05″	10″	15″	21″	27″	33″	40″	48″
2	03	07	13	18	24	30	36	44	52
3	06	13	18	24	30	36	44	52	1′02
4	17	22	28	35	42	50	1′00	1′11	1 26
5	39	47	57	1′07	1′20	1′37	2 00	2 32	3 25
	For Latitude 22° 30′.								
0 h.	02″	08″	13″	18″	24″	30″	36″	44″	52″
2	06	11	15	21	27	33	40	48	57
3	11	15	21	27	33	40	48	57	1′08
4	20	26	32	39	46	56	1′07	1′19	1 37
5	45	53	1′03	1′16	1′31	1′52	2 21	3 07	4 28
	For Latitude 25°.								
0 h.	05″	10″	15″	21″	27″	33″	40″	48″	57″
2	08	14	19	25	31	38	46	54	1′05
3	12	18	24	30	37	44	53	1′04	1 18
4	23	29	35	45	58	1′03	1′16	1 31	1 52
5	49	59	1′10	1′24	1′52	2 07	2 44	3 46	5 43

HOUR ANGLE.	DECLINATIONS.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
	For Latitude 27° 30′.								
0 h.	08″	13″	18″	24″	30″	36″	44″	52″	1′02″
2	11	16	22	28	34	41	49	1′00	1 10
3	17	22	28	35	42	50	1′00	1 11	1 26
4	28	35	42	50	1′00	1′11	1 26	1 43	2 09
5	54	1′05	1′18	1′34	1 54	2 24	3 11	4 38	8 15
	For Latitude 30°.								
0 h.	10″	15″	21″	27″	33″	40″	48″	57″	1′08″
2	14	19	25	31	38	46	54	1′05	1 18
3	20	26	32	39	47	55	1′06	1 19	1 36
4	32	39	46	52	1′06	1′19	1 35	1 57	2 29
5	1′00	1′10	1′24	1′52	2 07	2 44	3 46	5 43	13 06
	For Latitude 32° 30′.								
0 h.	13″	18″	24″	30″	36″	44″	52″	1′02″	1′14″
2	17	22	28	35	42	50	1′00	1 11	1 26
3	23	29	35	43	51	1′01	1 13	1 28	1 47
4	35	43	51	1′01	1′13	1 27	1 46	2 13	2 54
5	1′08	1′15	1′31	1 53	2 20	3 05	4 25	7 36	
	For Latitude 35°.								
0 h.	15″	21″	27″	33″	40″	48″	57″	1′08″	1′21″
2	20	25	32	38	46	55	1′05	1 18	1 35
3	26	33	39	47	56	1′07	1 21	1 38	2 00
4	39	47	56	1′07	1′20	1 36	1 59	2 32	3 25
5	1′07	1′20	1′38	2 00	2 34	3 29	5 14	10 16	
	For Latitude 37° 30′.								
0 h.	18″	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″
2	22	28	35	42	50	1′00	1 12	1 26	1 45
3	29	36	43	52	1′02	1 14	1 29	1 49	2 16
4	43	51	1′01	1′13	1 27	1 49	2 14	2 54	4 05
5	1′11	1′26	1 54	2 10	2 49	3 55	6 15	14 58	
	For Latitude 40°.								
0 h.	21″	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″
2	25	32	39	46	52	1′06	1 19	1 35	1 57
3	33	40	48	57	1′08	1 21	1 38	2 02	2 36
4	47	55	1′06	1′19	1 36	1 58	2 30	3 21	4 59
5	1′15	1′31	1 51	2 20	3 05	4 25	7 34	25 18	
	For Latitude 42° 30′.								
0 h.	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″
2	28	35	39	50	1′00	1 12	1 26	1 45	2 11
3	36	43	52	1′02	1 13	1 29	1 49	2 17	2 59
4	50	1′00	1′11	1 26	1 44	2 10	2 49	3 55	6 16
5	1′16	1 36	1 58	2 30	3 22	5 00	9 24		

HOUR ANGLE.	DECLINATIONS.								
	For Latitude 45°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h.	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″
2	32	39	46	52	1′06	1 19	1 35	1 57	2 29
3	40	47	56	1′07	1 21	1 38	2 00	2 34	3 29
4	54	1′04	1′16	1 33	1 54	2 24	3 11	4 38	8 15
5	1′23	1 41	2 05	2 41	3 40	5 40	12 02		
	For Latitude 47° 30′.								
0 h.	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″
2	35	42	50	1′00	1 12	1 26	1 45	2 01	2 51
3	43	51	1′01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1′09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1′27	1 46	2 12	2 52	4 01	6 30	16 19		
	For Latitude 50°.								
0 h.	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″
2	38	46	55	1′06	1 18	1 35	1 57	2 28	3 19
3	47	56	1′06	1 19	1 36	2 29	2 31	3 23	5 02
4	1′02	1′14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		
	For Latitude 52° 30′.								
0 h.	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″	3′05″
2	43	50	59	1 11	1 26	1 42	2 23	2 49	3 55
3	50	1′00	1′11	1 26	1 45	2 11	2 51	2 58	6 22
4	1′05	1 18	1 35	2 10	2 28	3 19	4 53	3 42	
5	1 34	1 56	2 27	3 16	4 47	8 52			
	For Latitude 55°.								
0 h.	40″	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″	3′33″
2	46	55	1′05	1 18	1 34	1 56	2 30	3 15	4 47
3	55	1′06	1 19	1 35	1 58	2 30	3 21	4 58	9 19
4	1′10	1 23	1 42	2 06	2 43	3 44	5 49	12 41	
5	1 37	2 01	2 34	3 28	5 15	10 18			
	For Latitude 57° 30′.								
0 h.	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″	3′05″	4′37″
2	50	59	1 11	1 25	1 43	2 09	2 47	3 51	6 04
3	58	1′10	1 24	1 42	2 07	2 43	3 45	5 50	12 47
4	1′11	1 25	1 43	2 10	2 50	3 55	6 14	14 49	
5	1 41	2 06	2 42	3 42	5 46	12 26			
	For Latitude 60°.								
0 h.	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″	3′33″	5′23″
2	54	1′04	1 17	1 33	1 54	2 24	3 12	4 38	8 15
3	1′08	1 15	1 30	1 51	2 20	3 04	4 24	7 31	24 44
4	1 18	1 34	1 56	2 28	3 18	4 50	8 53		
5	1 45	2 11	2 50	3 57	6 21	15 32			

182

184

$I = \frac{E}{R}$ $R = \frac{E}{I}$

$\frac{\text{Coul}}{\text{U}}$ $\frac{\text{ampere}}{\text{U}}$

E

$I = \frac{E}{R}$ $R = \frac{E}{I}$

$\frac{\text{Coul}}{U}$ $\frac{\text{ampere}}{U}$

E

503

$I = \frac{E}{R}$ $R = \frac{E}{I}$

$\frac{\text{Coul}}{\text{w}}$ $\frac{\text{ampere}}{\text{w}}$

E

$I = \frac{E}{R}$ $R = \frac{E}{I}$

$\frac{\text{Coul}}{U}$ $\frac{\text{ampere}}{U}$

E